

PRACTICAL
BUSINESS TREATISE.

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THE BRITISH AMERICAN COMMERCIAL ARITHMETIC,

FOR THE USE OF

SCHOOLS, COLLEGES, AND COUNTING-HOUSES,

EMBRACING

AN EXTENSIVE COURSE BOTH IN THEORY AND PRACTICE,

BY

T. A. BRYCE, M. A.,

TEACHER OF MATHEMATICS AND ENGLISH, IN THE TORONTO B. A. C. C.

AND

T. C. MUSGROVE AND H. C. WRIGHT,

PRINCIPALS AND PROPRIETORS

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PREFACE.

WE have for some time looked upon it as a kind of reproach that Canada has never produced a treatise on Arithmetic adequate to the wants of a commercial community such as this country has become. It is not enough that the school-boy should be provided with a course suited to his years; there must be supplied to him something higher as he advances in years and progress, and nears the period when he is to enter on real business life.

We have felt this keenly in our experience in conducting the BRITISH AMERICAN COMMERCIAL COLLEGES. We have hitherto been obliged to have recourse to United States' publications, which, without any disparagement to their intrinsic merit, we are forced to say, are not suited, in many important respects, to the wants of Canada. The great increase in our sphere of operations, and the grave responsibilities devolving on us thereby, have made us feel in duty bound to supply to our students a book such as they require. We have therefore undertaken the task of supplying the deficiency. As we proceeded with the work we found it necessary to extend our original programme considerably, and, therefore, also the limits of the book, so as to make it useful, not only to our own Colleges, but to the community at large.

In carrying out our plan we have endeavoured to unfold the theory of Arithmetic as a SCIENCE in as concise a manner as seemed consistent with clearness, and at the same time to show its applications as an ART. We have striven to make the business part so copious and practical as to afford the young student ample information and discipline in all the principles and usages of commercial intercourse. For the same reason, we have introduced some articles on Commercial Law, written by J. D. Edgar, Esq., Barrister-at-Law, a prominent part of our aim being to produce a work which shall be found useful, not only in the class-room and the learner's study, but also on the merchant's table and the accountant's desk. We have taken especial care not to enunciate any rule without explaining the *reason*, for, without a knowledge of the principle, the operator is a

mere calculating machine that can work but a certain round, and is almost sure to be at fault when novel cases arise. In giving those explanations we have not followed any predecessors, but have been guided entirely by our own experience in teaching. The great mass of the exercises likewise are entirely new, though we have not scrupled to make selections from some of the most approved works on the subject; but in doing so, we have confined ourselves to such questions as are to be found in almost all popular works, and which, therefore, are to be looked upon as the common property of science.

We have, as much as possible, avoided Algebraic forms and notation, as being unsuited to a large proportion of those for whom the book is intended, and to many altogether unintelligible. We have been encouraged to follow out this course from the reflection that those who understand Algebraic modes will have all the less difficulty in understanding the common Arithmetical ones. Even in the Mathematical parts, we have endeavoured to popularize the subject as much as possible.

We were compelled to follow a certain logical order in arranging the subjects treated of, but the teacher and learner will often find it necessary to depart from that order. (See Suggestions to Teachers.)

As we consider that rules and definitions should always be expressed in the smallest possible number of words consistent with perspicuity and accuracy, we have taken great pains to carry out this principle in every case.

We have appended copious exercises to each rule, especially to the most important, as Fractions, Proportion, Analysis, and Interest. Besides these, we have introduced extensive collections of mixed exercises throughout the body of the work, besides a large number at the end. The utility of such miscellaneous questions will be admitted by all, but the *reason* why they are of such importance seems strangely overlooked or misunderstood, even by writers on the subject. They are spoken of as mere *review* exercises. Their great value depends on something still more important. A class is working questions on a certain rule, and each member of the class has just heard the rule enunciated, and readily applies it. So far, one important object is attained, viz., freedom of operation. But something more is necessary. The learner must be taught to discern *what rule is to be applied* to the solution of each question proposed. The pupil, under careful teaching, may be able to understand fully every rule, and never confound any one with any other, and yet be

doubtful what rule to apply to an individual case. The miscellaneous problems, therefore, are intended, not so much as exercises on the *operations* of the different rules, as on the *mode of applying* those rules ; or, in other words, to practise the pupil in perceiving of what rule any proposed question is a particular case. To this we attach great importance, not only as regards readiness in real business, but also as a mental exercise to the young student.

We are far from supposing, much less asserting, that the work is complete, especially as the whole has been prepared in less than the short space of six months. We present it, however, to the public, in the confident expectation that it will meet, to a great extent at least, the necessities of the times. With this view, also, we have given the great mass of the examples and exercises, involving money, in dollars and cents, with, however, a number in pounds, shillings and pence, sufficient for the purpose of illustration. We have followed this course because we do not see any use in perpetuating the cumbrous system of the old currency, and we even hope to see the day when the decimal system will be universally adopted in British America. For this reason, too, in teaching Book-keeping, we uniformly employ the decimal notation.

We beg, also, respectfully to call the attention of the Government to the desirableness of taking steps to introduce the decimal system in weights and measures as well as in money. Its great simplicity must commend it to every intelligent mind. Some difficulty would, no doubt, for a time attend the change, but this would soon pass away, especially as the country is already familiarized with the notation by the use of the decimal coinage. We feel confident that a Parliamentary bill for this object, co-ordinate with the same movement now going on in Britain, would be most acceptable to the great majority of the people of Canada, if the matter were only taken up by some persons of energy and influence.

The rule for finding the Greatest Common Measure, though not new, is given in a new, and, we think, a concise and convenient form.

The rule for finding the Cube Root is a modification of that recommended by Dr. Hind, and will be found very ready and short.

On the subjects of Logarithms and Mensuration, we have only given the general principles, and a few of the most important cases. To do justice to these subjects would require separate books. In treating of Common Fractions we have placed Multiplication and

Division before Addition and Subtraction, for two reasons :—FIRST. In Common Fractions, Multiplication and Division present much less difficulty than Addition and Subtraction ; and, SECONDLY, as in Whole Numbers, Addition is the rule that regulates all others ; so in Fractions, which originate from Division, we see, in like manner, that all other operations result from Division, and, in connection with it, Multiplication. We hope this will be accounted reason sufficient for following so unwonted a course. By the ordinary routine the pupil is, in effect, called upon to apply rules and principles that he has not learned.

Several subjects, commonly treated of in works on Arithmetic, have been omitted, in order to leave space for more important matter bearing on commercial pursuits. Duodecimals, for example, have been omitted, as that mode of calculation is now virtually superseded by the use of Decimals.

Barter, too, has been passed by, as questions of that class can easily be solved by the rule of Proportion, which has been fully explained.

The subject of Analysis has been gone into at considerable length, and we expect that the new manner in which the explanations and solutions are presented, and the extensive collection of exercises appended, will contribute to make this a valuable part of the treatise.

We think that the view we have given of Decimal Fractions is the only true one, and is calculated to give to the student clear notions of the nature of the notation, and to show the great convenience and utility of Decimals. We have ignored the distinction sometimes made between *Decimals* and *Decimal Fractions*, as being “a distinction without a difference.” *Decimal* is merely a short way of writing *Decimal Fraction*. Thus : .7 is merely a convenient mode of writing $\frac{7}{10}$. These differ in form only, but otherwise are as perfectly identical as $\frac{3}{4}$ and $\frac{6}{8}$.

The contracted methods of Multiplication and Division will be found, after some practice, extremely useful and expeditious in Decimals expressed by long lines of figures.

We have, after some hesitation, introduced an article on Logarithms, and also Tables, as the logarithmic mode of computation saves, in many cases, an immense amount of time and labour, and will be found extremely useful in surveying and mechanical calculations, especially when angular and linear units have to be compared. As the book has expanded to much greater dimensions than we

anticipated, we have judged it better not to insert a table of Squares and Cubes, as we had intended. For the same reason, we found it impossible to introduce tables of Logarithmic Sines, Cosines, &c.

We have entrusted a great part of the composition of the work to the Teacher of Mathematics and English in our Toronto College, T. A. Bryce, M.A., Glasgow University, Scotland. As our own time was so much occupied in teaching, we were anxious to procure the assistance of a gentleman who was at once a sound Mathematician, an accomplished English scholar and an accurate writer, and at the same time an experienced practical teacher, believing that all these qualifications were needed for the composition of such a treatise. These requisites we found in Mr. Bryce. In the explanation of principles and framing of rules we think he has been peculiarly happy, and we take pleasure in acknowledging the valuable aid he has rendered us in the preparing of the book.

SUGGESTIONS TO TEACHERS AND STUDENTS.

WE would first refer to our remark in the Preface, that we did not expect the teacher to follow our logical arrangement, and even advised that he should not. We know by experience that the same course does not suit all students, any more than the same medical treatment suits all patients. The course requires to be varied according to age, ability and acquirements. The greatest difficulties generally present themselves at the earliest stages. What more serious difficulty, for example, has a child to encounter than the learning of the alphabet? Though this is, perhaps, the extreme case, yet others will be found to be in proportion. For beginners, therefore, we recommend the following course :

Let the elementary rules be carefully explained and illustrated by *simple examples*, and the pupil shown how to work *easy exercises*. This done, let the whole be reviewed, and exercises of a more difficult kind be proposed. The Decimal Coinage should then be taken up. In explaining this part of the subject the teacher ought to notice carefully that the operations in this case differ in no way from those already gone through, in reference to Abstract Numbers, except in the preserving of the position of the mark that separates the cents from the dollars, usually called the decimal point. The next step should be the whole subject of Denominate Numbers, and, in illustration and application, the rule of Practice. After a thorough review of all the ground now gone over, Simple Proportion may be entered upon, using only such questions as do not involve Fractions. Then, after a course of Fractions has been gone through, Proportion should be reviewed, and questions which involve Fractions proposed. After this, it will generally be found desirable to study Percentage, with all its applications, the most important of which is Interest. The order in which the rest of the course shall be taken is comparatively unimportant, as the student has now realized a capital on which he can draw for any purpose.

We would, in the strongest manner possible, impress on the minds of teachers and students the great utility of frequent reviews, and especially of constant exercise in the addition of money columns.

We have endeavoured, as far as possible, to make the exercises under each rule of progressive difficulty. We have also made it an object to give each exercise the semblance of a real question, for all persons, and especially the young, take greater interest in exercises that assume the form of reality than in such as are merely abstract; and, besides, this is a preparatory exercise to the application of the rules afterwards.

In the foregoing suggestions we have had in view the case of a child beginning from the very elements, but the judicious teacher will readily modify and apply these principles according to circumstances. We shall only briefly add the following cautions and hints:

At every stage the greatest care should be taken that the learner understands the meaning of each rule, and the terms and conditions of each question, before he attempts to solve it.

The teacher should never attempt to explain two things at the same time, and he should be satisfied that the first idea is fully grasped and stored before he approaches the second.

Unruffled temper and untiring patience are essential on the part of the teacher, in order that he may be able to discover the source of every difficulty that presents itself to the pupil's mind, and remove it by careful, and, when necessary, repeated explanation.

The teacher should not always be talking or working on the black-board; he should require the pupils to speak a good deal in answer to questions, and also to work much on their slates.

The pupils should never be made to work or listen till they are fatigued, or till their attention flags.

Finally, we would suggest to every teacher to keep constantly before his mind both of the two great works he has to accomplish—*First*, the development of the mental powers of his pupil; and, *secondly*, imparting to him such knowledge as he will require to use when he enters upon life, either as a professional man or a merchant. Some seem to consider these two objects incompatible, as if taking up time in mental training left insufficient time for the imparting of actual knowledge. We, however, consider this a palpable error, for the more the mind is cultivated, the more readily and rapidly will it take in any species of knowledge, and the more surely too will it retain what it has mastered. Mental culture is at once the foundation and

the means; the other is the superstructure raised on that foundation, and by that means; or it may be compared to a great capital judiciously embarked in trade, and often turned, and therefore yielding good profits. It frequently happens, however, from the peculiar circumstances of individuals and families, and even communities, that young men require to be hurried into business, so as to be able to support themselves; but even in such cases the desired object will be much more speedily and securely attained by such a course than by what is usually and not inappropriately called "*cramming*." We have striven to give the character here recommended to this book, especially in the explanatory portions.

We solicit the attention of the student, as well as the teacher, to these suggestions, and in particular of the self-taught student, whose wants also we have kept in view throughout.

MUSGROVE & WRIGHT,

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ARITHMETIC.

ARTICLE 1.—**ARITHMETIC** treats of numbers in theory and practice. In relation to theory it is a science, and in relation to practice it is an art.

All computations are made by fixing on a certain quantity, called a unit or one, and repeating that unit any required number of times. Various units are selected, according to the nature and extent of the quantity or space to be measured. For example, in measuring length or distance, if the extent is small, such as the length of a pane of glass, we select a small unit, called an inch, and repeating that unit any required number of times, say twelve, we say the pane is twelve inches long,—if a more extended space is to be measured, it is convenient to adopt a larger unit,—thus, if we wish to measure the length of a desk, we should probably select a unit called a foot, equal to twelve of the preceding units,—if we wish to measure the length of a room, we should select a still larger unit, called a yard, equal to three of the last,—again, if we wish to measure the length of a field, we should adopt a unit equal to five and a-half of the last, and called a perch or rod,—if we wish to note the distance between Toronto and Montreal, we have recourse to a still larger unit, called a mile, and equal to three hundred and twenty of the last,—finally, when astronomers are estimating the distance of any planet, say the earth, from the sun, they generally use a unit equal to a million of the last-mentioned, and they say that the earth is ninety-five millions of miles from the sun, but they simply note the distance as ninety-five; and in the same manner they mark the distance of Venus as sixty-nine, meaning in both cases that each unit is a million of miles. A similar illustration may be applied to every kind of measurement.

The symbols or characters now almost universally used to denote quantity or magnitude, are the Arabic figures, or digits 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. These, by various combinations, can be made to represent any quantity or magnitude whatsoever. The first nine are called significant figures, because they always denote some real quantity,—the last, called nought (often improperly ought), or cipher, or zero, simply indicates the absence of any significant figure.

NUMERATION.

2.—NUMERATION is the mode of marking and reading off any line of figures that has been written down, so as to ascertain its value readily and express that value in words. For this purpose every such line is divided into sets or lots of three figures each, counting from right to left, and each set is called a period,—thus, 88888888 forms three periods by marking the figures in threes from right to left by a character of the same form as the comma in composition,—thus, 888,888,888. The first period is called the period of units, the second the period of thousands, the third the period of millions, and so on,—billions, trillions, quadrillions, &c., &c., to any required extent, which seldom exceeds millions.

The first figure of each period denotes units* of that period, the second tens, and the third hundreds of that period. Thus, in the example given above, the first figure denotes eight units in the period of units, or eight ones, or, as it is usually read, simply eight; so, also, the fourth denotes eight units in the period of thousands; or eight times one thousand, or eight thousands; the seventh figure again denotes eight units in the period of millions, or eight times one million, or eight millions; again, the second, fifth, and eighth figures denote tens in the period of units, thousands and millions, respectively; lastly, the third, sixth and ninth figures denote hundreds in the periods of units, thousands and millions, respectively. Such a line, then, as 888,888,888 is read eight hundred and eighty-eight millions, eight hundred and eighty-eight thousands, eight hundred and eighty-eight.

Every period but the last must have three figures. Thus, in the line 43,279,865 the first and second periods have three digits each, units, tens and hundreds, but the third has only two, units and tens, but no hundreds, and therefore is read forty-three millions, two hundred and seventy-nine thousands, eight hundred and sixty-five.

RULE FOR NUMERATION.

Beginning at the right, count off periods of three digits each till not more than three are left; then read off each period from left to

* It is somewhat awkward that the term units is used for two purposes, viz. : as the name of the first period and also as the name of the first figure of each period. Though we cannot well change what usage has so long established, yet the teacher may obviate the difficulty by varying the expression occasionally, if not habitually, saying, E. G., units in the *unity* period, or the *place* of units in the units period.

right by naming as many hundreds, tens and units as each contains, and adding at the end of each period its proper name. The name of the unity period is usually omitted. When a cipher occurs no mention is made of that place in the period, but the cipher is counted as a digit; thus, in the line 360,708,091 each cipher is counted a digit, but the reading is three hundred and sixty millions, seven hundred and eight thousands and ninety-one.

EXERCISES

Divide into periods and read the following lines :

- | | | |
|------------------|---------------|---------------------|
| 1.—586729341 | 2.—976852734 | 3.—2178427385 |
| 4.—92879357485 | 5.—4638709120 | 6.—1111111111111111 |
| 7.—2822828228288 | 8.—10904870 | 9.—1010101010101 |

NOTATION.

3 —NOTATION is the mode of expressing any quantity or magnitude by the combination of conventional symbols or characters. Thus, by the Roman notation, the letter I. stands for *one*, II. for *two*, X. for *ten*, &c.; thus, XII. stands for *one* ten and *two* units. By the Arabic notation, any digit standing alone, as 5 in the margin, denotes simply five units, but if another digit (5) be placed to the right of it, then the new 5 denotes units and the other

5	5 becomes tens, so that appending a second digit makes
55	the first one ten times its original value; again, if
555	another digit (5) be subjoined, it takes the place of units,
5555	and the 5 next to it becomes tens and the third becomes
	hundreds, so that each of them has ten times the value in
	the third line that it had in the second; so also, if
	another digit (5) be added, each of the three to the left
	of it will have ten times the value that it had in the third line, and
	so on. Universally, every digit placed to the right makes every one
	to the left ten times its previous value.

The use of the tenth of the Arabic characters, the cipher (0) will be made more clear by the rule of notation than by numeration. If I am counting my cash and find that I have *eight* ten-dollar bills, and *eight* one-dollar bills, it is plain from Art. 2 that if I write 8 alone this must represent the one-dollar bills, and to represent the ten-dollar bills along with the one-dollar bills I must

write 88, for the figure to the left being ten times that to the right, will stand correctly for the ten-dollar bills, just as that to the right, being in the units' place, stands for the one-dollar bills.—But if I have no one-dollar bills and write 8, this would stand for only one-dollar bills, and hence the necessity for introducing a non-significant character and writing 80, for though the cipher represents no quantity, yet by being put in the place of units it throws the 8 to be in the place of tens, and therefore the 8 now stands fitly for the *eight* ten-dollar bills, and is written \$80.—Again, if I find that I have *two* one-hundred-dollar bills, *six* one-dollar bills, but *no* ten-dollar bills, and I write only 26, this would be plainly incorrect, for the 2 would stand for ten-dollar bills only, but by inserting a zero mark between the figures I throw the 2 into the place of hundreds, and \$206 represents correctly that I have *two* one-hundred dollar bills, and *six* one-dollar bills, but *no* ten-dollar bills. The superiority of this simple system over the cumbersome Roman one will be manifest from its simplicity and brevity by writing *eighty-eight* according to both systems—thus: LXXXVIII. and 88.

RULE FOR NOTATION.

Write the significant figures of the first period named in their proper places, filling up any places *not named* with ciphers, just as if you were writing the units period with nothing to follow; then, to indicate that something is to follow, place a comma to the right, and do the same for every period down to units, inclusive. For example, teacher says: "Write down one hundred and six millions;" pupil writes 106 and pauses; teacher adds, "ninety thousand;" pupil fills up thus: 106,090, and pauses; teacher concludes: "and eighteen;" pupil completes 106,090,018. If the teacher should say sixteen millions and the pupil write 016, the cipher would be manifestly superfluous, as it has no effect on figures placed to the right of it, but only on those placed to the left.

EXERCISES.

Write in figures and read the following quantities:

1. Ten millions, seven thousand and eleven.
2. Ninety billions, seven thousand and ten.
3. Eighteen millions, sixty thousand and nine hundred.
4. Forty thousand and nine hundred.

5. Eighty-seven millions and one.
6. Ninety thousand, seven hundred and eight.
7. Eleven millions, eight hundred thousand and twenty-four.
8. Six hundred and seven thousand and ninety-seven.
9. Eight hundred and seventy billions, sixty thousand and eighteen.
10. Eleven billions, eleven millions, eleven thousand and eleven.

A X I O M S.

4.—AXIOMS used in the sequel :

- I. Things that are equal to the same thing, or to equals, are equal.
- II. If equals be added to equals, the wholes are equal.

Corollary.—If equals be multiplied by the same, the products are equal.

- III. If equals be taken from equals, the remainders are equal.

Cor.—If equals be divided by the same, the quotients are equal.

- IV. The whole is greater than its part

Cor.—The whole is equal to all its parts taken together.

- V. Magnitudes which coincide, or occupy the same or equal spaces, are equal.

N. B.—This axiom is modified by, but still is the principle of, all business transactions, purchases, sales, barter, exchanges, &c., &c., where the articles traded in are not *equals*, but *equivalents*.

A D D I T I O N.

5.—ADDITION is the mode of combining two or more numbers into one. The operation depends on axiom II. The result is called the sum. Thus: $\$8 + \$9 + \$6 = \$23$. The sign plus (+) indicates addition.

To illustrate the operation, let it be required to find the sum of the five numbers of dollars noted in the margin.

\$287654	of the five numbers of dollars noted in the margin.
758287	First, the numbers are placed so that those of the
612873	same <i>name</i> are in vertical columns, <i>i. e.</i> , units under
494768	units, tens under tens, &c. Next, we find that the
836195	sum of the units' column is (Ax. IV., <i>Cor.</i>) 27, <i>i. e.</i> ,
	<i>two</i> tens and <i>seven</i> units. Next, we find that the
	sum of the tens' column is 35, but, as it is the <i>tens'</i>

27
350
2400
27000
260000
2700000

\$2989777

viz.: \$2,969,777. In practice the operation is much abbreviated in the following manner:—When the units' column has been added, and we find the sum to be 27, *i. e.*, 7 units and 2 tens, we write down the 7 units under the units' column, and add up (Art. 3) the 2 tens with the tens' column, and we find the sum is 35 *tens*, *i. e.*, 5 *tens* and 3 *hundreds*, and we place the 5 *tens* under the *tens*' column, and add up the 3 *hundreds* with the *hundreds*' column, and so on.

\$287654
758287
612873
494768
836195

\$2989777

The transferring of the tens, obtained by adding the *units*' column to the *tens*' column, and the *hundreds* obtained by adding the *tens*' column to the *hundreds*' column, &c., &c., is called carrying. In all such operations the learner should carefully bear in mind the principle explained in Art. 3., that every figure to the left is ten times the value that it would have if one place farther to the right.*

EXERCISES.

Find the sums of the following quantities :

(1)	(2)	(3)	(4)
895763		99876	
49176	987654231	63879	89765324
283527	123456789	54387	42356798
659845	908760504	789	56798423
7984	890705063	137568	23567989
31659	759086391	278652	79842356
968438	670998767	85945	65324897
-----	-----	-----	-----
2896392	4340661745	721096	357655787
-----	-----	-----	-----

* We would strongly recommend every one who wishes to become an expert accountant, to avoid the common practice of drawling up a column of figures in the manner that may be sufficiently illustrated by the adding of the units' column of the above example. Never say 5 and 8 are 13 ; 13 and 3 are 16 : 16 and 7 are 23 ; 23 and 4 are 27 ; but run up your column thus : 5, 13, 16, 23,

ADDITION.

19

(5)	(6)	(7)	(8)
			738
			659
			471
			897
78563			658
47986	12345		856
5798	67890	918273	789
19843	98765	651928	978
56479	43219	374859	654
28795	87654	263748	999
897	32169	597485	888
1984	78912	986879	777
68195	65439	98765	666
3879	98765	9876	555
698	43288	987	897
5879	77877	456879	978
17985	98989	345678	
336981	805312	4705357	12460
(9)	(10)	(11)	(12)
189			1298
976	98	47	764
85	89	96	5837
73	76	83	6495
338	67	59	789
793	281	74	638
49	592	82	546
75	678	97	98
218	58	68	475
365	67	75	394
113	98	49	89
279	149	76	157
67	67	54	638
76	54	78	594
84	72	69	789
1379	298	37	114
5159	2744	1044	19715

27, for that is the mode to secure both rapidity and accuracy. The same remark will apply equally to multiplication, and therefore to every arithmetical operation. To enforce this advice let us add a simple example to caution the student before he approaches multiplication. In multiplying 497 by 6, avoid the tediousness of saying 6 times 7 is 42—2 and carry 4—6 times 9 is 54, and 4 is 58—8 and carry 5—6 times 4 is 24, and 5 is 29 ; but practice the eye, aided by the memory, to take in at a glance 6 times 7 is 42, &c.—The quick operator uses the eye, and not the tongue.

There is no method of proving the correctness of any addition with positive certainty, but a very convenient mode of checking is to add each column both upwards and downwards. Another mode is, to add by parts and take the sum of those. This is a very secure method in the case of long columns, but not so ready as the former. If the same result is found by each method, the sum may be accounted correct.

SUBTRACTION.

6. SUBTRACTION is the converse of addition, *i. e.*, it is the mode of finding the difference between two numbers, or, in other words, the excess of one number above another. The number to be subtracted is called the subtrahend, and that from which it is to be taken the minuend, and the result is called the remainder, difference or excess. The sign used for subtraction is a line (—) called *minus*, or less. Let it be required to find the difference between \$578643957 and \$235412712. Having placed

$$\begin{array}{r} 578643957 \\ 235412712 \\ \hline 343231245 \end{array}$$

them in vertical columns, as in addition, it is obvious that 2 units taken from 7 units will leave 5 units, and that 1 ten taken from 5 tens will leave 4 tens, and so on. But if it is required to find the excess of \$513674208 above \$347895319, we find that each figure of the subtrahend, except the last, count-

ing from right to left, is greater than the corresponding one of the minuend, and therefore, to find the correct difference, we have recourse to a simple artifice, which is deduced from the principle of

$$\begin{array}{r} 333,333 \\ 177,777 \\ \hline 155,556 \end{array}$$

the notation, and may be illustrated in the following manner:—Taking the question in the margin, we are first required to subtract 7 units from 3 units. Now, though the algebraic notation furnishes the means of noting the difference directly, the ordinary arithmetical form does not, but still it furnishes the means of doing it indi-

rectly. By Art. 3 each figure to the left is ten times the value of the next to its right, therefore we take one of the 3 tens and call

$$\begin{array}{cccccc} 2(12)(12)(12)(12)(13) \\ 1\ 7\ \ 7\ \ 7\ \ 7\ \ 7 \\ \hline 1\ 5\ \ 5\ \ 5\ \ 5\ \ 6 \end{array}$$

it ten units, and add it to the 3 units, and thus we have 13 units, which let us enclose in a parenthesis or bracket, thus: (13), to indicate that the whole quantity, 13, is to occupy the units' place; when *one* of the *three* tens has been thus transferred to the units'

place, only *two* tens remain in the place of tens, and we are now required to take 7 tens from 2 tens; to do this we have recourse to the same artifice, by calling one of the hundreds *tens*, which gives 10 tens and 2 tens, and so on to the end, the last 3 necessarily

becoming 2. We can now subtract 7 from 13, &c., &c. This mode of resolution depends on the corollary to Axiom IV. The parts into which the whole is virtually resolved are shown in the margin. This artifice is popularly called borrowing. In practice the resolution can be effected mentally as we proceed, and as each figure from which we *borrow* is diminished by unity, it is usual to count it as it stands, and to compensate for this to increase the one below it by one, for, as in the example, 7 from 12 is the same as

200000
120000
12000
1200
120
13

333333

8 from 13, and 2 from 3 is the same as 1 from 2. We are now

prepared to answer the proposed question, as annexed, and we say 9 from 8, we cannot, and there are no tens to borrow from, we therefore take one of the hundreds and call it 10 *tens*, and one of the tens and call it 10 units, which with 8 units makes 18 units, and we take 9 from 18 and 9 remain. We have now only 9 tens left,

but we reckon them as ten, and to compensate for the surplus ten, we reckon the 1 below as 2, and say 2 from 10 and 8 remain. We proceed thus to the end, and find the whole remainder to be \$165778889.

\$513674208
\$347895319

\$165778889

EXERCISES. REMAINDERS.

- 1.—From 847639021 take 476584359=371054662.
- 2. “ 1010305061 “ 670685093=339619968.
- 3. “ 59638743 “ 18796854= 40841889.
- 4. “ 7813257 “ 3745679= 4067578.
- 5. “ 111111111 “ 98657293= 12453818.

In Subtraction, as in Addition, we have no method of proof that arrives at positive certainty, but either of the two following methods may be generally relied upon.

- 1.—Add the remainder and subtrahend, and if the sum is equal to the minuend, it is to be presumed that the work is correct.
- 2.—Subtract the remainder from the minuend, and if this second remainder is the same as the subtrahend, the work may be accounted correct.

MULTIPLICATION.

7.—MULTIPLICATION may be simply defined by saying that it is a short method of performing addition, when all the quantities to be added are the same or equal. Thus: $6+6+6+6+6+6+6+6$, means that eight sixes are to be added together, or that six is to be repeated as often as there are units in eight, and we say that 8 times 6 is 48, and write it thus: $8 \times 6 = 48$. So also $8+8+8+8+8$ gives 48. So that $6.8 = 8.6 = 48$, and thus we can construct a multiplication table. The number to be repeated is called the multiplicand, and the one that shows how often it is to be repeated is called the multiplier, and the result is called the product, or what is produced, and hence the multiplier and multiplicand are also called the factors or makers, or producers, and the operation may be called *finding* a product when the factors are given. Hence also the mode of carrying is the same in multiplication as in addition.

MULTIPLICATION TABLE.

Twice	3 times	4 times	5 times	6 times	7 times
1 is 2	1 is 3	1 is 4	1 is 5	1 is 6	1 is 7
2 — 4	2 — 6	2 — 8	2 — 10	2 — 12	2 — 14
3 — 6	3 — 9	3 — 12	3 — 15	3 — 18	3 — 21
4 — 8	4 — 12	4 — 16	4 — 20	4 — 24	4 — 28
5 — 10	5 — 15	5 — 20	5 — 25	5 — 30	5 — 35
6 — 12	6 — 18	6 — 24	6 — 30	6 — 36	6 — 42
7 — 14	7 — 21	7 — 28	7 — 35	7 — 42	7 — 49
8 — 16	8 — 24	8 — 32	8 — 40	8 — 48	8 — 56
9 — 18	9 — 27	9 — 36	9 — 45	9 — 54	9 — 63
10 — 20	10 — 30	10 — 40	10 — 50	10 — 60	10 — 70
11 — 22	11 — 33	11 — 44	11 — 55	11 — 66	11 — 77
12 — 24	12 — 36	12 — 48	12 — 60	12 — 72	12 — 84
8 times	9 times	10 times	11 times	12 times	
1 is 8	1 is 9	1 is 10	1 is 11	1 is 12	
2 — 16	2 — 18	2 — 20	2 — 22	2 — 24	
3 — 24	3 — 27	3 — 30	3 — 33	3 — 36	
4 — 32	4 — 36	4 — 40	4 — 44	4 — 48	
5 — 40	5 — 45	5 — 50	5 — 55	5 — 60	
6 — 48	6 — 54	6 — 60	6 — 66	6 — 72	
7 — 56	7 — 63	7 — 70	7 — 77	7 — 84	
8 — 64	8 — 72	8 — 80	8 — 88	8 — 96	
9 — 72	9 — 81	9 — 90	9 — 99	9 — 108	
10 — 80	10 — 90	10 — 100	10 — 110	10 — 120	
11 — 88	11 — 99	11 — 110	11 — 121	11 — 132	
12 — 96	12 — 108	12 — 120	12 — 132	12 — 144	

Regarding the following part of this table, see suggestions to Teachers :

13 times	14 times	15 times	16 times	17 times	18 times	19 times
2 is 26	2 is 28	2 is 30	2 is 32	2 is 34	2 is 36	2 is 38
3 -- 39	3 - 42	3 — 45	3 —, 48	3 — 51	3 - 54	3 — 57
4 — 52	4 — 56	4 — 60	4 — 64	4 — 68	4 — 72	4 — 76
5 — 65	5 — 70	5 — 75	5 — 80	5 — 85	5 — 90	5 — 95
6 — 78	6 — 84	6 — 90	6 — 96	6 — 102	6 — 108	6 — 114
7 — 91	7 — 98	7 — 105	7 — 112	7 — 119	7 — 126	7 — 133
8 — 104	8 — 112	8 — 120	8 — 128	8 — 136	8 — 144	8 — 152
9 — 117	9 — 126	9 — 135	9 — 144	9 — 153	9 — 162	9 — 171

We have in the above table corrected the gross grammatical blunder so common of saying eight times two ARE sixteen.

When more than two factors are given, the operation is called continued multiplication, as $6 \times 3 \times 2 \times 5 = 180$.

When the factors consist of more figures than one, the most convenient mode of operation is that shown by the annexed example, where the multiplicand is first repeated 8 times, then 60 times, or which is the same thing 6 times when the first figure of the second line is placed under the second figure of the first line, *i. e.* (art. 2,)

in the place of tens, and then the partial products are added, which (Ax. IV. Cor.) gives the full product.
Hence we deduce the

345186
268

2761488

2070916

690372

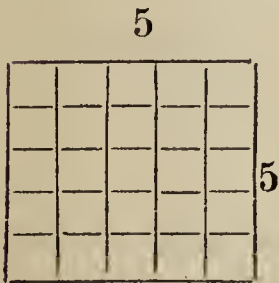
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RULE FOR MULTIPLICATION.

Place the multiplier under the multiplicand, units under units, tens under tens, &c., &c.,—commencing at the right, multiply each figure of the multiplicand by each figure of the multiplier in succession, placing the results in parallel lines, and units, tens, &c., in vertical columns,—add all the lines, and the sum of all the partial products will (Ax. IV. Cor.) be the whole product required.

As far as the learner has committed a multiplication table to memory, say to 12 times 12, the work can be done by a single operation.

When any number is multiplied by itself, the product is called the square or second power of that number, and the product of three equal factors is called a cube or third power, the product of four equal factors the fourth power, &c., &c. The terms *square* and *cube* are derived from superficial and solid measurement. The annexed square has each of its sides divided into 5 equal parts, and it will be found on inspection that the whole figure contains



25 ($=5 \times 5$) small squares, all equal in area, and having all their sides equal.—Hence because 5×5 represents the whole area, 25 is called the square of 5, or the second power of 5, because it is the product of the two equal factors 5 and 5. A cube is a solid body, the length, breadth and thickness of which are all equal, and hence, if these dimensions be each represented by 5, the whole solid will be represented by $5 \times 5 \times 5 = 125$, which is therefore called the cube or third power of 5. The terms square and cube are often used without any reference to superficial and solid measure. For example, in lineal measure an expression for distance in a straight line is often called the square and cube of a certain number, thus: 81 is called the square, and 729 the cube of 9, although these are only used to show that the distance is not 9 in either case, but in the one 9×9 , and in the other $9 \times 9 \times 9$. In such cases the terms second and third power are therefore to be preferred, and since no solid can have more than three dimensions, we have no term corresponding to square and cube for the product of four or more equal factors, and therefore we are obliged to use the words fourth power, fifth power, &c., &c.

CONTRACTIONS AND PROOF.

There are many cases in which multiplication may be performed by contracted methods, but the utility of these, for the purposes of accuracy, is, at least, doubtful. The most secure method in the great majority of cases, is to follow the general rule. Multiplication by 10, 100, &c., is effected at once by adding a cipher for ten, two for 100, &c., &c. The following is, next to the above, the most safe and useful contraction that can be adopted. It is exhibited in the subjoined examples, but purposely without explanation, as an exercise for the learner's reflection :

ORDINARY METHOD.	CONTRACTED METHOD.	ORDINARY METHOD.	CONTRACTED METHOD.
35697×17 17	35697×17 249879	35697×71 71	35697×71 249879
249879 35697	606849	35697 249879	2534487
606849		2534487	

The only practically useful proof of the correctness of the product, is the one subjoined, but even it, though it seldom fails, does not secure positive certainty :

Add together all the figures of each factor separately, rejecting 9 from all sums that contain it, and multiply the remainders together, rejecting every 9 from the result,—add the figures of the product in the same manner, and if the two remainders are equal, the work *may* be accounted as correct, but if they are not equal, the work *must* be wrong. The reason of this proof depends on the property of the number 9, that if any number be divided by 9, the remainder will be the same as if the sum of its digits were divided by 9.—Thus: $7422153 \div 9 = 824683 + 6$, and the sum of the digits is 24, and $24 \div 9 = 2 + 6$, *i. e.* 9 is contained in 24 twice with a remainder 6. Every 9 is rejected because 9 is contained in itself once evenly, and therefore cannot affect the remainder. Let it now be required to multiply 122 by 24. Now, $122 = 9 \times 13 + 5$, and $24 = 9 \times 2 + 6$, and if we multiply together the two factors thus resolved, we get $9 \times 13 \times 9 \times 2 + 9 \times 2 \times 5 + 9 \times 13 \times 6 + 6 \times 5$, and since 9 is a factor of all but the last, the last only will give a remainder when divided by 9, and therefore the whole product will give the same remainder when divided by 9, as $6 \times 5 \div 9$, which gives the remainder 3, for $6 \times 5 = 30$ and $30 \div 9$ gives 3 with a remainder 3. To test this by trial, we find $122 \div 9 = 13$ with a remainder 5, and $24 \div 9 = 2$ with a remainder 6, and the product of these remainders is $6 \times 5 = 30$, and $30 \div 9 = 3$ with a remainder 3. Again, $122 \times 24 = 2928$, and $2928 \div 9 = 325$ with a remainder 3, as in the case of the factors.

EXERCISES.

- | | |
|------------------------------------|---------------------------------------|
| 1. $7896 \times 5 = 39480$. | 8. $719864 \times 43 = 30954152$. |
| 2. $581967 \times 8 = 4655736$. | 9. $375967 \times 64 = 24061888$. |
| 3. $938746 \times 4 = 3754984$. | 10. $27859 \times 29 = 807911$. |
| 4. $193784 \times 7 = 1356488$. | 11. $679854 \times 83 = 56427882$. |
| 5. $391876 \times 9 = 3526884$. | 12. $759684 \times 187 = 142060908$. |
| 6. $987456 \times 6 = 5924736$. | 13. $5372 \times 1634 = 8777848$. |
| 7. $496783 \times 52 = 25832716$. | |

14. Find the second power of 389 ? Ans. 151321.

15. Find the third power of 538 ? Ans. 155720872.

16. Find the fourth power of 144 ? Ans. 429981696.

17. Find the cube of 99 ? Ans. 970299.

18. 5796 seamen have to be paid 169 dollars each ; what is the amount of the treasury order for that purpose ? Ans. \$979,524.

19. A block of buildings is 87 feet long; 38 feet deep, and 29 feet high ; how many cubic yards does it contain ? Ans. $3550\frac{8}{9}$ cubic yards:

20. If 29 oil wells yield 19 gallons an hour each ; how much will they all yield in a year ? Ans. 201115 gals.

21. If the rate on each of 1597 houses be \$19 ; what is the whole assessment ? Ans. \$30343.

22. If 1297 persons have paid up 9 shares each in a railway company, and each share is \$15 ; what is the working capital of the company ? Ans. \$172095

DIVISION.

8.—DIVISION is the converse operation to multiplication. It is the mode of finding a required factor when a product and another factor are given. It bears the same relation to subtraction that multiplication does to addition, as will be seen below. By Ax. IV. Cor. we may resolve any complex quantity into its component parts ; so division is resolving a certain quantity called the dividend into the number of parts indicated by another quantity called the divisor, (divider,) and the result is called the quotient (how often.) Let

8	240000	30000
	32000	4000
	7200	900
	640	80
	16	2
8	279856	34982

it be required to find how often 8 is contained in 279,856. We can resolve 279,856 as in the margin ; then dividing the lines separately by 8, we obtain the partial quotients, the sum of which is the whole quotient. But this resolution may be done mentally as we proceed. We first see that 8 is not contained in 2, therefore we take 27, and find that 8 is contained in it 3 times, with a remainder 3 ; next combining this 3 with the next figure 9, we get 39, in which 8 is contained 4 times, with a remainder 7 ; combining this 7 with the next figure 8, we have 78, in which 8 is contained 9 times, with a remainder 6 ; combining this with the 5 following, we obtain 65, and 8 is contained in it 8 times, with a remainder 1, which combined with the 6 makes 16, and 8 is contained twice in 16. The correctness of the result may be tested by multiplying the quotient by the divisor. When the divisor consists of more than one figure, the learner must have recourse to a trial quotient, but after some practice he will have little difficulty in finding each figure by inspection.

Let it be required to find how often 298 is contained in 431766.—The numbers being arranged in the convenient order indicated in the margin, we mark off to the right of the dividend blank spaces for the trial and true quotients. We readily see that 2 is contained *twice* in 4, but cannot so easily see whether the whole divisor 298 is contained twice in the same number of figures of the dividend, (viz. 431,) we therefore make trial, and place the 2 in the trial quotient, and multiply the divisor by 2 to find how much we shall have to subtract from 431. We find $298 \times 2 = 596$, larger than 431, and therefore we reject 2 and try 1. Now $298 \times 1 = 298$, less than 431, so we subtract and find a remainder of 133, and as this proves correct,

298)431766(2.1.5.4.5.4.9.8 trials.
298 1448 true quotient.

1337
1192

1456
1192

2646
2384

262

298

we place the 1 obtained in the true quotient. We find our next partial dividend by writing 7, the next figure of the dividend after the remainder 133. Our experience of the first case suggests to us that though 2 is contained 6 times in 13, yet on multiplying something will have to be carried from the 98 which we expect will make the result too large, and therefore we at once try 5, but we find that $298 \times 5 = 1490$, which is

larger than 1337, and so we try 4, and find $298 \times 4 = 1192$, which being less than 1337, we subtract and find a remainder of 145; and having placed the 4 in the true quotient, we bring down the next figure of the dividend, giving a partial dividend 1456. By inspection, as before, we see that 6 would be too large, owing to the carrying from 98, we try 5 and find $298 \times 5 = 1490$, which is larger than 1456; we try 4, and find $298 \times 4 = 1192$, which is less than 1456, so we subtract and find a remainder of 264. Having placed this 4 after the other 4 in the true quotient, we bring down 6, the last figure of the dividend, we try 9, and find $298 \times 9 = 2682$, which is greater than our last partial dividend, 2646; we try 8, and find $298 \times 8 = 2384$, and this being less than 2646, we subtract it from

that number, and find a final remainder of 262, and close the question by entering 8 in the true quotient. The mode adopted to indicate that the remainder 262 still remains to be divided, which cannot be actually done, as it is less than the divisor, is to write the 298 below

298	298000	=1000
	119200	= 400
	11920	= 40
	2384	= 8
Remainder	262	
		1448
Dividend	431766	

the 262, and draw a line between them, thus $\frac{262}{298}$, as also is seen in the margin. The resolution into partial dividends is also shown in the margin, where it will be seen that the partial dividends, including the remainder, make up the whole original dividend. So also the partial quotients are exhibited, making up the whole true quotient.

ent. That the trial quotient is not a single number, like the true quotient, but merely a succession of detached numbers, used as separate trials, is indicated by placing a full point between each pair. When we have multiplied the divisor by any figure in the trial quotient, and subtracted the product from the partial dividend, should the remainder be greater than the divisor, we perceive that the trial figure is too small, and we must try a larger.

From these illustrations we can deduce a

R U L E F O R D I V I S I O N .

(1.) Place the given numbers in the same horizontal line, putting the divisor to the left of the dividend, with a vertical line between them, draw another vertical line to the right of the dividend, and enter the quotient, figure by figure as obtained, to the right of that line. (2.) Find by the principles of multiplication, how often the divisor is contained in the same number of figures of the dividend; place the number thus obtained in the quotient, and multiply the divisor by it, and subtract the product from the corresponding partial dividend. (3.) To the remainder annex the next figure of the dividend, and proceed as before, and so on till all the figures of the dividend are exhausted. (4.) Should there be a remainder, write it and the divisor after the quotient, thus : $\frac{\text{remainder,}}{\text{divisor,}}$

The divisor is often written to the right of the dividend, and the quotient written below it, a horizontal line separating the two.

EXAMPLE OF FORM 1

$$\begin{array}{r}
 476 \overline{)8593504(18053\frac{276}{476}.} \\
 \underline{476} \\
 3833 \\
 \underline{3808} \\
 2550 \\
 \underline{2380} \\
 1704 \\
 \underline{1428} \\
 276
 \end{array}$$

EXAMPLE OF FORM 2.

$$\begin{array}{r|l}
 1860904 & 87 \\
 \hline
 174 & \underline{21389\frac{61}{87}} \\
 \hline
 120 & \\
 87 & \\
 \hline
 339 & \\
 261 & \\
 \hline
 780 & \\
 696 & \\
 \hline
 844 & \\
 783 & \\
 \hline
 61 &
 \end{array}$$

EXERCISES.

1. $1554768 \div 216 = 7198$.
2. $31884470 \div 779 = 40930$.
3. $57380625 \div 7575 = 7575$.
4. $12810098 \div 732 = 17500\frac{98}{732}$.
5. $9313702859 \div 4687319 = 1987\frac{9}{4687319}$.
6. $449148410476 \div 73885246 = 6079\frac{32}{73885246}$.
7. $109588282929 \div 1386 = 7902468\frac{279}{1386}$.
8. $35676210832 \div 79094451 = 764095\frac{1673987}{79094451}$.
9. $536818834 \div 907 = 591862$.
10. $170064915561 \div 759 = 2240644479$.
11. $554270297961 \div 7584 = 73084163\frac{5769}{7584}$.
12. $60435674634529 \div 764095 = 79094451\frac{97684}{764095}$.
13. How many bags, each containing 87 pounds, will 24,853,464 pounds of flour fill? Ans. 285,672.
14. 857 houses pay annually a tax of \$41136; what is the average on each per quarter? Ans. \$12.
15. \$9297175 of prize money are to be divided among 97,865 sailors; what is the share of each? Ans. \$95.
16. 120,815,231 pounds of cotton are made up in 233,879 bales; how many pounds in each bale? Ans 89.

DIVISION.

1. $49687532 \div 2 = 24843766$.
2. $57986327 \div 3 = 19328775\frac{2}{3}$.

3. $87965328 \div 4 = 21991332$.
4. $7963821 \div 5 = 1592764\frac{1}{5}$.
5. $6875324 \div 6 = 1145887\frac{1}{3}$.
6. $3987654 \div 7 = 569664\frac{6}{7}$.
7. $19876532 \div 8 = 2484566\frac{1}{2}$.
8. $2976854 \div 9 = 330761\frac{5}{9}$.
9. $4967532 \div 10 = 496753\frac{1}{5}$.
10. $46879352 \div 11 = 4261759\frac{3}{11}$.
11. $18765314 \div 12 = 1563777\frac{1}{6}$.
12. $78654246 \div 18 = 4369680\frac{1}{3}$.
13. $75088 \div 52 = 1444$.
14. $1674918 \div 189 = 8862$.
15. $31884470 \div 779 = 40930$.
16. $57380628 \div 7575 = 7575\frac{3}{7575}$.
17. $554270292198 \div 7584 = 73084163\frac{1}{1266}$.
18. $88789980979 \div 9584 = 9264397\frac{131}{9584}$.
19. $102030429729 \div 123456 = 826452\frac{95073}{123456}$.
20. $267817946000 \div 36500 = 10077204$.
21. 497 men fell 163798 trees; how many does each fell on an average? Ans. 329.
22. If 148 houses pay a tax of \$7844; what is the rate on each on an average? Ans. \$53.
23. If \$415143630 are levied from 4455 townships; what is the portion of each on an average? Ans. \$93186.
24. How many lots of 6754 each are contained in 396809151372? Ans. 58763718.
25. What quotient will be obtained by dividing 961504803 twice by 987? Ans. 987.

9.—TABLES of MONEY, WEIGHTS & MEASURES.

BRITISH OR STERLING MONEY.		DECIMAL COINAGE.	
4 farthings, or 2 half pennies, are.....	1 penny (d.)	10 mills (M) are.....	1 cent (ct.)
12 pence.....	1 shilling (s.)	10 cents.....	1 dime (d.)
20 shillings.....	1 pound (£)	10 dimes, or 100 cents...	1 dollar (\$)

AVOIRDUPOIS WEIGHT.

TABLE.			
16 drams make.....	1 ounce,	marked	oz.
16 ounces.....	1 pound,	"	lb.
25 pounds.....	1 quarter,	"	qr.
4 quarters.....	1 hundredweight,	"	cwt.
20 cwt.....	1 ton,	"	t.

NOTE.—This weight is used in weighing heavy articles, as meat, groceries, vegetables, grain, etc.

TROY WEIGHT.

TABLE.

24 grains (grs.) make.....	1 pennyweight, marked dwt.
20 pennyweights	1 ounce, " oz.
12 ounces.....	1 pound, " lb.

NOTE.—Troy weight is used in weighing the precious metals and stones.

APOTHECARIES' WEIGHT.

TABLE.

20 grains (grs.) make.....	1 scruple, marked scr.
3 scruples.....	1 dram, " dr.
8 drams.....	1 ounce, " oz.
12 ounces.....	1 pound, " lb.

NOTE.—Apothecaries and Physicians mix their medicines by this weight, but they buy and sell by Avoirdupois.

PRODUCE WEIGHT-TABLE FOR CANADA.

GRAIN.				SEEDS.			
Wheat.....	60	pounds	to the bushel.	Clover.....	60	pounds	to the bushel.
Oats	34	"	"	Flax	50	"	"
Corn.....	56	"	"	Timothy	48	"	"
Corn in cob.	80	"	"	Hemp	54	"	"
Barley	48	"	"	Blue grass..	14	"	"
Rye	56	"	"	Red Top.....	8	"	"
Buckwheat..	48	"	"	Hungarian {	48	"	"
Peas.....	60	"	"	grass ... }			
Beans.....	60	"	"	Millet.....	48	"	"
Tares.....	60	"	"	Rape.....	50	"	"

VEGETABLES.				VEGETABLES.			
Potatoes	60	pounds	to the bushel.	Castor Beans	40	pounds	to the bushel.
Parsnips	60	"	"	Malt.....	36	"	"
Carrots.....	90	"	"	DriedPeaches	33	"	"
Turnips.....	90	"	"	Dried Apples	22	"	"
Beets	60	"	"	Salt	56	"	"
Onions.....	60	"	"	Bran.....	20	"	"

LINEAR (OR LONG) AND SQUARE MEASURE.

LINEAR.		SQUARE.	
12 inches (in.) make..	1 foot (ft.)	I44 inches make.....	1 foot (ft.)
3 feet	1 yard (yd.)	9 feet	1 yard (yd.)
5½ yards.....	1 rod or perch.	30¼ yards	1 rod (rd.)
40 rods.....	1 furlong (fur.)	40 rods	1 rood (r.)
8 furlongs	1 mile (m.)	4 roods.....	1 acre (a.)

LAND MEASURE.

LENGTH.		AREA.	
7 ⁹² / ₁₀₀ inches make.....	1 link.	10,000 square links make	1 sq. chain
25 links.....	1 rod.	10 square chains	1 acre.
4 rods or 100 links.....	1 chain.		
80 chains.....	1 mile.		

In solid measure, *i. e.*, the measurement of solids, 1728 (the third power or cube of 12,) inches make 1 cubic foot, and 27 cubic feet (*i. e.* $3 \times 3 \times 3$,) make 1 cubic yard. In measuring timber, 40 cubic feet of round timber make what is called a ton, and the same name is given to 50 feet of hewn timber. A cord of firewood is 8 feet long, 4 feet wide, and 4 feet high, and therefore its solid content is $8 \times 4 \times 4 = 128$ feet.

Dry goods are measured by the yard, and fractions of a yard, the fractions used being one-quarter, one-eighth, and one-sixteenth.

MEASURES OF CAPACITY.

DRY.		LIQUID.	
2 pints make.....	1 quart (qt.)	4 gills make.....	1 pint (pt.)
4 quarts.....	1 gallon (gal.)	2 pints.....	1 quart (qt.)
2 gallons.....	1 peck (pk.)	4 quarts.....	1 gallon (gal.)
4 pecks.....	1 bushel (bu.)	63 gallons.....	1 hogshead (hhd.)
36 bushels.....	1 chaldron (ch.)	2 hogsheads.....	1 pipe (pi.)
The last is seldom used.		2 pipes.....	1 tun (tun.)

MEASURE OF TIME.		ANGULAR OR CIRCULAR MEASURE.	
60 seconds make.....	1 minute.	60 seconds make.	1 minute ($1'$.)
60 minutes.....	1 hour.	60 minutes.....	1 degree (1° .)
24 hours.....	1 day.	360 degrees.....	1 complete circle.
$365\frac{1}{4}$ days.....	1 year.		

There are other units applied to certain articles, *e. g.*, 12 articles, one dozen ; 20 articles, one score ; 144 articles, one gross ; 24 sheets of paper, one quire ; 20 quires, one ream,—14lbs., one stone. This last weight is varied in many places, 15lbs. and 16lbs., according to the nature of the article sold, *e. g.*,—potatoes, as an allowance for earth adhering.

THE CALENDAR MONTHS OF THE YEAR.

January.....	has 31 days.	July.....	has 31 days
February.....	“ 28 “	August.....	“ 31 “
March.....	“ 31 “	September.....	“ 30 “
April.....	“ 30 “	October.....	“ 31 “
May.....	“ 31 “	November.....	“ 30 “
June.....	“ 30 “	December.....	“ 31 “

Every fourth year is called Leap-year, in which February has 29 days.—If the last two figures denoting the year can be divided evenly by 4, it is Leap-year.

DECIMAL COINAGE.

10. THE principle of the decimal coinage is generally understood to depend on the rules of decimal fractions ; but as it is merely a separate and co-ordinate result of the common system of notation, we may explain it here, independently of the theory of decimal fractions.

We have already explained, that according to the Arabic notation, each digit has one-tenth the value that it would have if situated one place farther to the left. Thus, in the number 88, the digit to the right expresses 8 units, while that to the left expresses 8 tens. Now we cannot have any integer less than unity, but we may have to make calculations respecting quantities less than the unit under consideration, *e. g.*, in calculating by dollars, we may have to take cents into account, and as the cent is a sub-division of the unit, a dollar, some new character must be introduced to indicate this transition from the integral unit to a part of it. This is done very simply by interposing a mark like the period or full point (.) in printing.—This is usually called the decimal point, though it sometimes gets the vague and awkward name of the separatrix. This simple but admirable contrivance is ascribed to one Stevinus or Stevens, of the Netherlands, who gave his suggestion to the public about the year 1585. Its excellence consists in its being simply an extension of the common notation. The original system marks only the repetition of the unit of measure,—this applies the same principle to the sub-division of the unit into parts. To explain this, we have only to carry out the illustration already given regarding integers. We saw that the extreme right hand figure, 8 in our example, stood for 8 units, and was *one-tenth* of the preceding one; just in the same manner another figure, 8, placed to the right of the units' figure, will express *one-tenth* of those units, and the decimal point is used to mark this descending from integers to parts of the integral unit, and is written thus: 8.8, and means eight units, and eighth-tenths of that unit. If another 8 be added, thus: 8.88, it will express eight-tenths of the preceding unit, *i. e.*, eight-tenths of one-tenth, which is the same as eight one-hundredths of unity, and thus we have the descending scale by tenths towards the right of the decimal point, in the same manner as we had the ascending scale by tens towards the left. As a farther illustration, we may begin at the extreme right, as in 888.888, and we find throughout that each figure to the left is ten times that immediately to its right.

The decimal coinage adopts a certain unit called a dollar—the dollar is then sub-divided into ten equal parts, and each part is called a dime, the dime, in like manner, is divided into ten equal parts, and each part is called a cent; and the cent is divided into ten equal parts, and each part is called a mill. The mill enters into many calculations, though no coin of its value has ever been

issued. It is from this sub-division by ten, that the name *accimat*, derived from the latin *decem*, ten, is applied to this coinage. In the example 8.888, the first 8 means 8 dollars; the second, 8-tenths of a dollar, or 8 dimes, or 80 cents; the third, 8-tenths of a dime, or 8 one-hundredths of a dollar, or 8 cents; and the fourth 8-tenths of a cent, or 8 one-hundredths of a dime, or 8 one-thousandths of a dollar, or a mill, (from the latin *mille*, a thousand.) In naming any sum, it is not usual to mention either dimes or mills, but only dollars and cents. Thus: $\begin{smallmatrix} \$ & \text{d. c. m.} \\ 12.875 \end{smallmatrix}$ is written \$12.875, or \$12.87½ and is read twelve dollars eighty-seven and-a-half cents, which is perfectly correct, as 8 dimes make 80 cents, and 5 mills make half a cent. We noted, in treating of simple division, that when the terms in which a question is expressed require us to divide a less number by a greater, or in the case of remainders, the division is indicated by writing the dividend above the divisor, and separating them by a line,—thus: $7 \div 8$ is written $\frac{7}{8}$. So to indicate that 1 dollar is divided into 100 cents, we write $\$1\frac{1}{100}$, which means the one-hundredth part of a dollar, and therefore dollars and cents are sometimes written, especially in bills and drafts, in this manner, \$12. $\frac{25}{100}$, but the form \$12.25 is generally preferable. To show the reason of the form \$1.05, for one dollar and five cents, we have only to notice that the form \$1.5, would mean one dollar and five dimes, or fifty cents; whereas \$1.05 means one dollar, no dimes, and five cents.

From the foregoing explanations, it is plain that the rules for the addition, subtraction, multiplication and division of abstract numbers, or applicate numbers of only one denomination, apply also to dollars and cents, because they increase from right to left, and decrease from left to right, according to the same law, that is, in the former case by tens, and in the latter by tenths.

It would be of great benefit to the whole commercial community, and perhaps still greater to the farmer, if the decimal scale were adopted in weights and measures, as well as in money, as it would materially simplify and expedite all calculations. Every one must feel and admit the very great ease and rapidity with which every operation is effected, accounts made up, and books kept in dollars and cents, in comparison with the sub-division into pounds, shillings and pence, and the difference would be at least as great regarding weights and measures. It would also very much accelerate the learner's progress, for it would save him the heavy labour of committing to memory the formidable host of tables, through which he has now to cut his

way—the whole processes of reduction would be compressed into “nut-shell” dimensions, and the memory would not be over-taxed in after years to keep up the recollection of the tables conned in youth. Besides, by the plan we have suggested, the pupil could pass at once from the elementary rules to the higher ones, such as proportion and interest, and could either get into business in a much shorter time than is possible at present, or devote his time to higher and more important studies.

EXERCISES.

Addition of dollars and cents.

(1.)	(2.)	(3.)	(4.)
		\$85.50	\$116.20
	\$13.19	49.63	291.45
\$125.75	14.16	92.18	89.75
98.50	85.92	37.09	365.84
25.15	64.15	8.92	91.50
76.05	37.25	76.45	76.15
91.11 $\frac{1}{2}$	91.20	25.75	485.00
43.87 $\frac{1}{2}$	18.75	64.16	157.92
84.20	29.10	18.60	263.75
67.62 $\frac{1}{2}$	47.85	59.11	188.25
39.80	55.55	148.17	39.48
17.37 $\frac{1}{2}$	72.63	265.90	136.13
669.44	529.75	931.46	2301.42
(5.)	(6.)	(7.)	(8.)
\$11.27			\$55.63
45.15	\$44.50	\$296.75	17.75
54.72	67.23	176.84	84.18
31.30	89.75	518.50	29.88
49.50	27.63	369.63	45.13
16.75	95.13	627.45	38.81
84.28	38.88	258.13	67.25
14.85	17.45	591.18	96.20
9.44	56.64	179.25	77.63
28.09	73.85	567.42	8.75
345.35	511.06	3585.15	521.21

4. What is the difference between 2769 dollars and 50 cents, and 987 dollars $87\frac{1}{2}$ cents? Ans. \$1781.62 $\frac{1}{2}$.

5. The debit side of a ledger is \$1770.80, and the credit side \$876.50; what is the balance? Ans. \$894.30.

6. The credit side of a cash book is \$8795.88, and the debit side is \$10358.18; what is the balance? Ans. \$1562.30.

7. A firm owes \$227968.25, and the estate is worth \$98764.75; what is the state of the affairs of the firm? Ans.—The firm is unable to pay \$129,203.50 over and above the assets.

8. A ship and cargo were worth \$27509.50,—the ship was lost, and only \$6784.60 worth of the cargo saved; what was the loss? Ans. \$20724.90.

9. A cotton mill was totally destroyed by fire; the mill and its contents were worth \$78616; it was insured in one office for \$11760; in another, for \$9845; in another, for \$10800; and, in a fourth, for \$12685; did the owner lose, and if so, how much? Ans.—He lost \$33526.

10. I have sold for cash, during the last month, \$2786.88 worth of goods, I have received payment of S. Fulton's account, \$255.29 received proceeds of J. Jones' note, \$302.64; received interest on sundry debentures, \$278.50; sold my shares in the G. T. R. for \$785.75; received in cash, interest from Royal Canadian Bank, \$187.25; sold block of buildings in King street for \$1719.00, and shares in Rossin house for \$718.50. Paid S. Smellie's account, \$261.88; for sundry insurances, \$879.60; rent of office, \$150.00; for consignment to Liverpool, \$2786.50, and charges on same, \$175.63; what is the balance of receipts above expenditure? Ans. \$2780.20.

*Multiplication of dollars and cents. **

(1.)	(2.)	(3.)	(4.)
\$365.75	\$1873.47	\$865.63	\$24786.38
87	69	93	45
256025	1686123	259689	12393190
292600	1124082	779067	9914552
3182025	12926943	8050359	111538710

* We must here caution the tyro against such modes of expression as this, —“multiply \$85 by \$12.” Such an expression is simply absurd, for to say \$12 times \$85, might as well mean 1200 times \$85, or 12000 times \$85,

Division of dollars and cents.

1. $\$28642.14 \div 29 = \$987.66.$
2. $\$37133.34 \div 87 = \$426.82.$
3. $\$60509.68 \div 76 = \$796.18.$
4. $\$43009.75 \div 98 = \$438.87\frac{1}{2}.$
5. $\$1943243.55 \div 983 = \$1976.85.$
6. $\$31421.25 \div 63 = \$498.75.$
7. $\$28479.75 \div 78 = \$365.12\frac{1}{2}.$
8. $\$2595.37\frac{1}{2} \div 769 = \$3.37\frac{1}{2}.$

9. $\$2927.30$ a year; how much per day? Ans. $\$8.02.$

10. $\$3953.19$ a year; how much for every working day?
Ans. $\$12.63.$

11. 269 persons have to pay a tax of $\$1312.72$; what is the average tax on each? Ans. $\$4.88.$

12. A collection of $\$544.04$ is made by 1876 persons; how much did each give on an average? Ans. 29 cents.

To reduce currency money to the denominations of the decimal coinage. Since 100 cents make 1 dollar, and 4 dollars make 1 pound, 400 cents make 1 pound currency, and therefore to find the number of cents in any given number of pounds, we must multiply the pounds by 400. Again, since 20 cents make 1 shilling, or 12 pence, to find the number of cents in any given number of shillings, we must multiply the shillings by 20. Lastly, 5 cents, are equal to 3 pence; and 12 farthings are also equal to 3 pence, and (Ax. I.) things that are equal to the same thing, are equal to one another; therefore, 5 cents are equal to 12 farthings, and 1 farthing is the $\frac{1}{12}$ of 5 cents, or $\frac{5}{12}$ of 1 cent. Hence to find the number of cents in any number of pence and farthings, we multiply the number of farthings in the given pence and farthings by 5, and divide the product by 12. Having obtained the three results, we add them all together. Thus to change £48 18s.

$$\begin{array}{r} 48 \times 400 = 19200 \\ 18 \times 20 = 360 \\ 9\frac{3}{4} = 39\text{f.} \times \frac{5}{12} = 16\frac{1}{4} \\ \hline 19576\frac{1}{4} \end{array}$$

9 $\frac{3}{4}$ d. to dollars and cents, we multiply 48 by 400, 18 by 20, and take $\frac{5}{12}$ of 9 $\frac{3}{4}$, or 39 farthings, and add the three together, which gives us 19576 $\frac{1}{4}$ cents, or $\$195.76\frac{1}{4}.$

repeated 2 $\frac{1}{2}$ times, which would make 6s. 3d. (3.) The interpretation might be, that as 2s. 6d. is 30 pence, that the other 2s. 6d. is to be repeated 30 times, which would give £3 15s. 0d. (4.) The phrase may also be interpreted as meaning that 30d. was to be repeated 30 times, which would also give £3 15s. 0d. The last two interpretations are the same in two different forms, and give the same result. This is the only view in which the expression has any sense, and proves our statement, that whenever a denominate number is used as a multiplier, it ceases to be denominate, and becomes abstract. The same principle will apply to division.

EXERCISES.

(1.)

$$\begin{array}{r}
 £79 \times 400 = 31600 \\
 16 \times 20 = 320 \\
 6\frac{1}{2}d \times \frac{5}{12} = 10\frac{5}{6} \\
 \hline
 \$319.30\frac{5}{6} \\
 \hline
 \end{array}$$

(2.)

$$\begin{array}{r}
 £117 \times 400 = 46800 \\
 17 \times 20 = 340 \\
 8\frac{3}{4}d \times \frac{5}{12} = 14\frac{7}{12} \\
 \hline
 \$471.54\frac{7}{12} \\
 \hline
 \end{array}$$

- | | |
|---|--|
| 3. £87.14.10 $\frac{3}{4}$ = \$350.97 $\frac{1}{12}$. | 12. £137.16.8 = \$551.33 $\frac{1}{3}$. |
| 4. £29.19.9 = \$119.95. | 13. £236.19.2 $\frac{1}{2}$ = \$947.84 $\frac{1}{6}$. |
| 5. £67.13.4 $\frac{3}{4}$ = \$270.67 $\frac{1}{12}$. | 14. £19.16.8 = \$79.33 $\frac{1}{3}$. |
| 6. £279.15.10 $\frac{1}{2}$ = \$1119.17 $\frac{1}{2}$. | 15. £98.1.1 $\frac{1}{2}$ = \$392.22 $\frac{1}{2}$. |
| 7. £118.11.4 $\frac{1}{2}$ = \$474.27 $\frac{1}{2}$. | 16. £87.11.8 = \$350.33 $\frac{1}{3}$. |
| 8. £79.8.4 = \$317.66 $\frac{2}{3}$. | 17. £457.12.6 = \$1830.50. |
| 9. £37.18.8 = \$151.73 $\frac{1}{3}$. | 18. £219.4.7 $\frac{3}{4}$ = \$876.92 $\frac{1}{12}$. |
| 10. £57.8.11 $\frac{3}{4}$ = \$229.79 $\frac{7}{12}$. | 19. £49.9.4 $\frac{3}{4}$ = \$197.87 $\frac{1}{12}$. |
| 11. £49.7.6 = \$197.50. | 20. £287.18.10 $\frac{1}{2}$ = \$1151.77 $\frac{1}{2}$. |

To change dollars and cents to Halifax currency, we must reverse the above operation. Thus, to reduce \$195.76 $\frac{1}{4}$ to £. s. d.—

$$\begin{array}{r}
 400)19576\frac{1}{4}(48 \\
 \underline{400} \\
 3576\frac{1}{4} \\
 \underline{3200} \\
 20)376\frac{1}{4}(18 \\
 \underline{20} \\
 176 \\
 \underline{160} \\
 16\frac{1}{4} \\
 \underline{12} \\
 5)195(39
 \end{array}$$

First, reduce the dollars and cents to cents, then divide by 400, which gives 48, the even number of pounds, with a remainder of 376 $\frac{1}{4}$ cents; then divide this remainder by 20, which gives 18, the number of shillings, with a remainder of 16 $\frac{1}{4}$ cents, as in the converse operation, we multiplied by 5, and divided by 12, so now we multiply by 12, and divide by 5; thus, 16 $\frac{1}{4}$ $\times$ 12 = 195, and 195 $\div$ 5 = 39, the number of farthings, and this being reduced to pence and farthings, gives 9 $\frac{3}{4}$, so that \$195.76 $\frac{1}{4}$ = £48.18.9 $\frac{3}{4}$.

Or the work may be shortened by the following method. As \$4 make £1, the number of £'s in \$195.76 $\frac{1}{4}$, will be the same as the number of times that 4 is contained in the 195 dollars, which gives £48, and \$3 remain-

$$\begin{array}{r}
 \$195-76\frac{1}{4} \\
 \hline
 4)195 \\
 \hline
 £48-300 \\
 \hline
 20)376\frac{1}{4} \\
 \quad s18-16\frac{1}{4} \\
 \quad \quad 3 \\
 \hline
 \quad 5,48\frac{3}{4} \\
 \hline
 \quad 9\frac{3}{4}d.
 \end{array}$$

ing. Now, these three dollars are equivalent to 300 cents, which added to the remaining $76\frac{1}{4}$ cents, gives $376\frac{1}{4}$ cents; this divided by 20, will give the shillings, because 20 cents are equal to one shilling, and it is self-evident that the number of shillings in $376\frac{1}{4}$ cents, will be the same as the number of times 20 is contained in that number, which gives 18 shillings, and $16\frac{1}{4}$ cents remaining. Lastly, as 5 cents are equal to 3 pence, one cent will be equal to $\frac{1}{5}$ of 3 pence, which is $\frac{3}{5}$ of a penny; therefore, if one cent is equal to $\frac{3}{5}$ of a penny, the remaining $16\frac{1}{4}$ cents will be equal to $16\frac{1}{4}$ times $\frac{3}{5}$ of a penny, which is $9\frac{3}{4}d.$; hence we have $\$195.76\frac{1}{4}$ equal to $£48.18.9\frac{3}{4}$.

remaining $16\frac{1}{4}$ cents will be equal to $16\frac{1}{4}$ times $\frac{3}{5}$ of a penny, which is $9\frac{3}{4}d.$; hence we have $\$195.76\frac{1}{4}$ equal to $£48.18.9\frac{3}{4}$.

EXERCISES.

- | | |
|---|-------------------------------|
| 1. Reduce $\$119.95$ to Halifax currency. | Ans. $£29.19.9.$ |
| 2. Reduce $\$270.67\frac{1}{2}$ " " | Ans. $£67.13.4\frac{3}{4}.$ |
| 3. Reduce $\$474.27\frac{1}{2}$ " " | Ans. $£118.11.4\frac{1}{2}$ |
| 4. Reduce $\$197.50$ " " | Ans. $£49.7.6.$ |
| 5. Reduce $\$1119.17\frac{1}{2}$ " " | Ans. $£279.15.10\frac{1}{2}.$ |
| 6. Reduce $\$551.33\frac{1}{3}$ " " | Ans. $£137.16.8.$ |
| 7. Reduce $\$1830.50$ " " | Ans. $£457.12.6.$ |
| 8. Reduce $\$1151.77\frac{1}{2}$ " " | Ans. $£287.18.10\frac{1}{2}.$ |

MIXED EXERCISES.

- | | |
|--|-------------------------------|
| 1. Reduce $£436.7.8\frac{1}{2}$ to dollars and cents. | Ans. $\$1745.54\frac{1}{6}.$ |
| 2. Reduce $\$547.87$ to Halifax currency. | Ans. $£136.19.4\frac{1}{5}.$ |
| 3. Reduce $£783.13.5\frac{1}{4}$ to dollars and cents. | Ans. $\$3134.68\frac{3}{4}.$ |
| 4. Reduce $\$576.85$ to Halifax currency. | Ans. $144.4.3.$ |
| 5. Reduce $£606.19.8\frac{3}{4}$ to dollars and cents. | Ans. $\$2427.94\frac{7}{12}.$ |
| 6. Reduce $\$375.99$ to Halifax currency. | Ans. $£93.19.11\frac{2}{5}.$ |
| 7. Reduce $3s. 8\frac{1}{4}d.$ to dollars and cents. | Ans. $73\frac{3}{4}$ cents. |
| 8. Reduce 17 cents to Halifax currency. | Ans. $10\frac{1}{5}$ pence. |
| 9. Reduce $10\frac{3}{4}$ pence to dollars and cents. | Ans. $17\frac{11}{12}$ cents. |
| 10. Reduce 23 cents to old Canadian currency. | Ans. $13\frac{4}{5}$ pence |

THE FARMERS' RULE FOR REDUCING CENTS TO PENCE, AND PENCE TO CENTS.

QUESTION.

Said farmer A. to grocer B.
 There's something here that puzzles me ;
 I sold some butter here to-day,
 I sold by cents, by pence they pay ;
 How shall I change the cents to pence,
 And know the trick from this day hence ?

ANSWER.

Five cents are three pence you must know,
 As twenty cents to twelve pence go ;
 Three times the cents, the fifth of that
 Is just the thing you would be at ;
 And if you buy from grocers here,
 That other case is just as clear,
 Five times the pence, the third of it
 Will make you safe and always fit.

REDUCTION.

11.—REDUCTION is the mode of expressing any given quantity in terms of a higher or lower denomination, *e. g.*, expressing any given number of dollars as cents, and *vice versa*, any number of cents as dollars.

When a higher denomination is changed to a lower, (as dollars to cents,) the process is called reduction *descending*, and when a lower is changed to a higher, (cents to dollars,) it is called reduction *ascending*.

RULE.

To express any given quantity in terms of a lower denomination, multiply it by the number of units which it contains of the next lower denomination, and add in the given units of that denomination, and so on to the lowest denomination given. Thus, to express

$$\begin{array}{r}
 \text{£}25.11.4\frac{1}{2} \\
 20 \\
 \hline
 511 \\
 12 \\
 \hline
 6136 \\
 4 \\
 \hline
 24546
 \end{array}$$

8 dollars and 25 cents as cents, multiply 8 by 100, giving 800, and add the 25 cents, giving 825 cents.

So also, as in the margin, the pounds are multiplied by 20, for 20s.=£1, and the 11 shillings added in, giving 511 shillings, then these shillings are multiplied by 12, and the 4 pence added in, giving 6136 pence, and this finally is multiplied by 4, and the two farthings added in, giving 24546 farthings. So also $\$98 \times 100 = 9800$ cents. To express a lower in terms of a higher denomination, divide the lower by the number that denotes how many units of the lower are contained in one unit of the higher. Thus

to reduce 24546 farthings to £. s. d.—since 4 farthings make 1 penny, we divide by 4, and get 6136 pence, with a remainder of 2 farthings, or 1 half-penny. Again, since 12 pence make one shilling, we divide 6136 by 12, and get 511 shillings, with a remainder of 4 pence. Lastly we divide 511 by 20, and get 25 pounds, with a remainder of 11 shillings, so that 24546 farthings make £25 11s. 4½d. So also, since 100 cents make one dollar, to reduce 12579 cents to dollars, divide by 100, and $12579 \div 100 = \$125.79$. We thus see that cents can be changed to dollars and cents, by simply cutting off two figures from the right. So also dollars can be changed to cents by adding two ciphers, or dollars and cents can be changed to cents by removing the decimal point two places towards the right.

EXERCISES.

1. How many dollars are there in 47986 cents? Ans. \$479.86.
2. How many cents are there in 187 dollars? Ans. 18700.
3. How many pence are there in £87.12.8? Ans. 21032.
4. How many pence are there in £113.18.4? Ans. 27340.
5. How many farthings are there in £79.15.10½? Ans. 76602.
6. How many half pence in £97.17.6? Ans. 46980.
7. How many pounds, &c., are there in 7983 pence?
 Ans. £33.5.3.
8. How many pounds, &c., are there in 156793 farthings?
 Ans. £163.6.6¼.
9. How many pounds are there in 2 tons 16 cwt. 2 qrs. and 21
lbs.? Ans. 5671.
10. How many pounds are there in 18 cwt. and 22 lbs.?
 Ans. 1822.
11. Reduce 14796 lbs. to tons, &c.? Ans. 7 tons, 7 cwt., 3 qrs., 21 lbs.
12. Reduce 7643 quarters to tons, &c.? Ans. 95 tons, 10 cwt., 3 qrs.
13. How many drams are there in 18 lbs., 13 oz. and 15 drs.? Ans. 4831.
14. How many pounds are there in 2785 drams?
 Ans. 10 lbs., 14 oz., 1 dr.
15. How many grains are there in 17 lbs., 11 oz., 18 dwt. and
22 grains? Ans. 103654.
16. How many lbs. in 46891 grs.? Ans. 8 lbs., 1 oz., 13 dwt., 19 grs.

17. Reduce 98 miles, 5 furlongs and 30 rods to rods ?
Ans. 31590 rods.
18. How many inches from Toronto to Hamilton, (38 miles) ?
Ans. 2407680.
19. How many miles are there in 527168 feet ?
Ans. 99 miles, 6 fur., 29 pr., 2 yds., 3 ft., 6 in.
20. Reduce 57 acres, 3 roods and 24 rods to rods ?
Ans. 9264 rods.
21. How many square yards are there in 17 acres, 2 roods and 18 rods ?
Ans. 85244 $\frac{1}{2}$ yards.
22. Find the number of acres, &c., in 479685971 square inches ?
Ans. a. 76.1.35.19.2.119.
23. How many acres do 176984 square yards make ?
Ans. a. 36.2.10.21 $\frac{1}{2}$.
24. How many square links are there in 37 acres ?
Ans. 3,700,000 links.
25. How many acres, &c., in 479,863,201 square links ?
Ans. 4798 a., 6 ch., 3201.
26. 7,864,391 cubic inches ; how many cubic yards ?
Ans. yds. 168.15.263.
27. 9 cubic yards, 7 cubic feet, 821 cubic inches ; how many cubic inches ?
Ans. 432821 cubic inches.
28. How many gills does a tun contain ?
Ans. 8064 gills.
29. How many gallons, &c., do 479865 gills make ?
Ans. gals. 14995.3.0.1.
30. How many pints are there in 28 bu., 3 pecks and 1 gal. ?—
Ans. 1848 pints.
31. 27 yards, 3 qrs., 3 nails ; how many nails ?
Ans. 447 nails.
32. 286 nails ; how many yards, &c. ?
Ans. 17 yards, 3 qrs., 2 nls.
33. 36 ° 40' 25" ; how many seconds ?
Ans. 132025".
34. How many degrees, &c., in 49786" ?
Ans. 13 ° .49'.46".
35. The population of Toronto is 45,288 ; what would a poll tax of 5 cents each amount to ?
Ans. \$2264.40.
36. How long would it take a railway train to move a distance equal to that of the earth from the sun, (95 millions of miles,) at a speed of 52 miles an hour ?
Ans. 208 years, 201 days, 19 $\frac{1}{3}$ hours.
37. The area of Upper Canada is 94,720,000 acres ; how many square feet ?
Ans. 4,126,003,200,000 square feet.

38. Sound moves about 1130 feet in a second of time; how long would it be in moving from the earth to the sun?

Ans. 14 years, 27 days, 15 hours, 50 min., $5\frac{350}{130}$ sec.

39. How many seconds of this century had elapsed at the end of 1864, counting the day at 24 hours? Ans. 2,019,686,400".

40. The great bell of Moscow weighs 127,836 lbs.; how many tons, &c., does it weigh, the quarter being 28 lbs.?

Ans. 57t. 1c. 1q. 16lbs.

41. How many days from the 11th July, 1861, to the 1st of April, 1864? Ans. 995 days.

42. A congregation of 569 persons made a collection of £40.6.1; how many pence did each give on an average? Ans. 17d.

43. The British mint can strike off 20,000 coins in an hour; what is the value of all the pennies coined in one day of 12 hours' work? Ans. £1,000.

44. 417 tons of fish were caught at Newfoundland in one season, and sold by the stone of 14 lbs., at an average price of 42 cents a stone; what did they bring? Ans. \$25020.

45. How many feet from pole to pole, the earth's diameter being 7945 miles? Ans. 41949600 feet.

DENOMINATE NUMBERS.

12.—WHEN numbers are spoken of in general, without reference to any particular articles, such as money or merchandise, they are called *abstract*, but when they are applied to such articles they are sometimes called *applicate*, as being *applied* to some particular articles to express their quantity; sometimes they are called *concrete*, (growing together,) as attached to some particular substances, and sometimes they are called *denominate*, as denoting quantities that consist of different denominations, as dollars and cents,—pounds, ounces, &c. The elementary rules of addition, subtraction, multiplication and division, are performed on denominate numbers, exactly in the same way as on abstract numbers, with this single difference, that when a lower denomination is added, and gives a sum equal to one or more units of the next higher denomination, we carry that unit, or those units, to the next higher denomination. Thus: if the sum were 24 inches, we should call that two feet. In abstract and decimal numbers we always reduce, or carry, by tens.

E X A M P L E

Here we find the sum of the pence to be 28, and as 12 pence make 1 shilling, the number of shillings in 28 pence will be the same as the number of times that 12 is contained in 28, which is twice, with a remainder of 4 ; therefore we write the 4 pence under the pence column, and add up the 2 shillings with the shillings' column, and obtain 54 shillings, and as 20 shillings make 1 pound, the number of pounds in 54 shillings will be the same as the number of times that 20 is contained in 54, which is twice, with a remainder of 14, and therefore we write the 14 shillings under the shillings' column, and add up the 2 pounds with the units' column of pounds, and now since the remaining columns are all of the same denomination, we proceed as in simple addition, and find the whole to be £236.14.4. The same illustration will apply to the subtraction, multiplication and division of all kinds of denominate numbers.

In the exercises on the addition of denominate numbers, one question in abstract numbers is given to contrast with the denominate.

E X E R C I S E S .

(1.)	(2.)	(3.)	(4.)
		£76.18. 4	\$1967.87½
	\$857.63	17.11. 4½	2075.75
7865437	189.50	99.19. 9	3194.62½
198675	684.87½	11.11.11	7658.50
8476154	498.75	67.15.10½	8976.37½
1869538	867.12½	79.19. 9	2873.12½
4187643	365.37½	28.12. 1	1769.25
5768299	917.25	63. 8. 4½	2481.92
28365746	4380.50½	445.17. 5½	30997.42
(5.)	(6.)	(7.)	(8.)
lbs. oz. drs.	t. cwt. qrs. lbs.	lbs. oz. dwt. grs.	lbs. oz. drs. scr. grs.
13.14.10	26.17.3.21	3.11.16.21	5.11.7.2.19
15.11.10	18.11.0.19	5. 8. 7.11	4.10.4.1. 7
11. 4. 9	25.15.1.16	7. 9.18.23	3.11.6.2.14
8.12.13	13.17.2.20	11.10.15.17	1. 9.3.1.12
15. 7. 8	39. 4.1.23	12. 7. 9. 8	2. 4.5.0.10
10.13.11	28.16.3.14	16.10.11.22	6. 7.2.2. 9
8. 9. 6		18. 8.19.18	2. 8.1.1.13
4.15.15			
89.10. 2	153. 3.2.13	77. 8. 0. 0	28. 4.0.1. 4

(9.)	(10.)	(11.)	(12.)
m. fur. rods. yds.	yds. ft. in. l.	ac. roods, rd.	rods. yds. ft. in.
176.7.39.5	18.2.11.11	29.3.39	39.30.8.143
85.4.20.1	14.2. 7. 9	57.2.18	18.11.4. 68
79.6.29.3	8.1.10. 7	118.0.26	24. 4.7.118
42.3. 8.2	11.0. 7. 6	75.3.11	11.21.2. 96
67.1.11.2	7.2. 8. 5	51.1. 8	15.27.0.124
118.3.10.3	16.2. 9.10	94.1.19	27. 6.3. 87
81.2.31.1	8.1. 7. 6	63.2.21	19.25.2. 38
79.0.21.2		78.1.15	
18.3.33.3		19.3.33	
749.2. 6.0	87.0. 3. 6	589.0.30	157. 6.3. 98

(13.)	(14.)	(15.)	(16.)
a. ch. links.	ch. b. p. g. qt. pt.	tu. pi. hhd. gal. qt. pt. gl.	yds. qrs. nls.
79.9.9999	5.35.3.1.3.1	6.1.1.1.3.1.3	36.3.2
117.4.3650	7.18.2.0.1.1	4.0.1.1.2.0.2	19.1.3
47.5. 941	8. 7.1.1.0.1	5.1.0.0.1.1.1	87.2.1
56.2.1182	3.26.0.0.1.0	1.1.0.1.0	63.0.2
27.7.2813	4.18.0.1.0.1		74.2.2
36.1. 771			93.3.3
84.8.1160			
449.8. 516	29.34.0.0.3.0	16.1.1.5.0.0.2	375.2.1

(17.)	(18.)	(19.)	(20.)
cwt. qrs. lbs.		yrs. days. hrs. min. sec.	cwt. qrs. lbs.
87.3.11	359 ⁰ .59'.59"	33.364.23.59.59	18.1.18
49.1.18	153 .40 .45	28.113.11.48.48	22.3.11
28.3.15	270 . 0 . 0	17. 97.12. 0. 0	9.2.18
36.1. 8	179 .45 .30	1.307.23.48.49	12.1.15
88.1.16	81 .30 .10	12.114. 0. 0. 0	8.3.24
57.3.14	89 .59 .59		31.2. 0
348.3.7	1134 .56 .23	93.267.23.37.36	103.3.11

LEDGER ACCOUNTS.

The debit and credit sides of four folios of a ledger are as below, what are the balances ?

(21.) Dr.	(21.) Cr.	(22.) Dr.	(22.) Cr.
\$1214.75	\$2763.80	\$198.75	\$118.50
863.09	471.38	47.63	9.05
291.45	365.50	18.11	16.25
318.25	297.11	97.38	37.08
1789.87	584.88	85.88	19.13
947.63	963.15	76.20	47.75
2000.00	1257.75	4.50	65.92
798.38	189.60	181.60	32.40
2018.50	98.13	19.25	76.50
164.30	756.25	76.38	7.75
277.15	87.50	219.50	197.25
1165.20	163.63	48.75	15.75
367.40	1291.00	93.15	8.38
984.70	784.25	25.50	93.15
273.60	79.75	81.05	67.45
584.10	81.18	28.30	5.45
1200.00	318.50	69.08	18.09
68.75	1819.20	157.11	4.12
79.15	58.50	278.00	57.60
56.18	176.25	59.50	28.88
2860.14		11.25	
		941.12	
(23.) Dr.	(23.) Cr.	(24.) Dr.	(24.) Cr.
\$81.19	\$80.10	\$177.88	\$156.92
17.11	15.65	291.16	285.15
45.38	39.88	356.13	356.12
19.63	10.13	189.38	178.25
187.13	176.15	471.63	469.10
87.63	89.92	785.88	698.80
87.88	77.81	911.50	930.75
111.11	99.88	583.15	496.20
134.56	16.97	432.61	547.60
179.51	87.63	355.55	478.99
340.25	75.75	638.27	546.54
224.12	56.51	436.15	372.25
156.12	37.23	325.36	252.12
\$	\$	\$	\$

(25.)	Dr.	(25.)	Cr.	(26.)	Dr.	(26.)	Cr.
	\$176.93		\$1237.75		\$1087.63		4786.87
	27.85		2763.18		457.88		183.05
	79.37		194.25		190.37		97.75
	98.11		39.37		87.12		149.15
	35.40		8.25		94.25		13.25
	83.50		11.87		47.20		41.18
	1127.25		29.05		39.15		8.50
	48.18		63.20		8.75		9.75
	250.00		71.80		367.40		11.12
	779.63		13.10		18.93		183.62
	154.20		45.50		67.45		79.10
	59.75		25.20		21.63		814.00
	68.87		43.15		298.50		95.50
	18.75		7.50		78.60		218.00
	28.63		50.00		189.00		59.87
	71.38		87.75		47.15		18.05
	293.63		5.00		68.10		77.40
	185.10		31.60		54.30		38.87
	9.05		13.40		12.12		15.62
	64.20		90.75		89.75		9.87
	38.75		15.15		118.00		14.12
	45.45		67.63		69.50		89.50
	215.87		58.50		48.75		4.20
	7.75		67.05		36.12		67.37
	93.92		49.35		91.20		81.09
	81.88		21.25		87.63		7.05
	68.25		35.15		90.00		57.20
	99.99		20.13		100.75		114.25
	18.12		92.87		49.15		297.00
	27.13		35.28		87.63		78.75
	168.00		81.18		43.25		564.87
	75.75		10.80		81.37		961.34
	738.38		51.25		92.65		268.34
	18.24		67.54		37.49		567.84
	136.25		91.12		46.87		987.69
	126.72		18.35		91.13		356.78
	834.15		42.54		54.12		978.65
	128.71		16.21		64.54		546.37
	136.18		25.51		57.62		786.42
	178.16		53.99		38.94		428.97
	284.77		62.87		61.87		642.85
	326.54		91.54		93.89		529.64
	412.13		32.21		89.78		428.04
	391.15		54.12		21.46		106.70
	267.18		77.99		64.98		500.00
	125.13		42.51		73.75		250.09

SUBTRACTION.

(1.)	(2.)	(3.)
\$147985.87 $\frac{1}{2}$	£1573.11. 4 $\frac{1}{2}$	\$810731.37 $\frac{1}{2}$
86997.75	976.15.10 $\frac{1}{2}$	341876.62 $\frac{1}{2}$

4. I have taken this month in trade £1796.18.11, and paid £673.10.10 for fall goods, and expended for private purposes, £36.8.1, and lodged the rest in the Ontario Bank; how many dollars have I banked? Ans. \$4348.

5. I bought 47 tons, 17 cwt., 1 qr., 18 lbs. of grain, and have sold 29 tons, 18 cwt., 3 qrs., 22 lbs. of it; how much have I in store? Ans. 17 tons, 18 cwt., 1 qr. 21 lbs.

6. If the distance from Toronto to Quebec is 503 miles, 1 fur., 20 rods; and the distance from Montreal to Quebec is 180 miles, 2 fur., 35 rods: what is the distance from Toronto to Montreal?—
Ans. in .322.6.25.

7. A farmer possessed 1279 acres, 2 roods, 21 rods, and by his will left 789 acres, 3 roods, 36 rods to his eldest son, and the rest to the second; how much had the younger? Ans. 489 acres, 2 roods, 25 rods.

8. The latitude of London (England,) is 51°.30'.49" N., and that of Gibraltar 36°.6'.30" N.; how many degrees is Gibraltar south of London? Ans. 15°.24'.19".

9. The earth performs a revolution round the sun in about 365 days, 5 hours, 48 minutes and 48 seconds, and the planet Jupiter in about 4332 days, 14 hours, 26 minutes and 55 seconds; how much longer does it take Jupiter to perform one revolution than the earth? Ans. 3967 days, 8 h., 38 min., 7 sec.

10. I bought 54 lbs., 10 oz. of tobacco, and 11 oz. of it were lost by drying; and I sold 36 lbs., 12 oz. of it to A.; and 11 lbs., 9 oz. to B.; and used 3 lbs., 14 oz. myself; how much have I remaining, and how much did I get for what I sold, at 6 cents an ounce, and how much did my own consumption and drying come to at the cost price, which was 5 cents an ounce?

Ans. (1.) 1 lb., 12 oz. (2.) \$46.38. (3.) \$3.65.

MULTIPLICATION.

- | | |
|--|---|
| 1. \$1796 × 47 = \$84412. | 3. \$168.87 $\frac{1}{2}$ × 64 = \$10808. |
| 2. £2.19.2 $\frac{1}{4}$ × 144 = £426.3.0. | 4. £1.2.9 × 225 = £255.18.9 |

5. Find the duty on 97 consignments of merchandise at \$86.62 $\frac{1}{2}$ each ? Ans. \$8402.62 $\frac{1}{2}$.

It is often convenient to multiply denominate numbers by the *factors* of the multiplier. Thus: to multiply by 84 is the same as to multiply by 7 and 12. Thus, in the annexed examples, since $12 \times 7 = 84$, 18 tons, 12 cwt., 2 qrs., 11 lbs. $\times 84$, is the same as 18 tons, 12 cwt., 2 qrs., 11 lbs. $\times 12 \times 7$, &c.

(6.)	(7.)	(8.)
tons. cwt. qrs. lbs. 18.12.2.11 $\times 84$ 12	ac. roods. rds. 27.2.29 $\times 72$ 8	£3.15.6 $\times 150$ 5
223.11.1. 7 7	221.1.32 9	18.17.6 5
1564.19.0.24	1993.0. 8	94. 7.6 6
		566. 5.0

(9.)	(10.)
cwt. qrs. lbs. 23.3.22 $\times 49$ 7	lbs. oz. drs. 49.11.12 $\times 63$ 7
167.3. 4 7	348. 2. 4 9
1174.2. 3	3133. 4 .4

Questions, such as No. 10, may also be worked by multiplying each denomination separately by the whole given multiplier at once, reducing to the next higher denomination, and adding this to the next result.—Thus: £3.15.6, multiplied by 150, will give (1.) 900 pence=75 shillings,—(2.) 15 shillings multiplied by 150, will give 2250 shillings, which, added to the 75 shillings already found, will give 2325 shillings, which reduced will give 116 pounds, 5 shillings, and (3.) the 3 pounds multiplied by 150, will give 450 pounds, which added to what has already been found, will give a final result of £566.5.0, as already obtained by the method of factors.

£3.15.6
150
900
75.0
2250.0
2325.0
£116.5.0
450.0.0
£566.5.0

cwt. qrs. lbs.		cwt. qrs. lbs.
9.3.22+86	£2.13.11 $\frac{1}{4}$	1.2.17+27
86	125	27
857.1.17	£331.18.0 $\frac{1}{4}$	45.0. 9

SUPPLEMENT TO THE MULTIPLICATION OF DENOMINATE NUMBERS.

6. How many seconds has a person lived who has completed his twentieth year, the year consisting of 365 days, 5 hours, 48 minutes, and 48 seconds? Ans. 662688000.

7. Bought 7 loads of hay, each weighing 1 ton, 3 cwt., 3 qrs., 12 lbs; what did the whole weigh?

8. If a man can reap 3 acres and 35 rods per day, how much will he reap in 30 days? Ans. 96 acres, 90 rods.

9. If a staemboat ply across a channel, the breadth of which is equal to 2°, 25', 10'', what angular space has she traversed at the end of 20 trips? Ans. 48°, 23'.

10. If a man saves 3s. 9 $\frac{1}{2}$ d. a day, how much will he save in the year, omitting the Sabbaths? Ans. £59 6s. 9 $\frac{1}{2}$ d.

11. If 12 gallons, 3 quarts, 1 pint of molasses be used in a hotel in a week, how much would be used in a year at that rate?

Ans. 10 hhds., 39 gals., 2 qts.

12. If a man can saw one cord of wood in 8 hours, 45 minutes, 50 seconds, in what time will he saw 11 cords?

Ans. 4 days, 24 hours, 10 seconds.

13. If 13 waggons carry 3 tons, 15 cwt., 1 qr., 15 lbs. each, how much do they all carry? Ans. 49 tons, 0 cwt., 0 qr., 20 lbs.

14. If a man travel 20 miles, 5 furlongs, and 20 rods a day, how would he travel at that rate in a year?

Ans. 755 m., 7 fur., 20 rods.

15. There are 24 piles of wood, each containing 3 cords, 42 cubic feet; what is the whole quantity? Ans. 79 cords, 120 ft.

16. If 17 hhds. of sugar weigh 12 cwt., 1 qr., 20 lbs. each, how much will the whole weigh? Ans. 211 cwt., 2 qrs., 15 lbs.

17. Allowing 75 yards, 18 feet, for the surface of 9 rooms, how much paper would be required to cover the wall?

Ans. 693 sq. yards.

18. If 11 casks contain 54 gals., 3 qts., 1 pt., 2 gills each, how much would they all contain? Ans. 604 gals., 1 qt., 2 gills.

19. If the cars go 21 miles, 2 furlongs, 10 rods per hour, how far will they go in 15 hours? Ans. 319 miles, 1 fur., 30 rods.

20. If 1 silver cup weigh 3 oz., 15 dwts., 10 grs., how much will 10 such cups weigh? Ans. 3 lbs., 1 oz., 14 dwts., 4 grs.

DIVISION.

In Division, all remainders are to be reduced to the next lower denomination, and in that form divided, to get the units of that denomination.

EXERCISES.

1. A silversmith made half-a-dozen spoons weighing 2 lbs., 8 oz., 10 dwts.; what was the weight of each? Ans. 5 oz., 8 dwts., 8 grs.

2. If 45 waggons carry 685 bushels, 2 pecks, 4 quarts, how much does each carry on equal distribution? Ans. 15 bushels, $7\frac{5}{9}$ quarts.

3. If a labourer receives 149 lbs., 13 oz. of meat as payment for 26 days' work, how much is that per day, on an average?

Ans. 5 lbs., $12\frac{5}{26}$ oz.

4. If a steamer occupies 48 days, 17 hours, and 40 minutes, in making 121 trips, what is the average time? Ans. 9 h. 40 min.

5. If 98 bushels, 3 pecks, and 2 quarts of grain can be packed in 37 equal-sized barrels, how much will there be in each?

Ans. 2 bush., 2 pecks, $5\frac{17}{37}$ qts.

6. If a man has an income of £400 a year, how much has he each day? Ans. £1 1s. $11\frac{1}{3}$ d.

7. An English nobleman has £200,000 a year; how much has he a day? Ans. £547 18s. $10\frac{1}{2}$ d., nearly.

8. In a coal mine, 97 tons, 13 cwt., 2 qrs. were raised in 97 days; how much was that per day, on an average?

Ans. 13 cwt., 3 qrs., 22 lbs. +.

9. If 19 canisters of equal size contain 332 lbs., 8 oz., how much is in each? Ans. 17 lbs., 8 oz.

10. If \$15.50 be the value of 1 lb. of silver, what will be the weight of \$500000 worth?

Ans. 32258 lbs., 8 oz., 15 dwts., $11\frac{19}{31}$ grs.

11. If 1246 bushels of wheat are produced in a field of 16 acres what is the yield per acre? Ans. 77 bush., 3 pecks, 5 qts., $1\frac{3}{4}$ pts.

12. A gardener pulled 13500 bushels of apples off 60 trees; how many, on an average, were in each bushel? Ans. 230.

13. If 13 hogsheads of sugar weigh 6 tons, 8 cwts., 2 qrs., 7 lbs., what is the weight of each? Ans. 9 cwt., 3 qrs., 14 lbs.

14. What is the twenty-third part of 137 lbs., 9 oz., 18 dwts., 22 grs.? Ans. 5 lbs., 11 oz., 18 dwts., $5\frac{2}{3}$ grs.

15. A shipment of sugar consisted of 8003 tons, 17 cwt., 1 qr., 12 lbs., 10 oz., net weight; it was to be shared equally by 451 grocers; how much did each get?

Ans. 17 tons, 14 cwt., 3 qrs., 18 lbs. 14 oz.

16. If a horse runs 174 miles, 26 rods, in 14 hours, what is his speed per hour? Ans. 12 miles, 3 fur., 19 rods.

17. A farmer divided his farm, containing 322 acres, 2 roods, 10 rods, equally among his seven sons and 6 sons-in-law; what was the share of each? Ans. 24 acres, 3 roods, 10 rods.

18. If 132 bushels, 3 pecks, 7 quarts of corn be distributed equally among 23 poor persons; how much does each get?

Ans. 5 bushels, 3 pecks, 1 quart.

19. A man having purchased 119 cwt., 3 qrs., 23 lbs of hay, and drew home in 6 waggons; how much was on each waggon?

Ans. 19 cwt., 3 qrs., 23 lbs.

MIXED EXERCISES ON DENOMINATE NUMBERS.

20. A gentleman, by his will, left an estate worth \$2490, to be divided among his two sons and 3 daughters in the following proportions:—The widow was to receive *one-third* of the whole, less \$346; the younger son \$212 more than his mother; the older son as much as his mother and brother, lacking \$335.50, and the three daughters were to have the remainder, share and share alike; what was the share of each?

Ans. The widow got \$484; the older son got \$844 $\frac{1}{2}$; the younger son got \$696; each daughter got \$155 $\frac{1}{6}$.

21. A gentleman left a property in land, consisting of 448 acres, 3 roods, 24 rods, to be divided among his four children in the following proportions:—The youngest was to get 4 acres, 3 roods, 6 rods more than the eighth part; the second youngest was to get one-fifth of the remainder; the oldest but one was to get one-third of the remainder, and the oldest the residue; what was the share of each?

Ans. The youngest got 60 acres, 3 roods, 24 rods ; the next got 77 acres, 2 roods, 16 rods ; the next got 103 acres, 1 rood, $34\frac{2}{3}$ rods ; the oldest got 206 acres, 3 roods, $29\frac{1}{3}$ rods.

22. A ship made the following headway on six successive days : On Monday, $3^{\circ}, 8', 45''$ south, and $1^{\circ}, 51'$ east ; on Tuesday, $2^{\circ}, 36'$ south, and $2^{\circ}, 1', 15''$ east ; on Wednesday, $4^{\circ}, 0', 52''$ south, and 1° east ; on Thursday, $1^{\circ}, 48', 52''$ south, and $3^{\circ}, 16', 22''$ east ; on Friday, $1^{\circ}, 19'$ south, and $48', 29''$ east ; and on Saturday, $59', 30''$ south, and $3^{\circ}, 52', 11''$ east ; find her distances south and east from the port of departure.

Ans. South $13^{\circ}, 52', 59''$; East $12^{\circ}, 49', 17''$

23. A vintner sold in one week, 51 hogsheads, 53 gallons, 1 quart, 1 pint ; in the next week, 27 hogsheads, 39 gallons, 3 quarts ; in the next week, 19 hogsheads, 13 gallons, 3 quarts ; how much did he sell in the three weeks ?

Ans. 88 hogsheads, 43 gallons, 3 quarts, 1 pint.

24. In a pile of wood there are 37 cords, 119 cubic feet, 76 cubic inches ; in another there are 9 cords, 104 cubic feet ; in a third there are 48 cords, 7 cubic feet, 127 cubic inches, and in a fourth there are 61 cords, 139 cubic inches. Find the whole amount.

Ans. 156 cords, 102 feet, 342 inches.

25. The following cargo was landed at Montreal from Liverpool : 78 tons, 3 cwt., 2 qrs., 26 lbs. of Irish pork ; 125 tons, 15 cwt., 1 qr., 9 lbs. of iron ; 90 tons, 12 cwt., 2 qrs., 20 lbs. of West of England cloth goods ; 225 tons, 9 cwt., 12 lbs. of Scotch coal, and 106 tons, 1 qr. of Staffordshire pottery ; what is the whole amount of the consignment ?

Ans. 636 tons, 1 cwt., 16 lbs.

26. If a man can count 100 one-dollar bills in a minute, and keep working 10 hours a day ; how long will it take him to count a million ?

Ans. $16\frac{2}{3}$ days.

27. The earth's equatorial diameter is 41847426 feet ; how many miles ?

Ans. 7925 and 3426 feet.

28. The earth's polar diameter is 7899 miles, 900 feet ; how many feet ?

Ans. 41707620 feet.

29. Sound is calculated to move 1130 feet per second ; how far off is a cannon, the report of which is heard in $1' 9''$?

Ans. 77970 feet.

30. If the circumference of a waggon wheel be $14\frac{2}{3}$ feet ; how often will it turn round in a mile, (5280 feet) ?

Ans. 360 times.

GREATEST COMMON MEASURE.

13.—When any quantity is contained an even number of times in a greater, the greater is called a multiple of the less, and the less a submultiple, measure or aliquot part of the greater. Thus: 48 is a multiple of 2, 3, 4, 6, 8, 12, 16 and 24, and each of these is a submultiple of 48.

When one quantity divides two or more others evenly it is called a common measure of those quantities, and the greatest number that will divide them all is called the greatest common measure. Thus: 7 is a common measure of 63 and 49, and it is also the greatest common measure, for no larger number will divide both evenly.

When any quantity is measured evenly by two or more others, it is called a common multiple of them. Thus: 24 is a common multiple of 2, 3, 4, 6, 8 and 12.

A number which can be divided into two equal integral parts is called an *even number*, and one which cannot be so divided is called an *odd number*. Hence all numbers of the series 2, 4, 6, 8, 10, 12, &c., are even, while those of the series 1, 3, 5, 7, 9, 11, &c., are odd. Hence the sum of any number of even quantities is even; also, the sum of any even number of odd quantities is even; but the sum of any odd number of odd quantities is odd. This principle is of great use in checking additions.

A prime number is one which has no integral factors except itself and unity; a composite number is one that has integral factors greater than unity, and numbers which have no common factor greater than unity are said to be *prime to each other*. Of the first kind are 1, 2, 3, 5, 7, 11, &c., of the second, 4, 6, 8, 9, 10, 12, &c.; also, 2 and 7 are prime to each other, and so are 6 and 7.

If one quantity measure another it will measure any multiple of it. Thus: since 3 measures 6, it will also measure 12, 18, 24, &c., because it is a factor of all these.

If one quantity measure two or more others, it will also measure their sum and difference, and also the sum and difference of any

multiples of them, because it measures them when they are taken separately.

Hence, if one number divide the whole of another number, and also one part of it, it will divide the other part too. Thus: 6 divides 24 and 18, and so the other part, 6; 9 divides 45 and 27, and also the remainder, 18. Also, if a number be composed of several parts, each of which has a common factor, that factor will also measure their sum. Thus: 9 measures 18, 27, and 36, and their sum, 81.

From these principles we can deduce a rule for finding the greatest common measure of two or more quantities.

R U L E .

Divide the greater by the less, and then the less by the remainder, until nothing is left, and the last divisor will be the greatest common measure.

E X A M P L E .

2145	3471
1326	2145
819	1326
507	819
312	507
195	312
117	195
78	117
39	78
	78

A concise form of the work is exhibited in the margin. The quotients are omitted as unnecessary. The last divisor, 39, is the G. C. M., as may be proved by trial. If it is required to find the G. C. M. of more than two numbers, first find the G. C. M. of two of them, and then the G. C. M. of that and another, and so on.

E X E R C I S E S .

Find the G. C. M. of the following quantities:

- | | |
|-----------------------|--------------|
| 1. 247 and 323. | Ans. 19. |
| 2. 532 and 1274. | Ans. 14. |
| 3. 741 and 1273. | Ans. 19. |
| 4. 10416 and 25761 | Ans. 93. |
| 5. 468 and 1266. | Ans. 6. |
| 6. 285714 and 999999. | Ans. 142857. |
| 7. 15863 and 21489. | Ans. 29. |
| 8. 8280 and 11385. | Ans. 1035. |
| 9. 17222 and 32943. | Ans. 79. |
| 10. 19752 and 69132. | Ans. 9876. |

We may often find the G. C. M. by inspection. For example, in exercise 5, we see that 2 will measure both quantities (Art. 13), for both are even, and also that 3 will measure both, because it measures the sum of the digits (Art. 16.)

The least common multiple of two or more numbers is the smallest number that is divisible by all of them. Thus: 48 is a common multiple of 2, 3, 4, 6, 8 and 12, but 24 is the *least* common multiple of them.

It is plain that the least common multiple of quantities that have no common factor is their product. Thus: the L. C. M. of 5, 7, 6 is 210. But if the quantities have a common factor, that factor is to be taken only once. Thus: 96, 48, 24, are all common multiples of 2, 3, 4, 6, 8, 12, but the least of these, 24, contains only the factors 3 and 8, which are prime to each other, for 2, 3, 4, 6 are all contained in 12, and 8 and 12 have a common factor, 4, which being left out of one of them, 8, gives $2 \times 12 = 24$, or, being left out of the other, 12, gives $8 \times 3 = 24$. From this we derive the

R U L E :

$$\begin{array}{r}
 2 \dots 3 \dots 4 \dots 6 \dots 9 \dots 18 \dots 27 \dots 30 \\
 \hline
 2 \overline{) 4 \dots 18 \dots 27 \dots 30} \\
 2 \dots 9 \dots 27 \dots 15 \\
 \hline
 3 \overline{) 2 \dots 27 \dots 15} \\
 2 \dots 9 \dots 5 \\
 9 \\
 \hline
 45 \\
 2 \\
 \hline
 90 \\
 3 \\
 \hline
 270 \\
 2 \\
 \hline
 540
 \end{array}$$

Expunge all common factors and take the continued product of all the results and divisors. Thus, to find the L. C. M. of 2, 3, 4, 6, 9, 18, 27, 30, arrange them in a horizontal line, and as 2, 3, 6, 9 are all contained in 18, they may be omitted, as in the second line, then, as 2 is contained in 4, 18 and 30, it may be divided out, and as 9 in the third line is contained in 27, it may be omitted, as in the fourth line; and 27 and 15 being both divisible by 3, we obtain in the fifth line 2, 9, 5, all prime to each other, and the products of these and the divisors 3 and 2 is the L. C. M., 540.

EXERCISES

Find the L. C. M. of the following quantities :

1. 8, 12, 16, 24, 33. Ans. 528.
2. 35, 42, 45, 81, 100. Ans. 56700.
3. 2, 4, 8, 16, 32, 64, 128 Ans. 128.
4. 2, 3, 5, 7, 11. Ans. 2310.
5. 3, 9, 27, 81, 243, 729. Ans. 729.
6. 12, 16, 18, 30, 48. Ans. 720.
7. 3, 4, 5, 6, 7. Ans. 420.
8. 2, 3, 4, 5, 6, 7, 8, 9. Ans. 2520.
9. 2, 4, 7, 12, 16, 21, 56. Ans. 336.
10. 2, 9, 11, 33. Ans. 198.

EXAMPLES FOR PRACTICE.

1. What will 320 caps cost at \$7.50 each ? Ans. \$2400.
2. If you can purchase slates at 20 cents each ; how many can you buy for \$7.40 ? Ans. 37.
3. If you can walk 4 miles an hour ; how far can you go in 24 hours ? Ans. 96.
4. What will be the cost of 216 barrels of pork at \$7.50 per barrel ? Ans. \$1620.
5. How many sheep can be bought for \$560 at \$3.50 per head ? Ans. 160.
6. If 825 pounds of beef are consumed by a garrison in one day ; what will be the cost for 6 days at 11 cents per pound for beef ? Ans. \$544.50.
7. A farmer sold 185 acres of land at \$25 per acre, and received in payment 17 horses at \$70 each, and 12 cows at \$20 each ; how much remains due ? Ans. \$3195.
8. A merchant bought 120 yards of Canadian tweed at \$1.15 a yard ; 60 yards of flannel at 95 cents per yard, and 13 dozen pairs of gloves at 35 cents per pair ; what was the amount of his bill ? Ans. \$249.60.
9. At \$2 per gallon ; how much wine can be bought for \$84 ? Ans. 42 gals.
10. A boy had \$5.50, and he paid one dollar and five cents for a book ; how much had he left ? Ans. \$4.45.
11. What will 18 cords of wood cost at \$4.75 per cord ? Ans. \$85.50.

12. How many pounds of sugar can be bought for \$9.35, at 11 cents per pound ?
Ans. 85 lbs.

13. What will a jury of 12 men receive for coming from Kingston to Toronto at 10 cents a mile each ; the distance being 160 miles ?
Ans. \$192.

14. A grocer bought a hogshead of molasses at 32 cents per gallon ; but 18 gallons leaked out, and he sold the remainder at 55 cents per gallon ; did he make or lose, and how much ?
Ans. He gained \$4.59.

15. If a clerk's salary is \$600 a year, and his personal expenses \$320 ; how many years before he will be worth \$6600, if he has \$1000 at the present time ?
Ans. 20 years.

16. A speculator bought 200 bushels of apples for \$90, and sold the same for \$120 ; how much did he make per bushel ?
Ans. 15 cents.

17. A person sells 15 tons of hay at \$22 per ton, and receives in payment a carriage worth \$125, a cow worth \$45, a colt worth \$40, and the balance in cash ; how much money ought he to receive ?
Ans. \$120.

18. How many pounds of butter, at 20 cents per pound, must be given for 18 pounds of tea worth 75 cents per pound ?
Ans. $67\frac{1}{2}$ lbs.

19. A grocer bought 7 barrels of fish at \$18 per barrel ; but one barrel proved to be bad, which he sold for \$5 less than cost, and the remainder at an advance of \$3 per barrel ; did he gain or lose, and how much ?
Ans. Lost \$13.

20. A man bought a drove of cattle for \$18130, and after selling 84 of them at \$51 each, the rest stood him in \$43 each ; how many did he buy ?
Ans. 406.

21. What will 2 cwt. of cheese cost at $9\frac{1}{2}$ cents per pound ?
Ans. \$19 00.

22. A. is worth \$960, B. is worth five times as much as A., less \$600, and C. is worth three times as much as A. and B. and \$300 more ; what are B. and C. worth each, and how much are they all worth ?
Ans. B. \$4200 ; C. \$15780 ; all \$20940.

23. A boy bought a dozen knives at 15 cents each, and after selling half of them at the rate of \$2.22 per dozen, he lost three, and sold the balance at 25 cents each ; did he make or lose, and how much ?
Ans. Gained 6 cents.

24. A labourer bought a coat worth \$16, a vest worth \$3, and a

pair of pants worth \$5.50 ; how many days had he to work to pay for his suit ; his services being worth 50 cents per day ?

Ans. 49 days.

25. What will 14 bushels of clover seed cost at $12\frac{1}{2}$ cents per pound ?

Ans. \$105.

26. A farmer sold a load of oats weighing 1836 pounds, at 30 cents per bushel ; how much did he receive for the same ?

Ans. \$16.20.

27. A produce dealer bought at one time, one load of wheat weighing 3240 pounds, at \$1.05 per bushel ; one load of barley weighing 2400 pounds, at 85 cents per bushel ; one load of rye weighing 2800 pounds, at 65 cents per bushel ; two loads of pease, each 2400 pounds, at 68 cents per bushel ; three loads of buckwheat, each weighing 1400, at $55\frac{1}{2}$ cents per bushel ; and a quantity of oats weighing 578 pounds, at 33 cents per bushel ; what had he to pay for the whole ?

Ans. \$250.15 $\frac{1}{2}$.

28. A farmer has 12 sheep worth \$3.50 each ; 9 pigs worth \$4.65 each ; one cow worth \$35, and a fine horse valued at \$150. He exchanges them with his neighbour for a yoke of oxen worth \$75 ; two lambs worth \$1.925 each ; a carriage worth \$100, and takes the balance in calves at \$4.50 ; how many calves does he receive ?

Ans. 20.

29. A and B sat down to count their money, and found that they had together \$225, but A had \$15 more than B ; how much had each ?

Ans. A \$120, B \$105.

30. A miller bought 250 bushels of oats for \$85 and sold 225 bushels for \$70 ; what did the remainder cost him per bushel ?

Ans. 60c.

31. A widow lady has a farm valued at \$6720 ; also three houses, worth \$12530, \$11324, and \$9875. She has a daughter and two sons. To the daughter she gives one-fourth the value of the farm, and one-third the value of the houses, and then divides the remainder equally among the boys, how much did each receive ?

Ans. daughter \$12923, each son, \$13763.

32. A man went into business with a capital of \$1500 ; the first year he gained \$800, the second year \$950, the third year \$700, and the fourth year 625, when he invested the whole in a cargo of tea and doubled his money ; what was he then worth.

Ans. \$9150.

33. A boy paid out 30 cents for apples, at the rate of 6 for 3 cents ; how many apples did he purchase ?

Ans. 60.

34. A schoolboy bought 12 oranges at 3 cents each, and sold them for 12 cents more than he paid for them; how much did he sell them at each? Ans. 4c.

35. A clerk's income is \$2698 a year, and his expenses \$4.50 per day; how much will he save in two years? Ans. \$2111.

36. A speculator bought 200 acres of land at \$45 per acre, and afterwards sold 150 acres of it for \$11550; the balance he sold at a gain of \$5 per acre, and received in payment \$250 cash, and the balance in sheep at \$5 each; how many sheep did he receive? Ans. 450 sheep.

37. A butcher bought 9 calves for \$54, and 9 lambs for \$31.50; how much more did he pay for a calf than a lamb? Ans. \$2.50.

38. A farmer sold to a grocer 380 pounds of pork, at 7 cents per pound; 150 pounds of butter, at 17 cents per pound, and one cheese weighing 53 pounds, at 9 cents per pound; and received in payment 22 pounds of sugar, at the rate of 11 pounds for a dollar; 150 pounds of nails, at 6 cents per pound; 15 pounds of tea, at 65 cents per pound; one half-barrel of fish, at \$18 per barrel, and one suit of clothes worth \$27; did the farmer owe the grocer, or the grocer the farmer, and how much? Ans. the grocer owed the farmer 12 cents.

39. A milkman sold 120 quarts of milk, at 5 cents per quart, and took in payment, one pig worth \$1.50, and the balance in sheeting, at 10 cents per yard; how many yards did he receive? Ans. 45 yards.

40. How many pounds of cheese, at 9 cents per pound, must be given for 27 pounds of tea worth 80 cents per pound? Ans. 240.

FRACTIONS.

14.—VULGAR OR COMMON FRACTIONS.—When we have divided any number by a less, and find no remainder, the quotient is called an integer, or whole number. When we have divided any number by a less as far as possible, and find a remainder still to be divided, but less than the divisor, and therefore not actually divisible by it, we must have recourse to some method of indicating this. We have seen already that the conventional sign of division is this mark ($\div$); thus, $3 \div 4$ means that 3 is to be divided by 4, and this being impossible, we indicate the operation either as above or by writing the three in the place of the upper dot, and the 4 in the place of the lower, thus, $\frac{3}{4}$.

The nature of a fraction may be viewed in two ways. *First*, we may consider that a unit is divided into a certain number of equal parts and a certain number of these parts taken; or, *secondly*, that a number greater than unity is divided into certain equal parts, and *one* of these parts taken; thus, $\frac{3}{4}$ means either that a unit is divided into 4 equal parts and three of them taken, or that three is divided into 4 equal parts and one of them taken. For example, if a foot be divided into 4 equal parts, each of these parts will be 3 inches, and three of them will be nine inches; and since 3 feet make 36 inches, if we divide 3 feet into 4 equal parts, each of these parts will be 9 inches, and hence $\frac{3}{4}$ of 1 = $\frac{1}{4}$ of 3. The lower figure is called the denominator, because it shows the denomination or number of parts into which the unit is supposed to be divided, and the upper one is called the numerator, because it shows the number of those parts considered in any given question. When both are spoken of together they are called the *terms* of the fraction.

What may be considered the fundamental principle on which all the operations in fractions depend is this: that the form, but not the value of a fraction, is altered, if both the terms are either multiplied or divided by the same quantity. If we take the fraction $\frac{3}{4}$ and multiply its terms by 2, we get $\frac{6}{8}$. Now, the $\frac{1}{8}$ of a foot is an inch and-a-half, and therefore $\frac{6}{8}$ is 6 inches and 6 half-inches, or 9 inches; but we have seen that $\frac{3}{4}$ of a foot is 9 inches, therefore $\frac{3}{4}$ of a foot is the same as $\frac{6}{8}$ of a foot. So also $\frac{3}{4}$ of £1 and $\frac{6}{8}$ of £1 are both 15s. The same will hold good whatever the unit of measure may be, or whatever the fraction of that unit. Hence, universally the *form* of a fraction is altered if its terms be either multiplied or divided by the same number, but its *value* remains the same.

Again, if we multiply the numerator 3 by 2, but leave the denominator 4 unchanged, we obtain $\frac{6}{4}$, and, keeping to our first illustration, $\frac{6}{4}$ of a foot is 6 times three inches, or 18 inches, which is double of 9 inches, the value of $\frac{3}{4}$. We should have obtained the same result by taking $\frac{6}{8}$ and dividing its denominator by 2, without dividing its numerator. Hence, a fraction is multiplied by either multiplying its numerator or dividing its denominator. In like manner, if we take the fraction $\frac{6}{8}$ and divide its numerator by 2, we obtain $\frac{3}{8}$, and if we multiply the denominator of its equal $\frac{3}{4}$ by 2, we obtain the same result, $\frac{3}{8}$. Hence, $\frac{3}{8}$ is $\frac{1}{2}$ of $\frac{3}{4}$, and therefore a fraction is divided by either dividing its numerator or multiplying its denominator. These principles may also be referred to the obvious

fact that in dividing any quantity the greater the divisor the less the quotient, and the less the divisor the greater the quotient. As it is always desirable to have the smallest numbers possible to handle, let the operator observe this as a universal rule—*divide when you can*.

Fractions are classified in four different ways, according to four different circumstances.

I. They are divided into Proper and Improper Fractions.

A proper fraction is one whose numerator is less than its denominator. In strictness such alone is a fraction. An improper fraction is one whose numerator is greater than its denominator. Strictly this is not really a fraction, but only a certain quantity expressed in the fractional form.

II. Simple and Compound Fractions.

The term simple fraction, as opposed to compound fraction, expresses that the fraction is multiplied by unity alone, as $\frac{5}{8}$, which means either $\frac{5}{8}$ of 1 or $\frac{1}{8}$ of 5, or $\frac{5}{8} \times 1 = \frac{1}{8} \times 5$.

A compound fraction is one that is multiplied by some other quantity. A fraction is called compound if either multiplier or multiplicand, or both, be fractional. Thus: $\frac{3}{4}$ of $\frac{5}{6}$ and $\frac{7}{8}$ of 11 are both compound, and are written $\frac{3}{4} \times \frac{5}{6}$ and $\frac{7}{8} \times 11$.

III. Simple and Complex Fractions.

The term simple fraction, as opposed to complex fraction, means that there is only one division. Thus: $\frac{1}{1} \frac{5}{6}$ means that a single number, 15, is divided by a single number, 16.

A complex fraction is one of which either the numerator or denominator, or both, are fractional, that is, it indicates a division, when either the given product or given factor, or both, are fractional.

Thus: $\frac{3}{4} \div \frac{7}{11}$, or $\frac{\frac{3}{4}}{\frac{7}{11}}$ and $\frac{8}{\frac{5}{9}}$ and $\frac{\frac{1}{1} \frac{1}{2}}{7}$ are complex fractions and exhibit the only three possible forms.

IV. Vulgar, or Common, and Decimal Fractions.

Decimal fractions are those expressed with a denominator, 10, or a power of 10, *e. g.*, $\frac{7}{10}$, $\frac{19}{100}$, $\frac{21}{1000}$.

Any fraction not so expressed is called vulgar or common. Thus: $\frac{3}{4}$ would be called a common fraction, but its equivalent, $\frac{75}{100}$, would be called a decimal fraction, and is written .75, the denominator being omitted, but its existence being indicated by the mark (.), called the decimal point.

A mixed quantity is one expressed partly by a whole number and partly by a fraction, as $4\frac{7}{8}$, $12\frac{1}{2}$. This is not another kind of fraction, but simply another mode of writing an improper fraction, when the division indicated has been performed as far as possible. Thus: $\frac{39}{8}=4\frac{7}{8}$, and $\frac{25}{2}=12\frac{1}{2}$.

It is often said that there are six kinds of fractions—proper, improper, simple, compound, complex, and mixed. This is logically incorrect, for a proper fraction is simple, and a mixed quantity is an improper fraction in another form.

15.—OPERATIONS IN COMMON FRACTIONS.—From the principles laid down (Art. 21,) we can deduce rules for all the operations in fractions.

I. An improper fraction is reduced to a mixed quantity by performing the division indicated, as $\frac{217}{9}=24\frac{1}{9}$.

II. A mixed quantity is reduced to an improper fraction by multiplying the integral part by the denominator and adding in the numerator, as $12\frac{7}{8}=\frac{103}{8}$.

So also an integer may be expressed in the fractional form by writing 1 as a denominator, and multiplying the terms by whatever number will bring it to any required denomination. Thus: to reduce 7 to the same denomination as $\frac{5}{6}$, write $\frac{7}{1}$ and multiply the terms by 6, and the result, $\frac{42}{6}$, will be equivalent to the integer 7, and of the same form as $\frac{5}{6}$.

EXERCISES.

- | | |
|---|-----------------------------|
| 1. Express $\frac{441}{9}$ as a whole or mixed number. | Ans. 49. |
| 2. Express $\frac{87}{16}$ as a whole or mixed number. | Ans. $5\frac{7}{16}$. |
| 3. Express $\frac{781}{11}$ as a whole or mixed number. | Ans. 71. |
| 4. Express $\frac{129}{2}$ as a whole or mixed number. | Ans. $5\frac{1}{2}$. |
| 5. Express $\frac{19876}{3579}$ as a whole or mixed number. | Ans. $5\frac{1981}{3579}$. |
| 6. Express $\frac{859}{78}$ as a whole or mixed number. | Ans. $11\frac{1}{78}$. |
| 7. Express $\frac{365}{52}$ as a whole or mixed number. | Ans. $7\frac{1}{52}$. |
| 8. Express $\frac{89}{12}$ as a whole or mixed number. | Ans. $7\frac{5}{12}$. |
| 9. Express $\frac{1157}{13}$ as a whole or mixed number. | Ans. 89. |
| 10. Express $\frac{117}{11}$ as a whole or mixed number. | Ans. $10\frac{7}{11}$. |
| 11. Express $\frac{149}{63}$ as a whole or mixed number. | Ans. $2\frac{23}{63}$. |
| 12. Express $\frac{176}{9}$ as a whole or mixed number. | Ans. $19\frac{5}{9}$. |
| 13. Express $\frac{211}{17}$ as a whole or mixed number. | Ans. $12\frac{7}{17}$. |
| 14. Express $\frac{29}{7}$ as a whole or mixed number. | Ans. $4\frac{1}{7}$. |

15. Express $\frac{97}{4}$ as a whole or mixed number. Ans. $24\frac{1}{4}$.
16. Express $\frac{118}{21}$ as a whole or mixed number. Ans. $5\frac{13}{21}$.
17. Express $\frac{99}{31}$ as a whole or mixed number. Ans. $3\frac{6}{31}$.
18. Express $\frac{11}{2}$ as a whole or mixed number. Ans. $5\frac{1}{2}$.
19. Express $\frac{121}{4}$ as a whole or mixed number. Ans. $30\frac{1}{4}$.
20. Express $\frac{1331}{16}$ as a whole or mixed number. Ans. $83\frac{3}{16}$.
21. Express $\frac{181}{20}$ as a whole or mixed number. Ans. $9\frac{1}{20}$.
22. Express $27\frac{1}{2}$ as an improper fraction. Ans. $\frac{55}{2}$.
23. Express $66\frac{1}{9}$ as an improper fraction. Ans. $\frac{595}{9}$.
24. Express $15\frac{7}{9}$ as an improper fraction. Ans. $\frac{302}{9}$.
25. Express $7\frac{3}{4}$ as an improper fraction. Ans. $\frac{31}{4}$.
26. Express 49 as a fraction with the same denominator as $\frac{12}{13}$.
Ans. $\frac{637}{13}$.
27. Express 19s. as a fraction of £1. Ans. $\frac{19}{20}$.
28. Express 11 inches as a fraction of a foot. Ans. $\frac{11}{12}$.
29. Bring $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}$ to the same denomination.
Ans. $\frac{6}{12}, \frac{4}{12}, \frac{3}{12}, \frac{2}{12}, \frac{1}{12}$.
30. Express 11 as a fraction having the same denominator as $\frac{17}{701}$.
Ans. $\frac{7711}{701}$.

III. To reduce a fraction to its lowest terms or simplest form, divide the terms by their greatest common measure. This is often readily done by inspection, as $\frac{48}{72} = \frac{4}{6} = \frac{2}{3}$, but in such questions as $\frac{1092}{6552}$, the most secure and speedy method is to find the G. C. M. of the terms and divide them by it. Thus: the G. C. M. of the fraction $\frac{1092}{6552}$ is 1092, and the terms of the fraction divided by this give $\frac{1}{6}$, the simplest form.

EXERCISES.

1. Reduce $\frac{4536}{13608}$ to its lowest terms or simplest form. Ans. $\frac{1}{3}$.
2. Reduce $\frac{5971}{9383}$ to its lowest terms or simplest form. Ans. $\frac{7}{11}$.
3. Reduce $\frac{2920}{3285}$ to its lowest terms or simplest form. Ans. $\frac{8}{9}$.
4. Reduce $\frac{72000}{960000}$ to its lowest terms or simplest form.
Ans. $\frac{3}{40}$.
5. Reduce $\frac{3580}{4296}$ to its lowest terms or simplest form. Ans. $\frac{5}{6}$.
6. Reduce $\frac{87615}{175230}$ to its lowest terms or simplest form.
Ans. $\frac{1}{2}$.
7. Reduce $\frac{987}{12831}$ to its lowest terms or simplest form.
Ans. $\frac{1}{13}$.
8. Reduce $\frac{3333}{4444}$ to its lowest terms or simplest form. Ans. $\frac{3}{4}$.
9. Reduce $\frac{5034}{6712}$ to its lowest terms or simplest form. Ans. $\frac{3}{4}$.

10. Reduce $\frac{13594}{20391}$ to its lowest terms or simplest form. Ans. $\frac{2}{3}$.
11. Reduce $\frac{33595}{114223}$ to its lowest terms or simplest form. Ans. $\frac{5}{17}$.
12. Reduce $\frac{6935}{7300}$ to its lowest terms or simplest form. Ans. $\frac{19}{20}$.
13. Reduce $\frac{571428}{1999998}$ to its lowest terms or simplest form. Ans. $\frac{2}{7}$.
14. Reduce $\frac{31185}{50457}$ to its lowest terms or simplest form. Ans. $\frac{945}{1529}$.
15. Reduce $\frac{1638}{2106}$ to its lowest terms or simplest form. Ans. $\frac{7}{9}$.
16. Reduce $\frac{1827}{3045}$ to its lowest terms or simplest form. Ans. $\frac{3}{5}$.
17. Reduce $\frac{272}{425}$ to its lowest terms or simplest form. Ans. $\frac{16}{25}$.
18. Reduce $\frac{873}{3395}$ to its lowest terms or simplest form. Ans. $\frac{9}{55}$.
19. Reduce $\frac{41472}{96768}$ to its lowest terms or simplest form. Ans. $\frac{3}{7}$.
20. Reduce $\frac{63624}{69408}$ to its lowest terms or simplest form. Ans. $\frac{11}{12}$.
21. Reduce $\frac{789057010000}{1578114020000}$ to its lowest terms or simplest form. Ans. $\frac{1}{2}$.

IV. To multiply one fraction by another, multiply numerator by numerator and denominator by denominator.

Thus: $\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$. To illustrate that $\frac{1}{3}$ of $\frac{1}{3}$ is $\frac{1}{9}$, take a line and let it be divided into 3 parts, and each of those again into 3 parts, as in the margin, we find that the result is 9 parts, each, of course, being $\frac{1}{9}$ of the unit.

We have seen that a fraction is multiplied by multiplying the numerator or dividing the denominator. Now, if it were required to multiply $\frac{3}{4}$ by $\frac{5}{7}$, we could not divide the denominator, as 5 is not contained in 4, and therefore we multiply the numerator and obtain $\frac{15}{4}$, but we have multiplied by a quantity equal to 7 times the given one, and therefore we must divide the product by 7, *i. e.* (Art. 21,) we must multiply the denominator 4 by 7, which gives $\frac{15}{28}$ for the correct product.

EXERCISES.

1. Multiply $\frac{5}{12}$ by $\frac{13}{17}$? Ans. $\frac{65}{204}$.
2. What is the product of $\frac{3}{4}$ by $\frac{11}{13}$? Ans. $\frac{33}{52}$.
3. What is the product of $\frac{7}{12}$ by $\frac{5}{6}$? Ans. $\frac{35}{72}$.
4. What is the product of $\frac{5}{6}$ by $\frac{13}{14}$? Ans. $\frac{65}{84}$.
5. What is the product of $\frac{1}{2}$ by $\frac{19}{20}$? Ans. $\frac{19}{40}$.

6. What is the product of $\frac{7}{8}$ by $\frac{9}{10}$? Ans. $\frac{63}{80}$.
 7. What is the product of $\frac{99}{100}$ by $\frac{7}{10}$? Ans. $\frac{693}{1000}$.
 8. What is the product of $\frac{36}{55}$ by $\frac{7}{11}$? Ans. $\frac{252}{605}$.
 9. What is the product of $\frac{8}{9}$ by $\frac{4}{5}$? Ans. $\frac{32}{45}$.
 10. What is the product of $\frac{12}{13}$ by $\frac{7}{11}$? Ans. $\frac{84}{143}$.

When the product has been obtained it should be reduced to its lowest terms. Thus: the product of $\frac{7}{11}$ by $\frac{11}{13}$ is $\frac{77}{143}$, the terms of which are both divisible by 11, and so we get the equivalent fraction $\frac{7}{13}$. But we might as well have divided by 11 before multiplying, for by this method we should at once have found the fraction in its simplest form, viz., $\frac{7}{13}$. In the same manner any number or numbers which are factors of both numerator and denominator, may be omitted in the operation. This we call cancelling in preference to the excessively awkward term "cancellation." This method will be clearly seen in exercise 11.

If either the multiplier or multiplicand be a mixed quantity, it must be reduced to an improper fraction before the multiplication is performed. Thus: $8\frac{3}{4} \times 5\frac{5}{6} = \frac{35}{4} \times \frac{35}{6} = \frac{1225}{24} = 51\frac{1}{24}$.

11. What fraction is equal to $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$? Ans. $\frac{1}{9}$.
 12. What quantity is equal to $12\frac{1}{2}$ multiplied by $7\frac{5}{6}$? Ans. $97\frac{11}{12}$.
 13. What quantity is equal to $19\frac{1}{8}$ multiplied by $1\frac{15}{17}$? Ans. 36.
 14. What is the value of $\frac{5}{6}$ of $\frac{4}{5}$ of $\frac{18}{23}$ of $\frac{1}{8}$? Ans. $\frac{2}{46}$.
 15. What is the value of $\frac{1}{3}$ of $\frac{5}{7}$ of $\frac{8}{9}$ of $\frac{11}{13}$? Ans. $\frac{440}{2457}$.
 16. What is the product of $27\frac{5}{8}$ by $3\frac{8}{9}$? Ans. $107\frac{31}{2}$.
 17. What is the product of $\frac{18}{19}$ by $\frac{19}{20}$? Ans. $\frac{9}{10}$.
 18. What is the product of $5\frac{1}{2}$ by $5\frac{1}{2}$? Ans. $30\frac{1}{4}$.
 19. Find the square and cube of $\frac{17}{22}$? Ans. $\frac{289}{484}$ and $\frac{4913}{10648}$.
 20. What is the cube of $\frac{39}{40}$? Ans. $\frac{59319}{64000}$.
 21. Multiply 27 by $\frac{1}{27}$? Ans. 1.

V.—DIVISION OF FRACTIONS.

To divide one fraction by another, multiply by the reciprocal of the divisor; or, in other words, invert the divisor and multiply. In the language of science, the reciprocal of a fraction is the fraction with its terms inverted. Thus: $\frac{8}{7}$ is the reciprocal of $\frac{7}{8}$; $\frac{4}{3}$ of $\frac{3}{4}$. To find the reciprocal of a whole number, we must first

represent it as having a denominator 1,—thus $4 = \frac{4}{1}$; $6 = \frac{6}{1}$, and therefore the reciprocals are $\frac{1}{4}$ and $\frac{1}{6}$. The rule for division may be proved in two ways:

FIRST PROOF.—Let it be required to divide $\frac{7}{11}$ by $\frac{5}{6}$. If we had been required to divide by the whole number 5, we should either have divided (Art. 14,) the numerator, or multiplied the denominator,—as the numerator is not divisible by 5, we multiply the denominator, and obtain $\frac{7}{55}$; but we have divided by a quantity equal to six times the given one, and therefore, to compensate, we must multiply the result by 6, which gives $\frac{42}{55}$.

SECOND PROOF.—Write the question in the complex form— $\frac{\frac{7}{11}}{\frac{5}{6}}$, then (Art. 14,) multiply both terms by 11, and $\frac{7}{\frac{5}{6}}$ is obtained; and again multiply the terms by 6, and $\frac{42}{5}$ is the result as before.—The two operations are virtually the same, though exhibited in different forms, and both are equivalent to the technical rule, “Invert the divisor and multiply.”

Mixed quantities must be reduced to improper fractions as in multiplication. The expressions *multiplication* and *division*, as applied to fractions, are extensions of the ordinary meanings of those terms, for in their original meaning, the former implies increase, and the latter decrease; but when two proper fractions are multiplied together, the product is less than either of the factors, and when one proper fraction is divided by another, the quotient is greater than either the divisor or dividend. This will be seen by the annexed examples:

$\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$. But $\frac{3}{4} = \frac{24}{32}$ and $\frac{7}{8} = \frac{28}{32}$, both greater than $\frac{21}{32}$.
Also, $\frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{4}{3} = \frac{28}{24}$. But $\frac{7}{8} = \frac{21}{24}$ and $\frac{3}{4} = \frac{18}{24}$, both less than $\frac{28}{24}$.

If two fractions have a common denominator, their quotient is the quotient of their numerators. We have placed multiplication and division of fractions before addition and subtraction, because, as in whole numbers, multiplication and division are deduced from addition and subtraction, so conversely in fractions, addition and subtraction are to be deduced from multiplication and division, for a fraction is produced by division, and the multiplication of a fraction is merely the repeating of the divided unit a certain number of times. Thus: $\frac{1}{8}$ is a unit divided into 8 equal parts, and $\frac{7}{8}$ is that fraction repeated 7 times.

EXERCISES.

1. Divide $\frac{3}{11}$ by $\frac{2}{3}$; $\frac{3}{11} \div \frac{2}{3} = \frac{3}{11} \times \frac{3}{2}$. Ans. $\frac{9}{22}$
2. What is the quotient of $\frac{1\frac{3}{4}}$ divided by $\frac{1\frac{3}{5}}$? Ans. $\frac{1\frac{5}{4}}{1\frac{1}{4}}$
3. What is the quotient of $\frac{2\frac{3}{2}}$ divided by $\frac{1\frac{1}{3}}{5\frac{3}{5}}$? Ans. $\frac{2\frac{4}{8}}{4\frac{8}{6}}$
4. What is the quotient of $\frac{2\frac{9}{4}}$ divided by $\frac{2\frac{9}{3}}$? Ans. $\frac{1\frac{1}{5}}{1\frac{1}{5}}$
5. What is the quotient of $\frac{1\frac{1}{2}}{4\frac{5}{6}}$ divided by $\frac{2\frac{3}{7}}$? Ans. $\frac{4\frac{5}{8}}{8\frac{5}{2}}$
6. What is the quotient of 36 divided by $19\frac{1}{8}$? Ans. $1\frac{1\frac{5}{7}}{1\frac{5}{7}}$
7. What is the quotient of $3\frac{5}{9}$ divided by $2\frac{5}{8}$? Ans. $1\frac{6\frac{7}{9}}{1\frac{8}{9}}$
8. What is the quotient of $4\frac{1}{2}$ divided by 15? Ans. $\frac{3}{10}$
9. What is the quotient of $\frac{5\frac{0}{1}}{7}$ divided by $2\frac{1\frac{7}{8}}$? Ans. $\frac{9\frac{0}{0}}{9\frac{0}{1}}$
10. What is the quotient of $75\frac{7}{10}$ divided by 9? Ans. $8\frac{3\frac{7}{0}}{9\frac{0}{0}}$
11. What is the quotient of $6\frac{1\frac{9}{2}}{1}$ divided by $9\frac{2}{3}$? Ans. $\frac{5}{7}$
12. What is the quotient of $5\frac{4}{7}$ divided by $8\frac{7}{12}$? Ans. $\frac{4\frac{6}{8}}{7\frac{2}{1}}$
13. Divide the product of $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{9}$ by the product of $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{8}{11}$? Ans. $\frac{1\frac{1}{9}}{9} = 1\frac{2}{9}$
14. What is the quotient of $\frac{7}{11}$ of $\frac{1\frac{1}{3}}{8} \div \frac{3}{8}$ of $\frac{1\frac{1}{3}}$ of $\frac{7}{11} \div \frac{9}{10}$ of $\frac{5}{8}$? Ans. $4\frac{2\frac{0}{7}}{2\frac{0}{7}}$
15. How many $\frac{1}{34}$ are there in $\frac{5}{19}$? Ans. $8\frac{1\frac{0}{9}}{1\frac{0}{9}}$
16. What is the value of $\frac{5}{6}$ of $\frac{7}{8} \div \frac{3}{4}$ of $\frac{1\frac{1}{2}}$? Ans. $1\frac{2\frac{3}{3}}{3\frac{3}{3}}$
17. Divide 27 by $\frac{1}{27}$? Ans. 729.

Hence, any quantity divided by its reciprocal gives the square of that number, and exercise 21, of multiplication, shows that any quantity multiplied by its own reciprocal gives unity.

18. Divide $\frac{8\frac{0}{561}}$ by $\frac{4}{9}$, and the quotient by $\frac{3}{17}$? Ans. $1\frac{9}{11}$
19. Divide $\frac{4}{7}$ by $\frac{7}{11}$, and the quotient by $\frac{2\frac{2}{3}}$? Ans. $\frac{4\frac{6}{4}}{4\frac{6}{4}}$
20. Divide $\frac{7\frac{6}{7}}$ by $\frac{1\frac{3}{7}}$? Ans. $3\frac{4\frac{9}{10}}{10\frac{2}{7}}$
21. Divide $\frac{1\frac{1}{8}}$ by $\frac{4\frac{4}{36}}$? Ans. $\frac{1}{2}$

VI.—ADDITION OF FRACTIONS.

We have seen that no quantities can be added together except they are in the same denomination. We can add $\frac{4}{7}$, $\frac{3}{7}$, $\frac{5}{7}$ and $\frac{11}{7}$, as they are all of the same denomination, *sevenths*, and we find $2\frac{3}{7}$. We can easily see that to add $\frac{3}{4}$ and $\frac{7}{8}$, we have only to alter the form of $\frac{3}{4}$ to $\frac{6}{8}$, and we have both fractions of the same denomination, and therefore can add them, $-\frac{6}{8} + \frac{7}{8} = \frac{13}{8}$. So, also, $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{5}{6} + \frac{7}{12} = \frac{6}{12} + \frac{8}{12} + \frac{9}{12} + \frac{10}{12} + \frac{7}{12} = \frac{40}{12} = 3\frac{4}{3}$. But we cannot always tell thus by inspection, and therefore must be guided by some rule. To find the value of $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{2}{9} + \frac{7}{12}$.

By Art. 13 we find the L. C. M. of 4, 6, 8, 9, 12 to be 72, and the rest of the common operation is equivalent to multiplying the terms of each fraction by 72. Thus: if the terms of $\frac{3}{4}$ be both multiplied by 72, we get $\frac{3 \times 18}{4 \times 18} = \frac{54}{72}$, but we might as well have divided 72 by 4 before multiplying, and, to balance that, have multiplied the numerator 3, not by 72, but by the fourth part of 72, viz., 18, giving $\frac{54}{72}$, as the following scheme will show:— $\frac{3 \times 18}{4 \times 18} = \frac{3 \times 18 \times 4}{4 \times 18 \times 4} = \frac{3 \times 72}{4 \times 72} = \frac{54}{72}$. The other fractions being altered in the same manner, we get $\frac{54}{72} + \frac{60}{72} + \frac{63}{72} + \frac{16}{72} + \frac{42}{72}$, and as these are now all of the same denomination, though not altered in value, we can add them, and we find $\frac{3}{4} + \frac{5}{6} + \frac{7}{8} + \frac{2}{9} + \frac{1}{12} = \frac{54}{72} + \frac{60}{72} + \frac{63}{72} + \frac{16}{72} + \frac{42}{72} = \frac{235}{12}$. Hence the

R U L E .

Find the L. C. M. of all the denominators, which will be the common denominator; divide this common multiple by each denominator, and multiply the quotient by each numerator in succession for new numerators; add all these new numerators together, and place the common denominator below the sum, and the fraction thus obtained will be the sum of the given fractions. If the numerator, thus obtained, be greater than the denominator, the resulting fraction may be reduced to a whole or a mixed number by division.

EXERCISES.

1. Express $\frac{1}{15} + \frac{2}{15} + \frac{4}{15} + \frac{7}{15}$ as a single fraction? Ans. $\frac{14}{3}$.
2. Find the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$? Ans. $2\frac{3}{4}$.
3. Add together $4\frac{7}{8}$, $1\frac{17}{18}$, $2\frac{23}{24}$, $3\frac{29}{27}$ and $5\frac{1}{12}$? Ans. $18\frac{65}{108}$.
4. What fraction is equal to $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64}$? Ans. $\frac{63}{64}$.
5. What fraction is equal to $1\frac{1}{2} + 2\frac{2}{3} + 3\frac{3}{4} + 4\frac{4}{5} + 5\frac{5}{6} + 6\frac{6}{7}$?
Ans. $25\frac{171}{420}$.
6. Express $\frac{1}{2}$ of $\frac{3}{4} + \frac{2}{3}$ of $\frac{5}{6} + \frac{3}{4}$ of $\frac{1}{2}$ as a single fraction?
Ans. $\frac{47}{36} = 1\frac{11}{36}$.
7. Find the sum of $1\frac{14}{15}$, $8\frac{2}{5}$, $3\frac{9}{25}$ and $4\frac{3}{5}$? Ans. $18\frac{31}{64}$.
8. Find the sum of $\frac{1}{8}$ of $\frac{4}{5} + \frac{1}{12}$ of $\frac{4}{7} + \frac{3}{8}$ of $\frac{8}{9}$? Ans. $\frac{101}{180}$.
9. What single fraction is equivalent to $\frac{1}{2}$ of $\frac{1}{3} + \frac{1}{3}$ of $\frac{1}{4} + \frac{1}{4}$ of $\frac{1}{5}$?
Ans. $\frac{3}{10}$.
10. What single fraction is equivalent to $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{1}{5} + \frac{1}{5}$ of $\frac{2}{3}$ of $\frac{1}{4} + \frac{7}{8}$ of $\frac{1}{3}$ of $\frac{1}{7}$?
Ans. $\frac{3}{16}$.
11. What single fraction is equivalent to $\frac{3}{5}$ of $\frac{5}{6}$ of $\frac{7}{8} + \frac{2}{3}$ of $\frac{6}{8}$ of $\frac{5}{7}$?
Ans. $\frac{89}{112}$.

12. Simplify $\frac{4\frac{1}{2} + 8\frac{2}{5}}{4\frac{1}{5} + 8\frac{1}{10}}$? Ans. $1\frac{1}{7\frac{1}{3}0}$.
13. Find a single fraction equivalent to $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{2}{3} + \frac{5}{5}$ of $\frac{7}{8}$? Ans. $\frac{289}{60}$.
14. Divide the sum of $\frac{5}{11}$ and $\frac{3}{7}$ by the sum of $\frac{4}{5}$ and $\frac{3}{7}$? Ans. $\frac{340}{473}$.
15. Simplify $\frac{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{128}}{\frac{1}{2} + \frac{3}{4} + \frac{7}{8} + \frac{15}{16} + \frac{31}{32} + \frac{63}{64} + \frac{127}{128}}$? Ans. $\frac{127}{769}$.
16. Simplify $\frac{\frac{1}{3} + \frac{4}{7} + \frac{21}{32}}{\frac{2}{5} + \frac{23}{36} + \frac{3}{5}}$? Ans. $\frac{3147}{3304}$.

VII.—SUBTRACTION OF FRACTIONS.

What we have said of addition enables us to give at once the

RULE FOR SUBTRACTION.

Reduce the given fractions, if necessary, to new ones having a common denominator, as in addition, and subtract the numerator of the less from that of the greater, and place the common denominator below the remainder, and the resulting fraction will be the difference between the given fractions.

EXAMPLES.—(1.) To subtract $\frac{5}{11}$ from $\frac{17}{11}$. Here the denominations being the same, we can subtract at once, and find the difference to be $\frac{2}{11}$. (2.) To find the value of $\frac{8}{9} - \frac{6}{7}$. These fractions brought to a common denominator, as in addition, become $\frac{56}{63}$ and $\frac{54}{63}$, and therefore the difference is $\frac{2}{63}$. (3.) To find the excess of $12\frac{1}{2}$ above $7\frac{5}{8}$, we find new fractions with a common denominator, viz., $\frac{8}{24}$ and $\frac{15}{24}$, and we write $12\frac{8}{24} - 7\frac{15}{24}$. Now we are required first to subtract $\frac{15}{24}$ from $\frac{8}{24}$, but as we cannot do this directly, we take one of the 12 preceding units, and call it $\frac{24}{24}$, (for $\frac{24}{24} = 1$), then $\frac{24}{24} + \frac{8}{24} = \frac{32}{24}$, and $\frac{32}{24} - \frac{15}{24} = \frac{17}{24}$, then we subtract the 7 from the remaining 11; or, as in simple subtraction, 8 from 12, and we find the total excess to be $4\frac{17}{24}$. In practice it is most convenient to subtract 15 from 24, and add 8; thus $24 - 15 = 9$, and $9 + 8 = 17$, and the answer is $4\frac{17}{24}$.

EXERCISES.

1. $\frac{5}{6} - \frac{3}{6} = \frac{2}{6} = \frac{1}{3}$. 2. $\frac{9}{11} - \frac{5}{11} = \frac{4}{11}$. 3. $\frac{10}{13} - \frac{3}{13} = \frac{7}{13}$. 4. $\frac{7}{8} - \frac{19}{24} = \frac{1}{12}$.
5. What is the difference between $\frac{3}{8}$ and $\frac{29}{32}$? Ans. $\frac{17}{32}$.
6. What is the difference between $\frac{449}{756}$ and $\frac{65}{189}$? Ans. $\frac{1}{4}$.
7. What is the difference between $\frac{59}{60}$ and $\frac{8}{15}$? Ans. $\frac{9}{20}$.
8. What is the excess of $20\frac{4}{6}$ above $9\frac{19}{22}$? Ans. $10\frac{115}{98}$.

9. From $5\frac{2}{3}\frac{3}{2}$ take $3\frac{2}{7}$? Ans. $2\frac{9}{2}\frac{7}{4}$.
10. What is the difference between $5\frac{7}{12}$ and $6\frac{4}{2}\frac{3}{5}\frac{1}{20}$? Ans. $\frac{1}{2}\frac{4}{5}\frac{8}{20}$.
11. What is the value of $\frac{3}{4} + \frac{5}{6} - \frac{7}{8} + \frac{7}{12} - \frac{1}{2}$? Ans. $\frac{1}{2}\frac{9}{4}$.
12. What is the difference between $100\frac{3}{19}$ and $50\frac{4}{4}\frac{0}{1}$? Ans. $49\frac{1}{7}\frac{4}{7}\frac{2}{9}$.
13. What is the difference between $\frac{1}{2}$ of $\frac{1}{3}$ and $\frac{1}{3}$ of $\frac{1}{2}$? Ans. 0
14. What is the difference between $\frac{3}{8}$ of $\frac{7}{10}$ and $\frac{5}{6}$ of $\frac{4}{9}$? Ans. $\frac{2}{2}\frac{3}{1}\frac{3}{60}$.
15. What is the value of $\frac{1}{2} + \frac{2}{3} - \frac{3}{4} - \frac{5}{6} + \frac{1}{12}$? Ans. $\frac{1}{2}$.

VIII.—DENOMINATE FRACTIONS.

Hitherto we have treated of fractions abstractly, and we must now apply the principles laid down to denominate numbers, and show how a fraction may be transformed from one denomination to another of the same kind *e. g.*, how a fraction of a shilling may be expressed as a fraction of a pound, and *vice versa*.

R U L E .

(1.) *Reduce the given quantity to the lowest denomination which it expresses.* (2.) *Reduce the unit in the terms of which it is to be expressed to the same denomination, and* (3.) *make the former the numerator and the latter the denominator, and the fraction will be expressed in the required terms.*

E X A M P L E S .

1. To express 16s. 8d. as a fraction of £1: Reducing 16s. 8d. to pence, we get 200, and reducing £1 to pence, we get 240, and, therefore, 16s. 8d. is $\text{£}\frac{200}{240}$, which, in its lowest terms, is $\text{£}\frac{5}{6}$.

2. In like manner, to express 17s. 6d. as a fraction of £1, we reduce 17s. 6d. to pence, and find 210, which, divided by 240, the number of pence in £1, gives $\text{£}\frac{210}{240} = \text{£}\frac{7}{8}$.

3. So also, 12s. 6d., expressed as a fraction of £1, is $\text{£}\frac{5}{8}$.

4. 18s. 4d., expressed as a fraction of £1, is $\text{£}\frac{1}{1}\frac{1}{2}$.

5. 12s. 6d., expressed as a fraction of £1, is $\text{£}\frac{5}{8}$.

6. 13s. 4d., expressed as a fraction of £1, is $\text{£}\frac{2}{3}$.

E X E R C I S E S .

1. Express 3s. 9d. as a fraction of £1. Ans. $\text{£}\frac{5}{16}$.
2. Express 4s. 4d. as a fraction of £1. Ans. $\text{£}\frac{1}{6}\frac{3}{10}$.
3. Express $4\frac{1}{2}$ d. as a fraction of 1s. Ans. $\frac{3}{8}$ s.
4. Express 1 oz. troy as a fraction of 1 lb. Ans. $\frac{1}{12}$.

5. Express 40 lbs. as a fraction of 1 cwt. Ans. $\frac{2}{5}$ cwt.
6. Express 50 lbs. as a fraction of 1 ton. Ans. $\frac{1}{40}$ ton.
7. Express 72 lbs. as a fraction of 1 cwt. Ans. $\frac{18}{25}$ cwt.
8. A day is 23 hours, 56 minutes, 48 seconds, nearly; what fraction of this will 7 hours be? Ans. $\frac{525}{1796}$.
9. Express 95 square yards as a fraction of an acre. Ans. $\frac{19}{968}$.
10. Express 14 yards as a fraction of a mile. Ans. $\frac{7}{880}$.
11. What fraction of a year ($365\frac{1}{4}$ days) is one month (30 days)? Ans. $\frac{40}{487}$.
12. Express 100 yards as a fraction of a mile. Ans. $\frac{5}{88}$.
13. Express 45 cents as a fraction of a dollar. Ans. $\frac{9}{20}$.
14. Express 60 lbs. as a fraction of a cwt. Ans. $\frac{3}{5}$.
15. A man has an income of \$3610 a year and saves $\frac{3}{7}$ of it; how much does he spend? Ans. \$2062 $\frac{6}{7}$.

To find the value of a fraction in the denominations which the integer contains, reduce the numerator to the next lower denomination, and divide the result by the denominator; if there be a remainder, reduce to the next denomination, and divide again, and continue the same operation till there is either no remainder, or down to the lowest denomination by which the integer is counted. Thus, $\frac{6}{7}$ of £1 is 120 shillings divided by 7, which gives 17 shillings and 1 shilling, or 12 pence remainder, and $12 \div 7 = 1\frac{5}{7}$, so that $\text{£}\frac{6}{7} = 17$ shillings and $1\frac{5}{7}$ d.

EXERCISES.

1. What is the value of $\text{£}\frac{7}{12}$ Stg.? Ans. 11s. 8d.
2. What is the value of $\frac{9}{10}$ of a yard? Ans. 2 ft., $8\frac{2}{5}$ in.
3. What is the value of $\frac{13}{24}$ of a mile?
Ans. 4 fur., 13 rods, 1 yd., 2 ft., 6 in.
4. What is the value of $\frac{19}{20}$ of a shilling Stg.? Ans. $11\frac{2}{5}$ d.
5. What is the value of $\frac{4}{7}$ of a ton? Ans. 11 cwt., 1 qr., $17\frac{6}{7}$ lbs.
6. What is the value of $\frac{2}{3}$ lb. troy? Ans. 8 oz.
7. What is the value of $\frac{6}{13}$ of a shilling? Ans. $5\frac{7}{13}$ d.
8. What is the value of $\$ \frac{8}{9}$? Ans. $88\frac{8}{9}$ cts.
9. What is the value of $\frac{4}{5}$ of \$6? Ans. \$4.80.
10. What is the value of $\frac{17}{20}$ of \$8? Ans. \$6.80.

To change a fraction to one of a lower denomination, reduce the numerator to that denomination and divide by the denominator. Thus, to express $\text{£}\frac{7}{145}$ as a fraction of a shilling, reduce £7 to shillings, which gives 140 shillings, and $\frac{140}{145} = \frac{28}{29}$ of a shilling is obtained.

EXERCISES.

1. Express $\frac{7}{100}$ of a foot as a fraction of an inch. Ans. $\frac{2}{25}$.
2. Express $\frac{1}{120}$ of a cwt. as a fraction of a lb. Ans. $\frac{5}{6}$.
3. Express $\frac{1}{20}$ of a lb. as a fraction of an oz. Ans. $\frac{4}{5}$.
4. Express $\frac{3}{4}$ of $\frac{5}{12}$ of a yard as a fraction of a foot. Ans. $\frac{15}{16}$.
5. Express $\frac{4}{27}$ of a rod as a fraction of a yard. Ans. $\frac{2}{27}$.
6. Express $\frac{1}{2}$ of $\frac{1}{8}$ of an acre as a fraction of a rood. Ans. $\frac{1}{4}$.
7. Reduce $\frac{3}{26}$ cwt. to the fraction of a pound. Ans. $11\frac{7}{13}$ lb.
8. Reduce $\frac{1}{21}$ of a day to the fraction of a minute.
Ans. $68\frac{4}{7}$ min.
9. What part of a second is the *one-millionth* part of a day?
Ans. $\frac{54}{625}$ sec.
10. Reduce $\text{£}\frac{1}{36}$ to the fraction of a penny. Ans. $6\frac{2}{3}$ d.
11. Reduce $\frac{1}{24}$ of a pound avoirdupois to the fraction of an oz.
Ans. $\frac{2}{3}$ oz.

The reducing of a denominate fraction from one of a lower to one of a higher denomination being the converse of the last rule, we must perform the same operation on the denominator as was there performed on the numerator.

Thus, $\frac{5}{8}$ d. is $\text{£}\frac{1}{384}$, for $\text{£}\frac{5}{8 \times 12 \times 20} = \text{£}\frac{5}{1920} = \text{£}\frac{1}{384}$.

EXERCISES.

1. What part of 1 lb. troy is $\frac{3}{5}$ of a grain? Ans. $\frac{1}{5600}$.
2. What part of 4 days is $\frac{3}{4}$ of a minute? Ans. $\frac{1}{7680}$.
3. What part of 5 bushels is $\frac{2}{3}$ of $\frac{3}{4}$ of a pint? Ans. $\frac{1}{640}$.
4. What part of a rod is $2\frac{3}{4}$ of $\frac{7}{12}$ of an inch? Ans. $\frac{7}{864}$.
5. What part of 2 weeks is $\frac{5}{14}$ of a day? Ans. $\frac{5}{196}$.

DECIMAL FRACTIONS.

16.—We have seen already (Art. 3,) that every figure to the right is *one-tenth* the value it would have if removed one place to the left. Thus, resuming our former example, 8 standing alone means 8 *units*, but if we place another 8 after it, thus 88, it now means 8 *tens*, so that the last 8 is *one-tenth* of the first. Now, since the 8 to the right expresses units, another 8 placed to the right will express *eight-tenths* of the same unit, and another subjoined will express $\frac{8}{100}$ of the unit. Thus we see that the decimal notation is directly an extension of the Arabic. Hence arose the convenient mode of writing $8\frac{7}{10}$ in the form 8.7, by which is indicated that all

the figures before the decimal point (.) represent integers, and all after it fractions, each being *one-tenth* of what it would be if *one* place further to the left. Therefore 888.888 is *eight hundreds, eight tens, eight units,—eight-tenths, eight one-hundredths, and eight one-thousandths*; or, $\frac{8}{10} + \frac{8}{100} + \frac{8}{1000}$. These added will give $\frac{800}{1000} + \frac{80}{1000} + \frac{8}{1000}$, or $\frac{888}{1000}$, which, for brevity, is written .888, and may be read eight hundred and eighty-eight one-thousandths; or, as is usual, *point 888*, or *decimal 888*, but never properly eight hundred and eighty-eight. In the same manner as 80 means 8 tens and no units, so .08 means no tenths, but 8 hundredths, and .008 means no tenths, no hundredths, but eight one-thousandths, &c.—Hence we see that for every cipher in the denominator, which is always 10 of a power of 10, there must be a figure in the numerator when expressed decimally. Thus: $\frac{8}{1000}$ must be written decimally .008. From this we see that removing the decimal point one place to the right is the same as multiplying by 10, and removing it one place to the left is the same as dividing by 10; so, also, removing the point two places to the right is the same as multiplying by 100, and removing it two places to the left is the same as dividing by 100. This is the principle already laid down for the reduction of dollars to cents, and cents to dollars.

I.—REDUCTION OF COMMON FRACTIONS TO DECIMALS.—Let it now be required to express the common fraction $\frac{7}{8}$ as a decimal. We have seen (Art. 14,) that we may multiply the terms of any fraction by the same number without changing the value of the fraction. Let us then multiply the terms of $\frac{7}{8}$ by 1000, and we get $\frac{7000}{8000}$. On the same principle we can divide the terms by the same number without altering the value. Let us then divide by 8, and we get $\frac{625}{1000}$, where the denominator is a power of 10, and therefore the fraction is in the decimal form, and may be written .625, the denominator being omitted. But as it is not always apparent by what power of 10 we must multiply, so that when the terms are divided by the given denominator, that denominator may be transformed into 10 or a power of 10, *i. e.*, into 1 followed by a certain number of ciphers, we may as well add ciphers, one by one, as we proceed. This is exhibited in the annexed example. From these principles we can deduce a rule for reducing a common fraction to a decimal.

$$\begin{array}{r}
 8 \overline{) 500} \quad (0.625 \\
 \underline{48} \\
 20 \\
 \underline{16} \\
 40 \\
 \underline{40} \\
 0
 \end{array}$$

R U L E .

Divide the numerator, with a cipher or ciphers annexed, by the denominator. Thus $\frac{1}{16}$ will give, as in the margin, .6875. In the

$$\begin{array}{r}
 16 \overline{)110}(.6875 \\
 \underline{96} \\
 140 \\
 \underline{128} \\
 120 \\
 \underline{112} \\
 80 \\
 \underline{80} \\
 0
 \end{array}$$

examples given we find that the addition of three ciphers to the first, and four to the second, makes the numerator divisible by the denominator without remainder. Such fractions are called terminating decimals. From this we see that there are common fractions whose terms can be multiplied by such powers of 10 as will make the numerator divisible by the denominator without remainder, but it often happens that no power of ten will effect this, and that remainders occur which cannot be made divisible evenly by the denominator, by the addition of any

number of ciphers. Such fractions will never terminate, and therefore are called interminate, and the common fraction can never be expressed exactly in the decimal form, and all we can do is to make an approximation more or less close, according to the number of decimal places to which we carry it. Let us take the fraction $\frac{1}{9}$.—

$$\begin{array}{r}
 7 \overline{)10}(.142857 \\
 \underline{7} \\
 30 \\
 \underline{28} \\
 20 \\
 \underline{14} \\
 60 \\
 \underline{56} \\
 40 \\
 \underline{35} \\
 50 \\
 \underline{49} \\
 1
 \end{array}$$

First, 9 is not contained in 1, and therefore we place the decimal point in the quotient, and add a cipher to the numerator, and we find that 9 is contained *once* in 10, with a remainder 1,—annexing another cipher, we again obtain 1 in the quotient, and this will obviously continue *ad infinitum*.—This recurrence is marked by a dot or dash over the figure, thus: $\dot{.1}$ or $.1'$. If we express $\frac{1}{7}$ as a decimal, we find that after we have got six figures in the quotient, we have a remainder 1, the same as the original numerator, and therefore we should again obtain the same quotient .142857, and hence this is called a circulating or periodic decimal, and the first and last of the recurring figures are marked with a point or trait. Thus: $\dot{.1}4285\dot{7}$ or $.1'42857'$. Again, it often happens that some figures do not recur whilst others following them do, as in the annexed example, after we have got

five figures the 11500 which gave us the third figure 3, in the quoti-

$$\begin{array}{r}
 4111 \\
 33300 \overline{) 41110} \cdot 12345 \\
 \underline{33300} \\
 78100 \\
 \underline{66600} \\
 115000 \\
 \underline{99900} \\
 151000 \\
 \underline{133200} \\
 178000 \\
 \underline{165500} \\
 11500
 \end{array}$$

ent recurs, and by pursuing the division, we should find 345 recurring without end. When all the figures recur, the fraction is called a pure periodic decimal; when only some of them recur, it is called mixed, and the term repeater is applied when only one figure recurs, as

$$\frac{1}{9} = .1111, \text{ \&c.} = .\dot{1} \text{ or } \frac{7}{12} = .58333, \text{ \&c.} =$$

.583. Since the denominator is always 10, or a power of 10, and since 10 has no factors but 2 and 5, and therefore powers of 10 no factors but 2 and 5, or powers of these, it follows that no decimal will terminate except the denominator be expressed by either or both of these, or some power or product of them. Hence all terminating decimals are derived from common fractions having for denominator some figure of the series 2, 4, 8, 16, 32, &c., or 5, 25, 125, &c., or 10, 20, 40, 50, 60, 80, 100, &c.

Hence all terminating decimals are derived from common fractions having for denominator some figure of the series 2, 4, 8, 16, 32, &c., or 5, 25, 125, &c., or 10, 20, 40, 50, 60, 80, 100, &c.

EXERCISES.

- | | |
|---|---------------|
| 1. Reduce the common fraction $\frac{1}{4}$ to a decimal. | Ans. .25. |
| 2. Reduce the common fraction $\frac{1}{2}$ to a decimal. | Ans. .5. |
| 3. Reduce the common fraction $\frac{3}{4}$ to a decimal. | Ans. .75. |
| 4. Reduce the common fraction $\frac{1}{3}$ to a decimal. | Ans. .3. |
| 5. Reduce the common fraction $\frac{1}{9}$ to a decimal. | Ans. .1. |
| 6. Reduce the common fraction $\frac{1}{8}$ to a decimal. | Ans. .125. |
| 7. Reduce the common fraction $\frac{1}{6}$ to a decimal. | Ans. .16. |
| 8. Reduce the common fraction $\frac{1}{7}$ to a decimal. | Ans. .142857. |
| 9. Reduce the common fraction $\frac{1}{5}$ to a decimal. | Ans. .2. |
| 10. Reduce the common fraction $\frac{1}{10}$ to a decimal. | Ans. .1. |
| 11. Reduce the common fraction $\frac{1}{11}$ to a decimal. | Ans. .09. |
| 12. Reduce the common fraction $\frac{1}{12}$ to a decimal. | Ans. .083. |
| 13. Reduce the common fraction $\frac{2}{3}$ to a decimal. | Ans. .6. |
| 14. Reduce the common fraction $\frac{4}{5}$ to a decimal. | Ans. .8. |

15. Reduce the common fraction $\frac{5}{8}$ to a decimal. Ans. .83.
16. Reduce the common fraction $\frac{3}{8}$ to a decimal. Ans. .375.
17. Reduce the common fraction $\frac{5}{8}$ to a decimal. Ans. .625.
18. Reduce the common fraction $\frac{7}{8}$ to a decimal. Ans. .875.
19. Reduce the common fraction $\frac{4}{9}$ to a decimal. Ans. .4.
20. Reduce the common fraction $\frac{5}{7}$ to a decimal. Ans. .714285.
21. Reduce the common fraction $\frac{10}{11}$ to a decimal. Ans. .90.
22. Reduce the common fraction $\frac{11}{12}$ to a decimal. Ans. .916.
23. Reduce the common fraction $\frac{12}{13}$ to a decimal. Ans. .923076.
24. Reduce the common fraction $\frac{11}{14}$ to a decimal.
Ans. .7857142.
25. Reduce the common fraction $\frac{11}{16}$ to a decimal. Ans. .6875.
26. Reduce the common fraction $\frac{5}{6}$ to a decimal. Ans. .075.
27. Reduce the common fraction $\frac{13}{32}$ to a decimal. Ans. .34375.
28. Reduce the common fraction $\frac{5}{1025}$ to a decimal.
Ans. .00487804.
29. Reduce the common fraction $\frac{37}{9}$ to a decimal.
Ans. .4683544303797.
30. Reduce the common fraction $\frac{4}{909}$ to a decimal. Ans. .0044.
31. Reduce the common fraction $\frac{1}{49}$ to a decimal.
Ans. .020408163265306122448979591836734693877551.
32. Express $\frac{1}{99}$ decimally. Ans. .01.
33. Express $\frac{1}{999}$ decimally. Ans. .001.
34. Express $\frac{1}{9999}$ decimally. Ans. .0001.
35. Express $\frac{6}{10001}$ decimally. Ans. .00059994.

To reduce a denominate number to the form of a decimal fraction, *reduce it to the lowest denomination which it contains; reduce the integral unit to the same denomination, and divide the former by the latter.*

Thus, to express 18s. 4d. as a decimal of £1, we must reduce it to pence, the lowest denomination given, and divide it by 240, the number of pence in £1, which gives the fraction $\frac{220}{240} = \frac{11}{12}$, and this reduced to a decimal, gives .716 or £.916. In like manner 15s. 10½d. is reduced to half-pence, viz., 381, and the half-pence in £1 are 480, and $\frac{381}{480} = \frac{127}{160}$, which expressed decimally is .79375.

II.—REDUCTION OF DECIMALS TO COMMON FRACTIONS.—

To find the common fraction corresponding to any given decimal.—This involves three cases according as the fraction is a terminating decimal, a pure circulating decimal, or a mixed circulating decimal. The first case scarcely requires proof. We give it, however, in order to assist those unaccustomed to the algebraic notation, to understand more clearly the form of illustration used in the other cases.

Let us take the fraction .9375, and use d for decimal. We now write $d=.9375$, and multiplying both terms by 10000, we obtain $10000\ d=.9375$, and therefore $d=\frac{9375}{10000}$, which reduced to its lowest terms is $\frac{15}{16}$, the common fraction required. This is simply putting for denominator 1, followed by a cipher for each figure in the decimal.

$$\begin{array}{r} d=.666+ \\ \hline 9\ d=6 \end{array}$$

To find the value of a pure circulator, suppose $\dot{.6}$. Put $d=\dot{.6}$, or $d=.666$, and multiply by 10, which gives $10\ d=6.66$, and writing the former expression beneath, and subtracting, we get $9\ d=6$, and consequently $d=\frac{6}{9}$ or $\frac{2}{3}$, the common fraction sought.

$$\begin{array}{r} d=.7\dot{2} \\ 100\ d=72.7\dot{2} \\ \hline 99\ d=72 \end{array}$$

Let us now seek the vulgar fraction corresponding to $.7\dot{2}$. Put $d=.7\dot{2}$, multiply by 100, and subtract as before, and there results a remainder of $99\ d=72$, or $d=\frac{72}{99}=\frac{8}{11}$.

$$\begin{array}{r} d=.568\dot{1} \\ 10000\ d=5681.8\dot{1} \\ 100\ d=56.8\dot{1} \\ \hline 9900\ d=5625 \end{array}$$

Again, to find the vulgar fraction corresponding to $.568\dot{1}$. Multiply first by 10000, and then by 100, and subtract the latter from the former, and you obtain $9900\ d=5625$, and hence $d=\frac{5625}{9900}$, which reduced to its lowest terms is $\frac{25}{44}$.

From these investigations the three following rules for the three cases mentioned are derived :

I. *If the fraction be a terminating decimal make it the numerator, and for denominator write 1, followed by as many ciphers as there are figures in the decimal.*

II. *If the decimal be a pure circulator, make the digits of the decimal the numerator, and for denominator write as many nines as there are figures in the period*

III. *If the decimal be a mixed circulator, subtract the non-circulating part from the whole decimal to the end of the first period, both being treated as whole numbers; make the remainder the numerator, and for denominator write as many nines as there are circulating figures, and after them as many ciphers as there are non-circulating figures. In all cases reduce to the lowest terms.*

EXERCISES.

1. Find the vulgar fraction corresponding to $.0\dot{4}$. Ans. $\frac{4}{99}$.
2. Find the vulgar fraction corresponding to $.5\dot{4}$. Ans. $\frac{6}{11}$.
3. Find the vulgar fraction corresponding to $.24\dot{5}\dot{7}$. Ans. $\frac{811}{3300}$.
4. Find the vulgar fraction corresponding to $.1$. Ans. $\frac{1}{9}$.
5. Find the vulgar fraction corresponding to $.3$. Ans. $\frac{1}{3}$.
6. Find the vulgar fraction corresponding to $.7$. Ans. $\frac{7}{9}$.
7. Find the vulgar fraction corresponding to $.75$. Ans. $\frac{3}{4}$.
8. Find the vulgar fraction corresponding to $.47\dot{5}4\dot{3}$.
Ans. $\frac{3958}{8325}$.
9. Find the vulgar fraction corresponding to $.468354430379\dot{7}$.
Ans. $\frac{37}{9}$.
10. Find the vulgar fraction corresponding to $.49$. Ans. $\frac{1}{2}$.
11. Find the vulgar fraction corresponding to $.16\dot{2}$. Ans. $\frac{18}{111}$.
12. Find the vulgar fraction corresponding to $.14$. Ans. $\frac{13}{90}$.
13. Find the vulgar fraction corresponding to $.013\dot{8}$. Ans. $\frac{1}{72}$.
14. Find the vulgar fraction corresponding to $.568\dot{1}$. Ans. $\frac{25}{44}$.
15. Find the vulgar fraction corresponding to $.59\dot{2}$. Ans. $\frac{16}{27}$.

The last rule may be deduced from the other two in the following manner:—Let us take the mixed circulator $.41\dot{8}$, and this being multiplied by 10, the four becomes a whole number, and to preserve the same value, 10 is put as a divisor, which gives $\frac{4.1\dot{8}}{10}$ or $\frac{4+1\dot{8}}{10}$, but by rule II. we have $.1\dot{8} = \frac{18}{99}$, and hence the whole may be written $4 + \frac{18}{99} = \frac{396+18}{990} = \frac{414}{990} = \frac{23}{55}$, and this result corresponds to rule III.

IV.—ADDITION & SUBTRACTION OF DECIMALS.

From what has been said, it is plain that decimals can be added and subtracted just as whole numbers, care being taken to keep the decimal points in the same vertical line. In all operations into which repetends enter, it should be observed that in order to have a result true to any given number of places, it is generally desirable to carry out the repetend to one or two places more than the required number. It is often sufficient, however, to allow for what would be carried, which can usually be done by inspection. In all cases, respect should be had to the degree of exactness which the nature of the calculation requires. The figures beyond those required can be estimated and added in. Thus, if only five places are required, and the calculation be carried to six places, and the seventh figure is a large *one*, it should be added to the sixth figure.

This may be stated in the form of a

R U L E .

Add and subtract as in whole numbers, keeping the decimal prints in the same vertical line.

E X E R C I S E S .

(1.)	(2.)	(3.)
1.78645	8.58333333+	51.250000000
3.97863	17.74747474+	3.444444444+
7.84396	112.08080808+	7.637373737+
4.32782	6.12500000	.885555555+
9.54179	15.66666667	11.875000000
11.69857	.76969697	7.875875875+
5.48491	11.00000000	7.111111111+
44.66213	171.97297979	90.079360724

In exercise 2, the eighth figure of each of the fifth and sixth lines is made 7 instead of 6, which renders it unnecessary to make any allowance for the repetends that would follow, but this change is not made on any of the last figures of exercise 3, and therefore we add 2 for what would be carried from the tenth decimal place to the ninth.

4. Find in the decimal form the sum of $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{2}{5}$. Ans. 2.316.

5. Find in the decimal form the sum of $\frac{1}{5}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, $\frac{31}{32}$, $\frac{63}{64}$, $\frac{127}{128}$.
Ans. 6.0078125.

6. Find in the decimal form the sum of $\frac{2}{3}$, $\frac{2}{5}$, $\frac{7}{24}$, $\frac{1}{15}$.
Ans. 2.345.
7. Find in the decimal form the sum of $2\frac{3}{5}$, $4\frac{7}{8}$, $5\frac{3}{10}$.
Ans. 12.775.
8. What is the sum of .786425, .975324, .176009, .32, .62519375, .4?
Ans. 3.28295175.
9. Add to 6 places $18.127\dot{6}$, $11.34\dot{9}$, $12.14\dot{5}$, $8.64\dot{8}$, $15.2\dot{3}$.
Ans. 65.504414.
10. Find to 6 places the sum of $15.\dot{7}$, $12.\dot{4}$, $18.\dot{3}8\dot{7}$, $.41\dot{6}$, $.746\dot{8}\dot{7}$, $.9$, $.4\dot{5}$, $10.4\dot{5}$, $.12\dot{3}4\dot{5}$.
Ans. 59.351152.
11. What is the sum of $.7\dot{6}$, $.41\dot{6}$, $.4\dot{5}$, $.64\dot{8}$, $.2\dot{3}$ to five places of decimals?
Ans. 2.52087.
12. Reduce to decimals, and find the sum of $\frac{3}{8}$, $\frac{1}{22}$, $\frac{7}{24}$, $\frac{2}{88}$.
Ans. 1.416.
13. Find the sum of $.42\dot{7}$, $.41\dot{6}$, $1.32\dot{8}$, $3.02\dot{9}$, 5.476 to six places of decimals.
Ans. 10.678037.
14. Required the sum of 1.25, $1.\dot{4}$, $1.6\dot{3}\dot{7}$, $1.88\dot{5}$, 1.684 , 1.937 , 1.148 and 1.764085 .
Ans. 12.750458.
15. Find the sum of $.4632\dot{1}$, $.8153\dot{2}$, $.15492\dot{6}$, $.753\dot{2}$ to true to four places.
Ans. 2.1867.
16. From 3.468 subtract 1.2591, and you find the excess 2.2089.
17. What is the excess of 10.008576 above 5.789?
Ans. 4.219576.
18. From $11.\dot{4}$ take $1.4\dot{8}$, and there remains to six places 9.959596.
19. What is the excess of 7.8 above $1.37546\dot{5}\dot{8}$?
Ans. 6.4245341.
20. What is the difference between $9.4657\dot{4}$, and $4.1834\dot{5}$?
Ans. 5.28229.
21. Express, decimally, the difference between $\frac{2}{3} + \frac{2}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7}$, and $\frac{1}{3} + \frac{4}{7} + \frac{2}{3}\frac{1}{2}$.
Ans. 2.34613+.
22. What is the difference, according to the decimal notation, between $\frac{7}{9}$ and $\frac{1}{9}\frac{3}{4}$ true to six places of decimals?
Ans. .636363.

23. What is the difference between $\frac{1}{2}$ and $\frac{4}{7}$ expressed decimally true to six decimal places? Ans. .071428.

24. What is the difference between the vulgar fractions corresponding to .49 and .5? Ans. 0.

25. Find the value of $.786425 + .975324 + .176009 + .32 + .62519375 - 3.28295175 + .4$. Ans. 0.

26. What is the difference between 138.6012, and 128.8512? Ans. 9.75

27. What is the excess of 31.6322 above $5.674 + 1.83 + .3125 + 18.62 + 4.3 + .395 - .5$. Ans. 1.0007.

28. What is the excess, expressed decimally, of 5.83 above $4\frac{17}{9}$. Ans. 1.6582.

29. What is the difference between 8.375 and $7\frac{3}{7}$ true to six decimal places? Ans. .946428.

30. What is the value of $601.050725 - 441.001 - .00625 - 3.818475 - 156.1 + .125$. Ans. .25.

V.—MULTIPLICATION OF DECIMALS.

If we multiply a decimal by a whole number, the process is precisely the same as if the multiplicand were a whole number, but care

must be taken to keep the decimal point in the same relative position. Thus, in the annexed example, as

$$\begin{array}{r} 5.678 \\ 6 \\ \hline 34.068 \end{array}$$

there are three decimal places in the multiplicand, we make three also in the product. If we have to multiply a whole number by a decimal, we must mark off a decimal in the product for each decimal in the multiplier.—

$$\begin{array}{r} 5678 \\ .6 \\ \hline 3406.8 \end{array}$$

The reason of this will be manifest from the consideration that if we multiply 8 units by .6, or $\frac{6}{10}$, we get $\frac{48}{10}$, or 4.8, *i. e.*, 4 units and 8-tenths; and again, when we multiply 7 tens by .6 or $\frac{6}{10}$, we get $\frac{420}{10} = 42$ units, which with the 4 units already obtained, make 46 units, and we now have arrived at whole numbers. The same

illustration will apply to multiplying by .66, which requires two decimal places to be laid off from the right. Therefore, for every decimal place in the multiplier one must be cut off in the product, and we saw already that for every decimal place in the multiplicand, a deci-

mal place must be cut off in the product, and therefore we conclude that for every decimal place in both factors, a decimal place must be marked in the product. It may be well to vary the illustration by observing that as the tenth of a tenth is a one-hundredth, tenths multiplied by tenths give hundredths; so also the product of tenths and hundredths is thousandths, and so on. Thus: $.2$ or $\frac{2}{10}$, multiplied by $.3$ or $\frac{3}{10}$, is $\frac{6}{100}$. Now, $.6$ would not represent this, for that would mean $\frac{6}{10}$; hence, it is necessary to prefix a cipher, and write $.06$, and this agrees with what has been already noted (Art. 3) regarding whole numbers, viz., that we are compelled by the nature of the notation to introduce a zero character, and in the present instance the cipher means that there are no *tenths*, just as it indicated in the case referred to that there were no *tens*. So, also, $\frac{6}{1000}$ would be written decimally $.006$, which would mean that there are no tenths, no hundredths, but 6 thousandths. From these explanations we deduce the

R U L E :

Multiply, as in whole numbers, and cut off from the right a decimal place for every one in both multiplier and multiplicand.

E X A M P L E S .

- Multiply $.78$ by $.42$. Here we multiply as if the quantities were whole numbers, and in the product point off a decimal figure for each one in both multiplier and multiplicand. In
- (1.)
$$\begin{array}{r} .78 \\ .42 \\ \hline 156 \\ 312 \\ \hline .3276 \end{array}$$
 Ex. 1, the number of figures in the product is the same as the number in both factors, and therefore we have no whole number in the result, but four decimal places. In Ex. 2 there are four decimal places in the factors, and there are six figures in the product, and consequently two figures represent whole numbers. In Ex. 3, when we multiply 6 by 3, we obtain 18, but if we had carried the repitend out one place farther we should have had 5 to be multiplied by 3, and consequently 1 to carry, so we add 1 to the 18, and in like manner we must allow 2 when multiplying by 4, and 1 when multiplying by 2.
- (2.)
$$\begin{array}{r} .674 \\ 34.6 \\ \hline 4044 \\ 2696 \\ 2022 \\ \hline 23.3204 \end{array}$$
 (3.)
$$\begin{array}{r} 4.5\dot{6} \\ 2.43 \\ \hline 1369 \\ 1826 \\ 913 \\ \hline 11.0929 \end{array}$$

EXERCISES.

1. Multiply 7.49 by 63.1. Ans. 472.619.
2. Multiply .156 by .143. Ans. .022308.
3. Multiply 1.05 by 1.05, and the product by 1.05. Ans. 1.157625.
4. Find the continual product of .2, .2, .2, .2, .2, .2. Ans. .000064.
5. Multiply .0021 by 21. Ans. .0441.
6. Multiply 3.18 by 41.7. Ans. 132.606.
7. Multiply .08 by .036. Ans. .00288.
8. Multiply .13 by .7. Ans. .091.
9. Multiply .31 by .32 Ans. .0992.
10. Find the continual product of 1.2, 3.25, 2.125. Ans. 8.2875.
11. Multiply 11.4 by 1.14. Ans. 12.996.
12. Find the continual product of .1, .1, .1, .1, .1, .1. Ans. .000001.
13. Multiply 1240 by .008. Ans. 9.92.
14. Find the continual product of .101, .011, .11, 1.1 and 11. Ans. .001478741.
15. Multiply 7.43 by .862 to six places of decimals. Ans. .640839.
16. Multiply 3.18 by 11.7, and the product by 1000. Ans. 132606.
17. Multiply .144 by .144. Ans. .020736.
18. What is the continual product of 13.825, 5.128 and .001? Ans. .0708946.
19. What is the continual product of 4.2, 7.8 and .01? Ans. .3276.
20. What is the continual product of .0001, 6.27 and 15.9? Ans. .0099693.

CONTRACTED METHOD.—In many instances where long lines of figures are to be multiplied together, the operation may be very much shortened, and yet sufficient accuracy attained. We may instance what the student will meet with hereafter, calculations in compound interest and annuities, involving sometimes most tedious operations. By the following method the results in such cases may be obtained with great ease, and correct to a very minute fraction. If we are computing dollars and cents, and extend our calculation to four

places of decimals, we are treating of the one-hundredth part of a cent, or the ten-thousandth part of a dollar, a quantity so minute as to become relatively valueless. Hence we conclude that three or four decimal places are sufficient for all ordinary purposes. There are cases, indeed, in which it is necessary to carry out the decimals farther, as, for instance, in the case of Logarithms to be considered hereafter. The principle of the contracted method will be best explained by comparing the two subjoined operations on the same quantities.

Let it be required to find the product of 6.35642 and 47.6453, true to four places of decimals :

EXTENDED OPERATION.

$$\begin{array}{r}
 6.35642 \\
 47.6453 \\
 \hline
 19\,06926 \\
 317\,8210 \\
 2542\,568 \\
 38138\,52 \\
 444949\,4 \\
 2542568 \\
 \hline
 302.8535\,37826
 \end{array}$$

CONTRACTED OPERATION.

$$\begin{array}{r}
 6.35642 \\
 3546.74 \\
 \hline
 2542568 \\
 444949 \\
 38138 \\
 2542 \\
 317 \\
 19 \\
 2 \text{ carried.} \\
 \hline
 302.8535
 \end{array}$$

RULE FOR THE CONTRACTED METHOD.

Place the units' figure of the whole number under the last required decimal place of the multiplicand, and the other integral figures to the right of that in an inverted order, and the decimal figures, also in an inverted order, to the left of the integral unit ; multiply by each figure of the inverted multiplier, beginning with the figure of the multiplicand immediately above it, omitting all figures to the right, but allowing for what would have been carried if the decimal had been carried out one place farther—place the first figure of each partial product in the same vertical column, and the others in vertical columns to the left ; the sum of these columns will be the required product.

Thus, in the above example, we are required to find the product correct to four decimal places, therefore we set the units' figure, 7, under the fourth decimal figure, and the tens' figure, 4, to the right, and the decimal figures, 6453, to the left in reversed order ; then we

multiply the whole line by 4, and then we multiply by 7, omitting the 2 which stands to the right, but allowing 1 for what would have been carried, that is, we say 7 times 4 is 28, and 1 is 29, and we write the nine under the 8, the first figure of the first partial product. By comparing the contracted method with the figures of the extended form, which are to the left of the vertical line drawn after the fourth decimal figures, it will be seen that the figures of each column are the same but placed in reversed order, which makes no difference in the sum, as $5+3=3+5=8$. This is the same principle as the contracted method of multiplying by 17, 71, &c., suggested in the article on simple multiplication.*

The object of writing the multiplier in a reversed order is simply to make the work come in the usual form, as otherwise we should be crossing and recrossing, so to speak, as will be seen by the operation in the margin.—Beginning with the left hand figure of the multiplier, and the right hand figure of the multiplicand, we find the first partial product; then taking the second figure of the multiplier from the left, (7) and the second figure of the multiplicand from the right, we get the second partial product, and so on, moving one place each time towards the right in the multiplier, and one place towards the left in the multiplicand. This is so different from the ordinary mode of operation, as to be excessively awkward and puzzling, and this gave rise to the idea of reversing the order of the digits. We append this remark as most persons cannot at first sight comprehend the reason of the inversion.

$$\begin{array}{r}
 6.35642 \\
 47.6453 \\
 \hline
 2542568 \\
 444949 \\
 38138 \\
 2542 \\
 317 \\
 19 \\
 2 \text{ allowed.} \\
 \hline
 302.8535
 \end{array}$$

* Let the learner observe that all the figures of the first column are of the same rank, viz., ten-thousandths, and therefore may be added together, and as the value of each figure is increased or decreased 10 times according to its position to left or right, it follows that all figures at equal distances from the decimal point, whether to right or left, are of the same rank, *i. e.*, units will be under units, tens under tens, tenths under tenths, hundredths under hundredths, &c., &c. The contracted method is not of much use in terminating decimals which extend to only a few places, but it saves a vast deal of labour in questions which involve either repetends or terminating decimals expressed by a long line of decimal figures

ADDITIONAL EXERCISES.

21. Multiply .26736 by .28758 to four decimal places
Ans. .0769.
22. Multiply 7.285714 by 36.74405 to five decimal places.
Ans. 267.70665.
23. Multiply 2.656419 by 1.723 to six decimal places.
Ans. 4.578932.
24. What decimal fraction, true to six places, will express the product of $\frac{9}{14}$ multiplied by $\frac{3}{17}$?
Ans. .113445.
25. What decimal fraction is equivalent to $\frac{2}{3} \times \frac{3}{7} \times \frac{4}{8}$?
Ans. .46748.
26. What is the second power of .841 ?
Ans. .707281.
27. What is the product of 1.65 by 1.48, true to five places ?
Ans. 2.45975.
28. Express decimally $2\frac{9}{13} \times \frac{8}{9}$.
Ans. 2.393162.
29. What is the product of 73.6371 by 8.143 ?
Ans. 599.6272077.
30. .681472 $\times$.01286, true to five places, will give .00876.

$$\begin{array}{r}
 .681472 \\
 68210.0 \\
 \hline
 681 \\
 136 \\
 55 \\
 1 \\
 \hline
 .00876
 \end{array}$$

In the last exercise it must be observed that since there is no whole number, and five decimal places are required, we must place a cipher under the fifth decimal figure, and write .01286 in reversed order. That the result is a sufficiently close approximation will be evident from the consideration that the last figure 6 is only six one-hundred-thousandths of the unit, and consequently the next figure would be only one-millionth part of the unit.

VI.—DIVISION OF DECIMALS.

We have already seen (1) that we cannot perform any operation except the numbers concerned are of the same denomination, or one of them be abstract; (2) that when a denominate number is used either as a multiplier or a divisor, it ceases to be denominate, and becomes abstract, and (3) that the rules for addition, subtraction, multiplication and division of integers apply equally to decimals, the only additional requirement being the placing and moving of the decimal point.

Suppose then we are required to divide 1.2321 by 11.1, we must by (1) bring both quantities to the same denomination. Now the dividend is carried down to ten-thousands for $1.2321 = 1 + \frac{2321}{10000}$, and therefore we express 11.1 in the corresponding form, ten-thousandths or $11 + \frac{1000}{10000}$, or 11.1000, so that we change the form, but not the value of 11.1, the divisor. Again, by (2) the .1, which originally expressed *a tenth* of some unit, and therefore was in reality denominate, now becomes abstract as one of the figures of the given factor of 1.2321, by means of which we are to find the other factor. Hence by (3) we can now divide 1.2321 by 11.1000, as if both were whole numbers, and this is the reason for omitting the decimal point when we have made the number of decimal places equal. Beginners generally feel a difficulty in conceiving how a fraction divided by a fraction can give a whole number. The difficulty may be easily removed by noticing that $\frac{1}{4}$ is contained *twice* in $\frac{1}{2}$ for $\frac{2}{4} = \frac{1}{2}$, *e. g.*, a half dollar contains, or is equivalent to, two quarter dollars. Thus the fraction $\frac{1}{2}$ divided by the fraction $\frac{1}{4}$, gives the whole number 2. So, also, $\frac{1}{8}$ is contained 4 times in $\frac{1}{2}$, and therefore $\frac{1}{2} \div \frac{1}{8} = 4$, a whole number. Hence, when we have reduced the divisor and dividend to the same denomination, we may omit the decimal point, as we have only to find *how often the one is contained in the other*. Hence the

R U L E .

If the number of decimal places in the divisor and dividend be not equal, make them equal by supplying ciphers or repetends, and then divide as in whole numbers, and the quotient so far will be a whole number, but if there is a remainder, annex ciphers or repetends, and the part of the quotient thus obtained will be a decimal.

The decimal places may be supplied as the work proceeds, as it is easy to see how many ciphers or repetends must be supplied; for we have seen in multiplication that the number of decimal places in any product must be equal to all the decimal places in the factors, and, since a dividend must always be viewed as a product, it follows that the difference between the number of decimal places in dividend and divisor will indicate how many ciphers or repetends must be supplied.

E X E R C I S E S .

1. Divide 47.58 by 26.175 to six decimal places.

Ans. 1.817765.

2. Divide 70.8946 by 13.825 to three places.

Ans. 5.128.

3. Divide 468.7 by 3.365 to six places of decimals.
Ans. 139.309889.
4. Express decimally $1 \div 7\frac{3}{10}$.
Ans. 233.3.
5. Express in the decimal form $\frac{7}{8}$ of $\frac{6}{7} \div \frac{4}{5}$ of $\frac{8}{9}$ true to six places of decimals.
Ans. 1.054687.
6. Divide the whole number 9 by the fraction .008. Ans. 1125.
7. What is the quotient of $5.\dot{0}\dot{9}$ by $6.\dot{2}$?
Ans. $\dot{8}\dot{1}$ nearly.
8. Divide .54439 by 7777.
Ans. .00007.
9. What decimal is obtained by dividing 1 by 10.473654?
Ans. .09547766.
10. What is the difference between $\frac{2}{3} \div \frac{5}{7}$ and $\frac{5}{6} \div \frac{1}{11}$ in the decimal form?
Ans. .24583.

CONTRACTED METHOD.

The work may often be much abbreviated in the manner exhibited by the following example :

$$\begin{array}{r}
 .14736).23748(1.611 \\
 \underline{14736} \\
 90120 \\
 \underline{38416} \\
 17040 \\
 \underline{14736} \\
 23040 \\
 \underline{14736} \\
 8304
 \end{array}$$

$$\begin{array}{r}
 14736)23748(1.611 \\
 \dots 14736 \\
 \hline
 9012 \\
 8842 \\
 \hline
 170 \\
 147 \\
 \hline
 23 \\
 15 \\
 \hline
 8
 \end{array}$$

Here it is required to divide .23748 by .14736. Since both divisor and dividend contain the same number of decimal places, no alteration is needed, and so we can at once reject the decimal point, and divide as in whole numbers. The principle of the contraction is simply what has been already explained, viz., that all we look for in such calculations is a sufficiently close approximation, by which we mean an approximation sufficient for all practical purposes. For this reason, when we have obtained the integral part of the quotient, we may omit one figure of the divisor in succession after each operation, as the value of each figure decreases in a tenfold degree as we descend towards the right, and after three decimal figures the error,

or deficit rather, becomes only thousandths, which are very rarely worth taking into account. For example, if the calculation regards dollars and cents, the error at the fourth decimal place would be only the *one-thousandth* part of a cent.

R U L E .

Arrange the fractions as in the ordinary mode ; find the first figure of the quotient and the first remainder ; then, instead of annexing a periodic figure or a cipher, cut off the right hand figure of the divisor, and use the remaining figures to find the next figure of the quotient, and so on.

It is usual to mark the figures as they are successively cut off by placing a point below each. In multiplying by each figure of the quotient, allowance must be made for what would have been carried from the figure of the divisor last cut off, had it been used in the division.

The vertical line drawn through the ordinary form shows how closely the two modes correspond. As has already been remarked, it is desirable, in order to secure accuracy, to carry the figures of repetends to one or two places more than are required.

E X E R C I S E S .

$$\begin{array}{r} (1.) \\ 43232323 \overline{) 73640000} (170.3355. \\ \dots\dots 43232323 \end{array}$$

$$\begin{array}{r} 30407677 \\ 30262626 \end{array}$$

$$\begin{array}{r} 145051 \\ 129697 \end{array}$$

$$\begin{array}{r} 15354 \\ 12970 \end{array}$$

$$\begin{array}{r} 2384 \\ 2162 \end{array}$$

$$\begin{array}{r} 222 \\ 216 \end{array}$$

$$6$$

(2.)

$$\begin{array}{r} 54637 \overline{) 43682} (.7995 \\ \dots\dots 38246 \end{array}$$

$$\begin{array}{r} 5436 \\ 4917 \end{array}$$

$$\begin{array}{r} 519 \\ 491 \end{array}$$

$$\begin{array}{r} 28 \\ 27 \end{array}$$

$$1$$

Divide 73.64 by .432, and .43682 by .54637 to 4 decimal places each. To show that there will be three integral places in the

quotient of Ex. 1, we must consider that there are two places of whole numbers in the dividend and none in the divisor, and, therefore, if we divide 73 and 6, the first decimal place of the dividend by .4, the first figure of the divisor, we get three integral places. Hence, since we are to have four decimal places, we shall have seven figures in all. This contraction is extremely useful when there are many decimal places.

3. Find the quotient of $8.6134 \div 7.3524$ to four decimal places.
Ans. 1.1715.
4. Divide .61 by 13.543516 to five decimal places. Ans. .04549.
5. Divide .58 by 77.482 to five decimal places. Ans. .00756.
6. Divide .812.54567 by $7.3\bar{4}$ to three decimal places.
Ans. 110.649.
7. Divide 1 by 10.473654 to six decimal places. Ans. .09547.
8. Divide 7.126491 by .531 to six decimal places.
Ans. 13.420887.
9. Divide 1.77975 by the whole number 25425. Ans. .00007.
10. Divide to eight places .879454 by .897. Ans. .98043924.

VII.—DENOMINATE DECIMALS.

To express one denominate number as a fraction of another of the same kind, reduce both to the lowest denomination contained in either, make the former the numerator and the latter the denominator of a common fraction, and reduce the fraction so found to a decimal in the manner already pointed out.

EXAMPLES.

To express 16 cents as a fraction of a dollar: Here the lowest denomination mentioned is cents, and we reduce a dollar to cents and write $\frac{16}{100} = \frac{4}{25}$, and, dividing 4 by 25, we get .16. To express 11s. $4\frac{1}{2}$ d. as a decimal of £1, we reduce both to half-pence, and obtain $\frac{27\frac{1}{2}}{480} = \frac{91}{160}$, which, reduced to a decimal, is .56875.

EXERCISES.

1. Reduce 5s. $10\frac{1}{2}$ d. to the decimal of £1. Ans. £ .29375.
2. Reduce $10\frac{1}{2}$ d. to the decimal of £1. Ans. £ .04375.
3. Reduce 15s. $9\frac{3}{4}$ d. to the decimal of £1. Ans. £ .790625.
4. Express 3 roods and 11 rods as a decimal of an acre.
Ans. .81875.

5. Express 3 cwt., 1 qr., 7 lbs., as a decimal of a ton.

Ans. .166.

6. Reduce 37 rods to the decimal of a mile. Ans. .115625.

7. Reduce 7 oz., 4 dwts., to the decimal of a pound. Ans. .6.

8. Reduce a pound troy to the decimal of a pound avoirdupois; correct to six decimal places.* Ans. .822857+.

9. Reduce 5 hours, 48 minutes, 49.7 seconds, to the decimal of a day, taken as 24 hours. Ans. .2422419.

10. Express an ounce avoirdupois as a decimal of a pound troy.

Ans. .9114583.

VIII.—REDUCTION TO DENOMINATIONS.

To find the value of a fraction in the lower denominations, expressed as a decimal of any given denomination, multiply in succession by the numbers which express the given and lower denominations, and after each multiplication cut off from the right as many decimal figures as are contained in the given decimal, and the figures to the left of the decimal point will give the required value.

EXAMPLES.

1. To find the value of £.79375 ; multiply by 20, by 12 and by

$$\begin{array}{r}
 20 \\
 \hline
 15.87500 \\
 12 \\
 \hline
 10.50000 \\
 2 \\
 \hline
 1.00000
 \end{array}
 \begin{array}{l}
 2, \text{ which gives } £0.15.10\frac{1}{2}. \\
 \text{Repetends must either be re-} \\
 \text{duced to common fractions or} \\
 \text{found approximately.}
 \end{array}$$

2. To find the value of £.7 : $\begin{array}{l} .77777 \\ 20 \end{array} \left. \vphantom{\begin{array}{l} .77777 \\ 20 \end{array}} \right\} \text{carry 1.}$

$$\begin{array}{r}
 15.55550 \\
 12 \\
 \hline
 6.66600 \\
 4 \\
 \hline
 2.66400
 \end{array}
 \quad £0.15.6\frac{3}{4} \text{ nearly.}$$

* The standard pounds are meant here, viz. : troy, 5760 grains, and avoirdupois 7000 grains. Taking the ounces would give $\frac{1}{16} = \frac{3}{4} = .75$.

EXERCISES.

1. What is the value of £.475 ? Ans. 9s. 6d.

2. What is the value of .7 of a cwt. ?

Ans. 3 qrs., 3 lbs., 1 oz., $12\frac{4}{7}$ drs.

3. What is the value of .5416 of a shilling sterling ? Ans. $6\frac{1}{2}$ d.

4. What is the value of .6845 of a cwt. ?

Ans. 2 qrs., 20 lbs., 10 oz., $9\frac{1}{2}\frac{3}{5}$.

5. What is the value of .4 of 9s. $4\frac{1}{2}$ d ? We have $.4 = \frac{4}{10}$ and 9s. $4\frac{1}{2}$ d., multiplied by 4, and the product, divided by 9, gives 4s. 2d., the exact value.

6. What is the value of .026 of $1^\circ 15'$? Reducing .026 to a vulgar fraction, we get $\frac{26}{1000} = \frac{13}{500}$, and multiplying $1^\circ 15'$ by 2, and dividing by 75, we find $2'$.

RATIO AND PROPORTION.

17.—RATIO is the relation which one quantity bears to another of the same kind with respect to magnitude, or the number of times that the less is contained in the greater. Thus, the ratio 7 to 21 is 3, because 7 is contained 3 times in 21, or 21 is 3 times 7. The same result is obtained if we divide 7 by 21, for we then find $\frac{7}{21} = \frac{1}{3}$, which means that 7 is $\frac{1}{3}$ of 21, and this expresses the very same relation as before; for, to say that 7 is $\frac{1}{3}$ of 21 is precisely the same as to say that 21 is 3 times 7. (See note under Inverse Proportion.) And, therefore, 3 is called the measure of the ratio. The numbers thus compared are called the terms of the ratio—the first the antecedent and the second the consequent, and the relation is written 7 : 21. The sign (:) originally indicated division.

That the magnitudes must be of the same kind will be obvious from the consideration that 7 bags of flour could have no ratio to 21 dollars, for multiplying 7 bags of flour by 3 would not make them 21 dollars, but 21 bags of flour, and multiplying 7 dollars by 3 would not make them 21 bags of flour, but 21 dollars. Hence, the less could not be increased to make the greater, except they are homogeneous, or of the same kind.

Proportion is the equality of ratios.

The ratio of 9 to 27 is 3, but we have seen that the ratio of 7 to 21 is also 3, therefore the ratios of 7 to 21 and of 9 to 27 are the

same, or $7 \div 21 = 9 \div 27$, and these quantities are, therefore, called proportionals. The sign ($::$) was formerly used for equality, and is still retained for equality of ratios, and the sign ($=$) is used for the actual equality of quantities, though occasionally used for equality of ratios. Hence, the usual mode of writing the equality of two ratios is $7 : 21 :: 9 : 27$. Such a statement is called a proportion, or an analogy, and is read—7 is to 21 as 9 to 27, *i. e.*, 27 exceeds 9 as many times as 21 exceeds 7, and this is expressed by saying 27 is the same multiple of 9 that 21 is of 7, or that 9 is the same sub-multiple, measure, or aliquot part of 27 that 7 is of 21. The four quantities are called the *terms* of the proportion; the first and last are called the *extremes*, and second and third the *means*; also, the first and third are called *homologous*, or of the same name, *i. e.*, both are antecedents, and so the second and fourth are homologous, for they are both consequents. The last term is called a fourth proportional to the other three, and we shall denote it by F. P. There are two simple ways of testing the correctness of an analogy. The first is to divide the second term by the first, and the fourth by the third, and if the quotients are equal, the analogy is correct. This is manifest from what has been already said. The second principle is, that, if the analogy be correct, the product of the extremes is equal to the product of the means. To prove this, let us resume the analogy, $7 : 21 :: 9 : 27$. We have seen that $21 \div 7 = 27 \div 9$, or $3 = 3$. Now, if each be multiplied by 63, we have (by Ax. II., Cor.,) $189 = 189$. But 189 is the product of 27 by 7, the extremes, and also of 21 by 9, the means—these products then are always equal. From this simple principle we readily deduce a rule for finding a fourth proportional to three given quantities. Let the quantities be 48, 96, and 132, written thus: $48 : 96 :: 132 :$, the required quantity. Now, $132 \times 96 = 12672$, the product of the means are therefore equal to the product of the extremes. We have, therefore, a product, 12672, and one of its factors, 48, hence, dividing this product by the given factor, we find the other factor to be 264, which is therefore the fourth proportional, or fourth term of the proportion, and we can now write the whole analogy, thus:— $48 : 96 :: 132 : 264$. To prove the correctness of the operation, multiply 264 by 48, and 12672 is obtained, the same as before. Hence,

THE RULE.

Divide the product of the second and third terms by the first, and the quotient will be the required fourth term.

To show the order in which the three given quantities are to be arranged, let it be required to find how much 730 yards of linen will cost at the rate of \$30 for 50 yards. It is plain that the answer, or fourth term, must be dollars, for it is a price that is required, and in order that the third term may have a ratio to the fourth, the \$30 must be the third term. Again, since 730 yds. will cost more than 50 yds., the fourth term will be greater than the third, and therefore the second must be greater than the first, and therefore the statement is

yds. yds. \$
 $50 : 730 :: 30 : 4\text{th proportional}$, and by the rule $\frac{730 \times 30}{50} = \frac{21900}{50} = 438$, the fourth term, and we can now write the whole analogy, 50 yds : 730 yds :: \$30 : \$438.

This may be called the ascending scale, for the second is greater than the first, and the fourth greater than the third. If the question had been to find what 50 yards of linen will cost at the rate of \$438 for 730 yards, we still find that the answer will be dollars, and that therefore, as before, dollars must be in the third place, but we see that the answer will now be less than 438, as 50 yards, of which the price is required, will cost much less than 730 yards, of which the price is given, and that therefore the second term must be less than the first. Hence the statement is 730 yds : 50 yds :: \$438 : F. P., and by the rule $\frac{438 \times 50}{730} = 30$, the fourth proportional. We now have the full analogy 730 yds : 50 yds :: \$438 : \$30. As the second is less than the first, and the fourth less than the third, this may be called the descending scale. If the first should turn out to be equal to the second, and therefore the third equal to the fourth, we should say that the quantities were to each other in the ratio of equality.

RULE FOR THE ORDER OF THE TERMS.

If the question implies that the consequent of the second ratio must be greater than the antecedent, make the greater term of the first ratio the consequent, and the less the antecedent, and vice versa.

The questions hitherto considered belong to what is called *Direct Proportion*, to distinguish it from another kind called *Inverse Proportion*; because, in the former, the greater the number given, the less will be the corresponding number required, and *vice versa*;

whereas, in the latter, the greater the number given, the less will be the number required, and *vice versa*. To illustrate this, let it be required to find how long a stack of hay will feed 12 horses, if it will feed 9 horses for 20 weeks. Here the answer required is time, and therefore 20 weeks will be the antecedent of the second ratio; but the greater the number of horses, the shorter time will the hay last, and therefore the fourth term will be less than the third, and therefore the statement will not be $9 : 12$, but the reverse, $12 : 9$; and hence the name INVERSE, because the term 9, for which the time (20 weeks,) is given, and which therefore we should expect to be in the first place, has to be put in the second; and the term 12, for which the time is required, and which therefore we should expect to be in the second place, has to be put in the first, and thus the whole analogy is $12 : 9 :: 20 : 15$.*

The principal changes that may be made in the order of the terms, will be more readily and clearly understood by the subjoined scheme, than by any explanation in words :

Original Analogy : $8 : 6 :: 12 : 9$ for $8 \times 9 = 72 = 6 \times 12$.

Alternately : $8 : 12 :: 6 : 9$ for $8 \times 9 = 72 = 6 \times 12$.

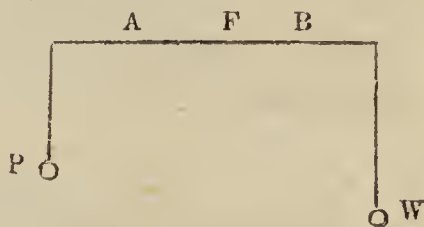
By Inversion : $6 : 8 :: 9 : 12$ for $6 \times 12 = 72 = 8 \times 9$.

By Composition : $8+6 : 6 :: 12+9 : 9$ or $14 : 6 :: 21 : 9$ for $14 \times 9 = 126 = 6 \times 21$.

By Division : $8-6 : 6 :: 12-9 : 9$ or $2 : 6 :: 3 : 9$ for $2 \times 9 = 18 = 6 \times 3$.

By Conversion : $8 : 8-6 :: 12 : 12-9$ or $8 : 2 :: 12 : 3$ for $8 \times 3 = 24 = 2 \times 12$.

Simple transposition is often of the greatest use. Let us take an easy practical example. In calculating what power will balance a given weight, when the arms of the lever are known, let P be the power, W the weight, A the arm of power, and B the arm of weight. The rule is, that the power and weight are inversely as the arms. This solves all the four possible cases by transposition.



* Inverse ratio is sometimes spoken of, but in reality there is no such thing. It is true that Inverse Proportion requires the terms of one of the ratios to be inverted, but that is a matter of analogy, not of ratio, for we have seen already that $7 \div 21$ expresses the very same relation as $21 \div 7$.—(See in-

$A : B :: W : P$, gives the power when the others are known,

$B : A :: P : W$ gives the weight when the others are known,

$W : P :: A : B$ gives the arm of weight when the others are known,

$P : W :: B : A$ gives the arm of power when the others are known.

The work may often be contracted in the following manner :—
Resuming our example $48 : 96 :: 132 : \text{fourth proportional}$, we see that 96 is double of 48, and therefore the ratio of 48 to 96 is the same as that of any two numbers, the second of which is double the first, and $48 : 96$ is the same as $1 : 2$, and we reduce the analogy to the simple form of $1 : 2 :: 132 : 4\text{th prop.}$, and we have $\frac{132 \times 2}{1} = 264$, the term required, as before. In the example $50 : 730 :: 30 : 4\text{th term}$, we have $\frac{730 \times 30}{50} = \frac{73 \times 30}{5} = \frac{73 \times 6 \times 5}{5} = 73 \times 6 = 438$. This is equivalent to dividing the first and second by 10, and the first and third by 5. Hence we may divide the first and second, or first and third by any number that will measure both. The same principle will also be illustrated by the consideration that the second and third are multipliers, and the first a divisor; and if we first multiply, and then divide by the same quantity, the one operation will manifestly neutralize the other. Thus: $48 : 96 :: 132 : \text{F. P.}$ may be written $1 \times 48 : 2 \times 48 :: 132 : \text{F. P.}$; where it is plain that since by first multiplying 132 by 48, and then dividing by the same, the one operation would neutralize the other, both may be omitted.

In proportion, when the means are equal, such as $4 : 12 :: 12 : 36$, it is usual to write the analogy thus— $4 : 12 : 36$, and 12 is called a mean proportional between 4 and 36. To prove the correctness of this statement, we multiply 36 by 4 and 12 by itself, and as both give 144, the analogy is correct. Now, as 144 is the square or second power of 12, so 12 is called the second root, or square root of 144, or that which produced it, or the *root* from which it grew; hence, to find a mean proportional between two given quantities, we have the following

R U L E .

Multiply them together, and take the square root of the product.

Thus, in the above example, $4 \times 36 = 144$, the square root of which is 12. Again, to find a mean proportional between 9 and 49, we mul-

troductory remarks.) The term *Reciprocal Ratio* is liable to the same objection, for though 3 and $\frac{1}{3}$ are reciprocals, yet they express the same relation. When the expression *Inverse Ratio* is legitimately used, it does not refer to a *single* ratio, but means that *two* ratios are so related that *one* of them must be inverted.

multiply 49 by 9, which is 441, the square root of which is 21, which is a mean proportional between 9 and 49, *i. e.*, $9 : 21 : 49$, or, written at full length, $9 : 21 :: 21 : 49$. Proof: $49 \times 9 = 441$ and $21 \times 21 = 441$. As the learner is not supposed, at this stage, to know the method of finding the roots of quantities beyond the limits of the multiplication table, we append a table of squares and roots at the end of the book.

When each quantity in a series is a mean proportional between two adjacent quantities, the quantities are said to be continued, or continual proportionals. Thus: $2 : 4 : 8 : 16 : 32 : 64 : 128$, and $3 : 9 : 27 : 81 : 243$, are series in which each is a mean proportional between two adjacent ones. Let us take 16 and the two adjacent ones, 8 and 32—the analogy is $8 : 16 :: 16 : 32$. Proof: $8 \times 32 = 256$, and $16 \times 16 = 256$. So also, 27 and the adjacent terms, 9 and 81. The analogy is $9 : 27 :: 27 : 81$, and the proof, $9 \times 81 = 729$, and $27 \times 27 = 729$.

This subject will be treated of at length in a subsequent part of the work, but this explanation has been introduced here to fill up the outline and let the learner understand the *nature* of continued proportionals.

EXERCISES.

1. If 6 barrels of flour cost \$32, what will 75 barrels cost?
Ans. \$400.
2. If 18 yards of cloth cost \$21, what must be paid for 12 yards?
Ans. \$14.
3. How much must be paid for 15 tons of coal, if 2 tons can be purchased for \$15?
Ans. \$112.50.
4. If you can walk 84 miles in 28 hours, how many minutes will you require to walk 1 mile?
Ans. 20.
5. What will 14 horses cost, if 3 of the average value can be bought for \$270?
Ans. \$1260.
6. What must be paid for a certain piece of cloth, if $\frac{2}{3}$ of it cost \$9.
Ans. \$13.50.
7. If 5 men are required to build a wall in 5 days, how many men will do the same in $2\frac{1}{2}$ days?
Ans. 10.
8. If 16 sheep are $\frac{2}{3}$ of a flock, how many are there in the same?
Ans. 24.
9. What must be paid for $4\frac{1}{2}$ cords of wood, if the cost of 3 cords is \$10?
Ans. \$15.

10. What is the height of a tree which casts a shadow of 125 feet, if a stake 6 feet high produces a shadow of 8 feet? Ans. $93\frac{3}{4}$.

11. How long will it take a train to run from Toronto to Hamilton (a distance of 39 miles), at the rate of 5 miles in $15\frac{5}{13}$ minutes? Ans. 2 hours.

12. If 15 men can build a bridge in 10 days, how many men will be required to erect three of the same dimensions in $\frac{1}{2}$ the time? Ans. 90.

13. If a man receive \$4.50 for 3 days' work, how many days ought he to remain in his place for £6 4s. 6d.? Ans. $16\frac{3}{5}$.

14. How much may a person spend in 94 days if he wishes to save 18 guineas out of a salary of \$500 per annum? Ans. \$109.297+.

15. If 3 cwt. 3 qrs. 14 lbs. of sugar cost \$36.50, what will 2 qrs. 2 lbs. cost? Ans. \$4.879+.

16. 5 men are employed to do a piece of work in 5 days, but after working 4 days they find it impossible to complete the job in less than 3 days more, how many additional men must be employed to do the work in the time agreed upon at first? Ans. 10.

17. A watch is 10 minutes too fast at 12 o'clock (noon) on Monday, and it gains 3 minutes 10 seconds a day, what will be the time by the watch at a quarter past 10 o'clock, A. M., on the following Saturday? Ans. 10 h. 40m. $36\frac{7}{8}$ s.

18. A bankrupt owes \$972, and his property, amounting to \$607.50, is distributed among his creditors; what does one receive whose demand is \$11.33 $\frac{1}{3}$? Ans. \$7.083+.

19. What is the value of .15 of a hhd. of lime, at \$2.39 per hhd.? Ans. \$.3585.

20. A garrison of 1200 men has provisions for $\frac{3}{4}$ of a year, at the rate of $\frac{7}{8}$ of a pound per day; how long will the provisions last at the same allowance if the garrison be reinforced by 400 men? Ans. $6\frac{3}{4}$ months.

21. If a piece of land 40 rods in length and 4 in breadth make an acre, how long must it be when it is 5 rods $5\frac{1}{2}$ feet wide? Ans. 30 rods.

22. A borrowed of B £175 5s. for 102 days, and afterwards would return the favor by lending B £210 6s.; for how long should he lend it? Ans. 85 days.

23. If a man can walk 300 miles in 6 successive days, how many miles has he to walk at the end of 5 days? Ans. 50.

24. If 495 gallons of wine cost \$394; how much will \$72 pay for? Ans. 90 gal.

25. If 112 head of cattle consume a certain quantity of hay in 9 days; how long will the same quantity last 84 head? Ans. 12 days.

26. If 171 men can build a house in 168 days; in what time will 108 men build a similar house? Ans. 266 days.

27. It has been proved that the diameter of every circle is to the circumference as 113 : 355; what then is the circumference of the moon's orbit, the diameter being, in round numbers, 480,000 miles? Ans. $1,507,964\frac{68}{113}$ m.

28. A round table is 12 ft. in circumference; what is its diameter? Ans. 3 ft. $9\frac{297}{55}$ in.

29. A was sent with a warrant; after he had ridden 65 miles, B was sent after him to stop the execution, and for every 16 miles that A rode, B rode 21; How far had each ridden when B overtook A? Ans. 273 miles.

30. Find a fourth proportional to 9, 19 and 99. Ans. 209.

31. A detective chased a culprit for 200 miles, travelling at the rate of 8 miles an hour, but the culprit had a start of 75 miles; at what rate did the latter travel? Ans. 5 miles an hour.

32. How much rum may be bought for \$119.50, if 111 gallons cost \$89.625? Ans. 148 gallons.

33. If 110 yards of cloth cost \$18; what will \$63 pay for? Ans. 385 yards.

34. If a man walk from Toronto to Hamilton, a distance of (say) 38 miles in 13 hours, 18 minutes; in what time will he walk at the same rate from St. Catherines to London, supposing the distance to be 102 miles? Ans. 35 hours, 42 min.

35. A butcher used a false weight $14\frac{3}{4}$ oz., instead of 16 oz. for a pound, of how many lbs. did he defraud a customer who bought 112 just lbs. from him? Ans. $9\frac{29}{55}$ lbs.

36. If 123 yards of muslin cost \$205; how much will 51 yards cost? Ans. \$85.

37. In a copy of Milton's Paradise Lost, containing 304 pages, the combat of Michael and Satan commences at the 139th page; at what page may it be expected to commence in a copy containing 328 pages?

Ans. The fourth proportional is $149\frac{37}{55}$; and hence the passage will commence at the foot of page 150.

38. Suppose a man, by travelling 10 hours a day, performs a

journey in four weeks without desecrating the Sabbath; now many weeks would it take him to perform the same journey, provided he travels only 8 hours per day, and pays no regard to the Sabbath?

Ans. 4 weeks, 2 days.

39. A cubic foot of pure fresh water weighs 1000 oz., avoirdupois; find the weight of a vessel of water containing $217\frac{1}{2}$ cubic in.

Ans. 7 lbs., $13\frac{1}{4}\frac{5}{4}$ oz.

40. Suppose a certain pasture, in which are 20 cows, is sufficient to keep them 6 weeks; how many must be turned out, that the same pasture may keep the rest 6 months?

Ans. 15.

41. A wedge of gold weighing 14 lbs., 3 oz., 8 dwt., is valued at £514 4s.; what is the value of an ounce?

Ans. £3.

42. A mason was engaged in building a wall, when another came up and asked him how many feet he had laid; he replied, that the part he had finished bore the same proportion to one league which $\frac{3}{17}$ does to 87; how many feet had he laid?

Ans. $32\frac{1}{14}\frac{9}{79}$.

43. A farmer, by his will, divides his farm, consisting of 97 acres, 3 roods, 5 rods, between his two sons so that the share of the younger shall be $\frac{3}{4}$ the share of the elder; required the shares.

Here the ratio of the shares is 4 : 3, and we have shown that if four magnitudes are proportionals, the first term increased by the second is to the second as the third increased by the fourth is to the fourth. Now, 97 acres, 3 roods, 5 rods, being the sum of the shares, we must take the sum of 4 and 3 for first term, and either 4 or 3 for the second, and therefore $7 : 4 :: 97 \text{ acres, 3 roods, 5 rods} : \text{F.P., i. e.,}$ the sum of the numbers denoting the ratio of the shares is to one of them as the sum of the shares is to one of them. This gives for the elder brother's share, 55 acres, 3 roods, 20 rods, and the younger's share is found either by repeating the operation, or by subtracting the share thus found from the whole, giving 41 acres, 3 roods, 25 rods.

44. A legacy of \$398 is to be divided among three orphans, in parts which shall be as the numbers 5, 7, 11, the eldest receiving the largest share; required the parts?

$23 : 5 :: 398 : 86\frac{1}{3}$, the share of the youngest.

$23 : 7 :: 398 : 121\frac{2}{3}$, the share of the second.

$23 : 11 :: 398 : 190\frac{8}{11}$, the share of the eldest.

45. Three sureties on \$5000 are to be given by A, B and C, so that B's share may be one-half greater than A's, and C's one-half greater than B's; required the amount of the security of each?

Ans. A's share, \$1052.63 $\frac{2}{9}$; B's, \$1578.94 $\frac{4}{9}$; C's, \$2368.42 $\frac{2}{9}$.

46. Suppose that one man starts from Montreal, and walks 6 miles an hour, and another at the same time from Newtonville, (5 miles west of Port Hope), at the rate of 5 miles an hour, when will they meet, the whole distance being 285 miles?

Ans. $\frac{5}{11}$ of a mile west of Gananoque, which is 155 miles west of Montreal by the Grand Trunk line.

47. A certain number of dollars is to be divided between two persons, the less share being $\frac{2}{3}$ of the greater, and the difference of the shares \$800; what are the shares, and what is the whole sum to be divided? Ans. Less share, \$1600; greater, \$2400; total, \$4000.

48. A certain number of acres of land are to be divided into two parts, such that the one shall be $\frac{3}{4}$ of the other; required the parts and the whole, the difference of the parts being 716 acres?

Ans. the less part 537 acres; the greater, 1253 acres; the whole, 1790.

49. A mixture is made of copper and tin, the tin being $\frac{1}{3}$ of the copper, the difference of the parts being 75; required the parts and the whole mixture? Ans. tin, 37 $\frac{1}{2}$; copper, 112 $\frac{1}{2}$; the whole, 150.

50. Pure water consists of two gasses, oxygen and hydrogen; the hydrogen is about $\frac{2}{15}$ of the oxygen; how many ounces of water will there be when there are 764 $\frac{2}{7}$ oz. of oxygen more than of hydrogen?

Ans. 1000 oz.

COMPOUND PROPORTION.

Proportion is called simple when the question involves only one condition, and compound when the question involves more conditions than one. As each condition implies a ratio, simple proportion is expressed, when the required term is found, by two ratios, and compound, by more than two. Thus, if the question be, How many men would be required to reap 65 acres in a given time, if 96 men, working equally, can reap 40 acres in the same time? Here there is but one condition, viz., that 96 men can reap 40 acres in the given time, which implies but one ratio, and when the question has been stated 40 : 65 :: 96 : F.P., and the required term is found to be 156, and the proportion 40 : 65 :: 96 : 156, we have the proportion, expressed by two ratios. But, suppose the question were, If a man, walking 12 hours a day, can accomplish a journey of 250 miles in 9 days. how many days would he require, walking at the

same rate, 10 hours each day, to travel 400 miles? Here there are two conditions, viz. : *first*, that, in the one case, he travels 12 hours a day, and in the other 10 hours; and, *secondly*, that the distances are 250 and 400 miles. The statement, as we shall presently show, would be $10 : 12 \left\{ \begin{array}{l} \\ 250 : 400 \end{array} \right. :: 9 : 17\frac{7}{5}$. Here each condition implies one ratio, 10 : 12 and 250 : 400, and when the required term, which is $17\frac{7}{5}$, is found, there are four ratios, viz., the two already noted, and $9 : 17\frac{7}{5}$, gives two more, one in relation to 10 : 12, and one in relation to 250 : 400. This will be evident, when we have shown the method of statement and operation.

EXPLANATORY STATEMENT
AND OPERATION.

$$\begin{array}{l} 11 : 33 :: 12 : \text{F. P.} \\ 1 : 3 : 12 : 36 \\ \hline 18 : 5 :: 36 : \text{F. P.} \\ 1 : 5 :: 2 : 10. \end{array}$$

PRACTICAL STATEMENT
AND OPERATION.

$$\begin{array}{l} 11 : 33 \left\{ \begin{array}{l} \\ 18 : 5 \end{array} \right\} :: 12 : \text{F. P.} \\ \hline 1 : 3 \left\{ \begin{array}{l} \\ 3 : 5 \end{array} \right\} :: 2 : \text{F. P.} \\ \hline 1 : 1 \left\{ \begin{array}{l} \\ 1 : 5 \end{array} \right\} :: 2 : 10. \end{array}$$

Let the question be, How many men would be required to reap 33 acres in 18 days, if 12 men, working equally, can reap 11 acres in 5 days?

We first proceed, as on the left margin, as if there were only one condition in the question; or, in other words, as if the number of days were the same in both cases, and the question were—If 12 men can reap 11 acres in a given time, how many men will be required to reap 33 acres in the same time. This, then, is a question in simple proportion, and by that rule we have the statement— $11 : 33 :: 12 : \text{F. P.}$, which, by contraction, becomes $1 : 3 :: 12 : \text{F. P.}$; and thus, we find F. P. to be 36, the number of men required, if the time were the same in both cases. The question is now resolved into this: How many men will be required to reap, in 18 days, the *same quantity* of crop that 36 men can reap in 5 days? This is obviously a case of inverse proportion, for the longer the time allowed the less will be the number of men required, and hence the statement, $18 : 5 : 36 : \text{F. P.}$, which, by contraction, becomes $1 : 5 :: 2 : \text{F. P.}$, which gives 10 for the number of men. The work may be shortened by making the two statements at once, as on the right margin. We first notice that the last term is to represent a

certain number of men, and, therefore, we place 12 in the third place; next, we see that, *other things being equal*, it will take more men to reap 33 than to reap 11 acres, and that, therefore, as far as that is concerned, the fourth term will be greater than the third, and so we put 11 in the first place, and 33 in the second. Again we see that, *other things being equal*, a less number of men will be required when 18 days are allowed for doing the work, than when it is required to be done in 5 days, and that therefore the fourth term, as far as that is concerned, will be less than the third, and therefore we write 18 : 5 below the other ratio as on the margin. Then by contraction we get $\frac{1}{3} : \frac{3}{5} \} :: 2 : \text{F. P.}$ Now, as 3 in the first term is to be a multiplier, and 3 in the second a divisor, we may omit these also, and we obtain $\frac{1}{1} : \frac{1}{5} \} :: 2 : 10$, the answer as before.

$$\begin{array}{r} 11 \times 18 : 33 \times 5 \\ \hline \end{array}$$

$$\begin{array}{r} 198 : 165 :: 12 : \text{F. P.} \\ 165 \times 12 = 10 \\ \hline 198 \end{array}$$

The full uncontracted operation would be to multiply 18 by 11, which gives 198, then to multiply 33 by 5, which gives 165, then multiply 165, the product of the two second terms, by 12, and divide the result, 1980, by 198, the product of the two first terms, which gives 10 as before.

Because in the analogy 198 : 165 :: 12 : 10, the first two terms are products, this kind of proportion has been called *compound*, and the ratio of 19 to 165 is called a *compound ratio*. We can show the strict and original meaning of the term *compound ratio* more easily by an example, than by any explanation in words. Let us take any series of numbers, whole, fractional or mixed, say 5, $\frac{7}{8}$, $\frac{3}{5}$, 19, 12, 1, 17, 11, $\frac{15}{16}$, 25, then the ratio of the first to the last is said to be compounded of the ratio of the first to the second, the second to the third, the third to the fourth, &c., &c., &c., to the end. Now the ratio of 5 to 25 is $\frac{25}{5} = 5$, and the several ratios are in this

order, $\frac{7}{8} \times \frac{3}{5} \times \frac{19}{12} \times \frac{1}{1} \times \frac{17}{11} \times \frac{15}{16} \times \frac{25}{5}$ which leaving finally $\frac{25}{5} = 5$ as before. If we took them in reverse order, viz., $\frac{5}{25} = \frac{1}{5}$, it is obvious that all therein could be cancelled, as each would in succession be a multiplier and a divisor.

We would also remark that compound proportion is nothing else than a number of questions in simple proportion solved by one opera-

tion. This will be evident from our second example by comparing the two operations on the opposite margins. Again, we remarked that every condition implies a ratio, and that therefore the third and fourth terms of our first example really involve two ratios, one in relation to each of the preceding. Hence universally the number of ratios, expressed and implied, must always be double the number of conditions, and therefore always even. As the third ratio is only written once, the number of ratios appears to be odd, but is in reality even.

R U L E :

Place, as in simple proportion, in the third place the term that is the same as the required term. Then consider each condition separately to see which must be placed first, and which second, other things being equal.

E X A M P L E .

1. If \$35.10 pay 27 men for 24 days; how much will pay 16 men 18 days? Here we first observe that the answer will be money, and therefore \$35.10 must be in the third place. Again, it will take less money to pay 16 men than 27 men, and therefore, other things being equal, the answer, as far as this is concerned, will be less than \$35.10, and therefore we put the less quantity, 16, in the second place. So also because it will take less to pay any given number of men for 18 days than for 24 days, therefore we put the less quantity in the second place, which the statement shows in the margin.
- | | |
|--------------------|--|
| 27 : 16 :: \$35.10 | |
| 24 : 18 | |
| 3 : 2 | |
| 3 : 2 | |
| 9 : 5 :: \$35.10 | |
| 4 | |
| 9)140.40 | |
| Ans. \$15.60 | |

E X E R C I S E S .

1. If 15 men; working 12 hours a day, can reap 60 acres in 16 days; in what time would 20 boys, working 10 hours a day, reap 98 acres, if 7 men can do as much as 8 boys in the same time?

Ans. $26\frac{2}{5}$ days.

2. If 15 men, by working $6\frac{2}{3}$ hours a day, can dig a trench 48 feet long, 8 feet broad, and 5 feet deep, in 12 days; how many hours a day must 25 men work in order to dig a trench 36 feet long, 12 feet broad, and 3 feet deep, in 9 days?

Ans. $3\frac{2}{5}$.

3. If 48 men can build a wall 864 feet long, 6 feet high, and 3 feet wide, in 36 days; how many men will be required to build a wall 36 feet long, 8 feet high, and 4 feet wide, in 4 days. Ans. 32.

4. In what time would 23 men weed a quantity of potato ground which 40 women would weed in 6 days, if 7 men can do as much as 9 women? Ans. $8\frac{8}{9}$ days.

5. Suppose that 50 men can dig in 27 days, working 5 hours a day, 18 cellars which are each 48 feet long, 28 feet wide, and 15 feet deep; how many days will 50 men require, working 3 hours each day, to dig 24 cellars which are each 36 feet long, 21 feet wide, and 20 feet deep. Ans. 45 days.

6. If 15 bars of iron, each 6 ft. 6 in. long, 4 in. broad, and 3 in. thick, weigh 20 cwt., 3 qrs., (28 lbs.) 16 lbs.; how much will 6 bars 4 ft. long, 3 in. broad, and 2 in. thick, weigh?

Ans. 2 cwt., 2 qrs., 8 lbs.

7. If 112 men can seed 460 acres, 3 roods, 8 rods, in 6 days; how many men will be required to seed 72 acres in 5 days?

Ans. 21.

8. If the freight, by railway, of 13 cwt. for 65 miles be 45 shillings; how far should $35\frac{5}{4}$ be carried for 75 shillings?

Ans. 40 miles.

9. If a family of 9 persons can live comfortably in London, England, for 1560 guineas a year; what will it cost a family of 8 to live in London, C. W., in the same style, for seven months, prices supposed to be $\frac{3}{5}$ of what they would be in England?

Ans. £509.12

10. If 63 lbs. of tea cost £20 10s. 6d.; what will 70 lbs. of a different quality cost, 9 lbs. of the former being equal in value to 10 lbs. of the latter?

Ans. £20 10s. 6d.

11. If 120 yards of carpeting, 5 quarters wide, cost \$60; what will be the price of 36 yards of the same quality, but 7 quarters wide?

Ans. \$25.20

12. If 48 men, in 5 days of $12\frac{1}{2}$ hours each, can dig a canal $139\frac{3}{4}$ yards long, $4\frac{1}{2}$ yards wide, and $2\frac{1}{2}$ yards deep; how many hours per day must 90 men work for 42 days to dig $491\frac{1}{16}$ yards long, $4\frac{7}{8}$ yards wide, and $3\frac{1}{5}$ yards deep?

Ans. 4.

13. A, standing on the bank of a river, discharges a cannon, and B, on the opposite bank, counts six pulsations at his wrist between the flash and the report; now, if sound travels 1142 feet per second.

and the pulse of a person in health beats 75 strokes in a minute, what is the breadth of the river? Ans. 1 mile, $201\frac{3}{5}$ feet.

14. If 264 men, working 12 hours a day, can make 240 yards of a canal, 3 yards wide, and 12 yards deep, in 5 days; how long will it take 24 men, working 9 hours a day, to make another portion 420 yards long, 5 yards wide, and 3 yards deep?

Ans. $320\frac{5}{8}$ days.

15. If the charge per freight train for 10800 lbs. of flour be \$16 for 20 miles; how much will it be for 12500 lbs. for 100 miles?

Ans. $\$92\frac{1}{2}$.

16. If \$42 keep a family of 8 persons for 16 days; how long, at that rate, will \$100 keep a family of 6 persons? Ans. $50\frac{5}{8}$ days.

17. If a mixture of wine and water, measuring 63 gallons, consist of four parts wine, and one of water, and be worth \$138.60; what would 85 gallons of the same wine in its purity be worth?

Ans. \$233.75.

18. If I pay 16 men \$62.40 for 18 days work; how much must I pay 27 men at the same rate? Ans. \$140.40.

19. If 60 men can build a wall 300 feet long, 8 feet high, and 6 feet thick, in 120 days, when the days are 8 hours long; in what time would 12 men build a wall 30 feet long, 6 feet high, and 3 feet thick, when the days are 12 hours long? Ans. 15 days.

20. If 24 men, in 132 days, of 9 hours each, dig a trench of four degrees of hardness, $337\frac{1}{2}$ feet long, $5\frac{3}{5}$ feet wide, and $3\frac{1}{2}$ feet deep; in how many days, of 11 hours each, will 496 men dig a trench of 7 degrees of hardness, 465 feet long, $3\frac{2}{3}$ feet wide, and $2\frac{1}{3}$ feet deep?

Ans. $5\frac{1}{2}$.

21. If 50 men, by working 3 hours each day, can dig, in 45 days, 24 cellars, which are each 36 feet long, 21 feet wide, and 20 feet deep; how many men would be required to dig, in 27 days, working 5 hours each day, 18 cellars, which are each 48 feet long, 28 feet wide, and 15 feet deep? Ans. 50.

22. If 15 men, 12 women, and 9 boys, can complete a certain piece of work in 50 days; what time would 9 men, 15 women, and 18 boys, require to do twice as much, the parts performed by each, in the same time, being as the numbers 3, 2 and 1? Ans. 104 days.

23. If 12 oxen and 35 sheep eat 12 tons, 12 cwt. of hay, in 8 days; how much will it cost per month (of 28 days,) to feed 9 oxen and 12 sheep, the price of hay being \$40 per ton, and 3 oxen being supposed to eat as much as 7 sheep? Ans. \$924.

24. A vessel, whose speed was $9\frac{1}{4}$ miles per hour, left Belleville at 8 o'clock, a. m., for Gananoque, a distance of 74 miles. A second vessel, whose speed was to that of the first as 8 is to 5, starting from the same place, arrived 5 minutes before the first; what time did the second vessel leave Belleville? Ans. 55 min. past 10 o'clock, a. m.

25. If 9 compositors, in 12 days, working 10 hours each day, can compose 36 sheets of 16 pages to a sheet, 50 lines to a page, and 45 letters in a line; in how many days, each 11 hours long, can 5 compositors compose a volume, consisting of 25 sheets, of 24 pages in a sheet, 44 lines in a page, and 40 letters in a line? Ans. 16 days.

MISCELLANEOUS EXERCISES ON THE PRECEDING RULES.

1. What is the value of .7525 of a mile?

Ans. 6 fur., 0 rd, 4 yds, 1 ft., $2\frac{2}{5}$ in.

2. What is the value of .25 of a score?

Ans. 5.

3. Reduce 1 ft. 6 in. to the decimal of a yard.

Ans. .5.

4. What is the value of 14 yards of cloth, at \$3.375 per yard?

Ans. \$47.15.

5. What part of 2 weeks is $\frac{5}{14}$ of a day?

Ans. $\frac{5}{98}$.

6. What part of £1 is 13s. 4d?

Ans. $\frac{2}{3}$.

7. Reduce $\frac{9}{75}$ of a day to hours, minutes and seconds.

Ans. 2 hours, 52 min., 48 sec.

8. Add $\frac{2}{3}$ of a furlong to $\frac{8}{9}$ of a mile.

Ans. 7 fur., 31 rds, 0 yd., 1 ft., 10 in.

9. What is the value of $.857\frac{1}{7}$ of a bushel of rye?

Ans. 48 pounds.

10. Reduce 47 pounds of wheat to the decimal of a bushel.

Ans. $.783\frac{1}{3}$.

11. Reduce 9 dozen to the decimal of a gross.

Ans. .75.

12. Add $\frac{7}{10}$ of a cwt. to $\frac{2}{5}$ of a quarter.

Ans. 3 qrs., 10 lbs.

13. Subtract $\frac{7}{8}$ of a day from $\frac{5}{7}$ of a week.

Ans. 4 days, 3 hrs.

14. From $\frac{1}{18}$ of 5 tons take $\frac{3}{7}$ of 9 cwt.

Ans. 2 tons, 17 cwt., 1 qr., $\frac{2}{3}\frac{5}{3}$ lbs.

15. How many yards of cloth, at $\$3\frac{1}{2}$ a yard, can be bought for \$48 $\frac{1}{7}$?

Ans. $13\frac{3}{4}\frac{7}{9}$ yards.

16. A man bought $\frac{7}{8}$ of a yard of cloth for \$2.80; what was the rate per yard?

Ans. \$3.20.

17. How many tons of hay, at $\$16\frac{1}{2}$ per ton, can be bought for \$196 $\frac{1}{8}$?

Ans. $11\frac{3}{4}\frac{9}{4}$ tons.

18. At $\$17\frac{3}{5}$ per week, how many weeks can a family board for $\$765\frac{3}{5}$? Ans. $43\frac{1}{2}$ weeks.

19. What number must be added to $26\frac{3}{5}$, and the sum multiplied by $7\frac{3}{4}$, that the product may be 496? Ans. $37\frac{2}{5}$.

20. A man owns $\frac{3}{4}$ of an oil well. He sells $\frac{2}{3}$ of his share for $\$3500$; what part of his share in the well has he still, and what is it worth at the same rate? Ans. $\frac{1}{4}$ left, and worth $\$875$.

21. How long will $119\frac{3}{7}$ hhds. of water last a company of 30 men, allowing each man $\frac{2}{5}$ of a gallon a day? Ans. 627 days.

22. Reduce $\frac{2}{7}$ of $2\frac{4}{5}$, $\frac{8}{12}$ of $1\frac{5}{6}$, and $3\frac{1}{2}$ of $2\frac{2}{7}$, to equivalent fractions having the *least* common denominator. Ans. $\frac{36}{45}$, $\frac{55}{45}$, $\frac{360}{45}$.

23. From $\frac{2}{7}$ of $2\frac{4}{5}$ of 4, take $\frac{9}{17}$ of $6\frac{4}{5}$ of $\frac{1}{9}$. Ans. $2\frac{4}{5}$.

24. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$? Ans. $1\frac{2089}{2520}$.

25. What is the sum of $\frac{2}{3}\frac{1}{5}$ of $3\frac{5}{8} + \frac{1}{4}\frac{1}{5}$ of 85? Ans. $22\frac{2491}{520}$.

26. How long will it take a person to travel 442 miles, if he travels $3\frac{1}{4}$ miles per hour, and $8\frac{1}{2}$ hours a day? Ans. 16 days.

27. Find the sum of $2\frac{1}{2}$ of $\frac{7}{10}$, $3\frac{1}{3}$ of $\frac{7}{9}$ of $\frac{6}{17}$ of $4\frac{1}{4}$ and $\frac{1}{2}$.

Ans. $6\frac{5}{8}$.

28. A has $2\frac{1}{7}$ times $8\frac{5}{6}$ dollars, and B $6\frac{1}{2}$ times $9\frac{3}{4}$ dollars; how much more has B than A? Ans. $\$44\frac{25}{6}$.

29. If I sell hay at $\$1.75$ per cwt.; what should I give for $9\frac{3}{5}$ tons, that I may make $\$7$ on my bargain. Ans. $\$329$.

30. If 7 horses eat $93\frac{1}{3}$ bushels of oats in 60 days; how many bushels will one horse eat in $87\frac{3}{7}$ days? Ans. $19\frac{3}{7}$.

31. Bought $14\frac{7}{10}$ yards of broadcloth for $\$102.90$; what was the value of $87\frac{3}{7}$ yards of the same cloth? Ans. $\$612$.

32. How many bushels of wheat, at $\$2\frac{2}{3}$ per bushel, will it require to purchase $168\frac{8}{9}$ bushels of corn worth 75 cents per bushel?

Ans. $47\frac{3}{11}$.

33. If in $82\frac{1}{2}$ feet there are 5 rods; how many rods in one mile?

Ans. 320.

34. Suppose I pay $\$55$ for $\frac{5}{8}$ of an acre of land; what is that per acre? Ans. $\$88$.

35. If $\frac{5}{8}$ of a pound of tea cost $\$1.66\frac{1}{3}$; what will $\frac{7}{9}$ of a pound cost? Ans. $\$1.55\frac{11}{15}$.

36. Subtract the sum of $2\frac{1}{2}$ and $1\frac{1}{12}$, from the sum of $\frac{3}{4}$, $7\frac{4}{9}$ and 3, and multiply the remainder by $3\frac{3}{11}$. Ans. $24\frac{10}{11}$.

37. If $\frac{7}{8}$ lb. cost $23\frac{3}{4}$ cents; what will $2\frac{1}{12}$ cost?

Ans. $77\frac{8}{11}$ cents.

38. What is the difference between $2\frac{1}{10} \times 3\frac{1}{9}$ and $2\frac{1}{9} \times 3\frac{1}{10}$?

Ans. $\frac{1}{90}$.

39. If $\frac{7}{8}$ lb. cost $\$ \frac{3}{8}$; what will $\frac{1}{12}$ lb. cost? Ans. $39\frac{2}{7}$ cents.

40. What is the difference between $\frac{3}{4}$ of $\frac{1}{3} + \frac{1}{5} + \frac{1}{7} \times \frac{1}{9}$, and $\frac{1}{4} + \frac{1}{6} + \frac{1}{8}$? Ans. $\frac{1}{2520}$.

41. If $4\frac{7}{11}$ yards cost $\$1\frac{1}{3}$, what will $2\frac{1}{8}$ yards cost?

Ans. $47\frac{2}{9}$ cents.

42. Bought $\frac{3}{7}$ of 2000 yards of ribbon, and sold $\frac{2}{3}$ of it; how much remains? Ans. $285\frac{5}{7}$ yards.

43. Divide the sum of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{15}{16}$, $\frac{31}{32}$, $\frac{63}{64}$, $\frac{127}{128}$ by the sum of $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$, and divide the quotient by $6\frac{7}{127}$, and multiply the result by $\frac{3}{4}$ of $\frac{5}{6}$. Ans. $\frac{5}{8}$.

44. I bought $\frac{7}{8}$ of a lot of wood land, consisting of 47 acres, 3 roods, 20 rods, and have cleared $\frac{1}{2}$ of it; how much remains to be cleared? Ans. 20 acres, 3 roods, $31\frac{1}{4}$ rods.

45. What is the difference between $1\frac{25}{128}$ and $1\frac{2}{3}$? Ans. $\frac{247}{504}$.

46. If $\$1\frac{1}{2}$ pay for a $1\frac{1}{2}$ st. of flour; for how much will $\$ \frac{5}{8}$ pay? Ans. $1\frac{1}{4}$ st.

47. Mount Blanc, the highest mountain in Europe, is 15,872 feet above the level of the sea; how far above the sea level is a climber who is $\frac{3}{16}$ of the whole height from the top, *i. e.*, $\frac{3}{16}$ of perpendicular height? Ans. 12896 feet.

48. What will 45.94375 tons cost if 12.796875 tons cost \$54.64? Ans. \$196.17.

49. If I gain \$37.515625 by selling goods worth \$324.53125; what shall I gain by selling a similar lot for \$520.6635416? Ans. \$60.1884.

50. If 52.815 cwt. cost \$22.345; what will 192.664 cwt. cost at the same rate? Ans. \$81.512+

51. Required, the sum of the surfaces of 5 boxes, each of which is $5\frac{1}{2}$ feet long, $2\frac{1}{4}$ feet high, and $3\frac{1}{6}$ feet wide, and also the number of cubic feet contained in each box. The box supposed to be made from inch lumber? Ans. Surface, $390\frac{5}{6}$ ft.; each box $42\frac{863}{864}$ C. F.

52. If I pay $\$ \frac{9}{10}$ for sawing into three pieces wood that is 4 ft. long; how much more should I pay, per cord, for sawing into pieces of the same length, wood that is 8 feet long? Ans. $22\frac{1}{2}$ cents.

53. A sets out from Oshawa, on a journey, and travels at the rate of 20 miles a day; 4 days after, B sets out from the same place, and travels the same road, at the rate of 25 miles per day; how many days before B will overtake A? 8 Ans. 16.

54. A farmer having $56\frac{1}{2}$ tons of hay, sold $\frac{3}{5}$ of it at $\$10\frac{5}{8}$ per ton, and the remainder at $\$9.75$ per ton; how much did he receive for his hay? Ans. $\$580\frac{4}{8}\frac{3}{0}$.

55. If the sum of $87\frac{1}{12}$ and $117\frac{1}{2}\frac{6}{7}$ is divided by their difference; what will be the quotient? Ans. $6\frac{5}{6}\frac{9}{4}\frac{3}{1}$.

56. If $8\frac{3}{4}$ yards of silk make a dress, and 9 dresses be made from a piece containing 80 yards; what will be the remnant left?

Ans. $1\frac{1}{4}$ yards.

57. A merchant expended $\$840$ for dry goods, and then had remaining only $\frac{3}{4}\frac{7}{9}$ as much money as he had at first; how much money had he at first? Ans. $\$3430$.

58. If a person travel a certain distance in 8 days and 9 hours, by travelling 12 hours a day; how long will it take him to perform the same journey, by traveling $8\frac{3}{4}$ hours a day? Ans. 12 days.

59. If 15 horses, in 4 days, consume 87 bushels, 6 qrts. of oats; how many horses will 610 bushels, 1 peck, 2 qrts, keep for the same time? Ans. 105.

60. Reduce 1 pound troy, to the fraction of one pound avoirdupois. Ans. $\frac{1}{1}\frac{4}{7}\frac{4}{5}$.

61. Reduce $\frac{\frac{2}{5} \times \frac{1}{12}}{4\frac{1}{2} \text{ of } \frac{2}{9}}$ to a simple fraction. Ans. $\frac{1}{3}$.

62. What will be the cost of 8 cwt., 3 qrs., $12\frac{1}{2}$ lbs. of beef, if 4 cwt. cost $\$34$? Ans. $\$75\frac{7}{16}$.

63. If 4 men, working 8 hours a day, can do a certain piece of work in 15 days; how long would it take one man, working 10 hours a day, to do the same piece of work? Ans. 48 days.

64. Divide $\$1728$ among 17 boys and 15 girls, and give each boy $\frac{7}{11}$ as much as a girl; what sum will each receive?

Ans. Each girl, $\$66\frac{6}{7}\frac{6}{1}$; each boy, $\$42\frac{4}{7}\frac{2}{1}$.

65. If A can cut 2 cords of wood in $12\frac{1}{3}$ hours, and B can cut 3 cords in $17\frac{1}{2}$ hours; how many cords can they both cut in $24\frac{1}{2}$ hours?

Ans. $8\frac{3}{18}\frac{2}{5}$.

66. If it requires 30 yards of carpeting, which is $\frac{3}{4}$ of a yard wide, to cover a floor; how many yards, which is $1\frac{1}{4}$ yards wide, will be necessary to cover the same floor? Ans. 18.

67. A person bought 1000 gallons of spirits for $\$1500$; but 140 gallons leaked out; at what rate per gallon must he sell the remainder so as to make $\$200$ by his bargain? Ans. $\$2$ per gallon.

68. What must be the breadth of a piece of land whose length is $40\frac{1}{2}$ yards, in order that it may be twice as great as another piece of

land whose length is $14\frac{5}{8}$ yards, and whose breadth is $13\frac{2}{3}$ yards?

Ans. $9\frac{1}{2}$ yards.

69. If 7 men can reap a rectangular field whose length is 1,800 feet, and breadth 960 feet, in 9 days of 12 hours each; how long will it take 5 men, working 14 hours a day, to reap a field whose length is 800 feet, and breadth 700 feet?

Ans. $3\frac{1}{2}$ days.

70. 124 men dug a trench 110 yards long, 3 feet wide, and 4 feet deep, in 5 days of 11 hours each; another trench was dug by one-half the number of men in 7 days of 9 hours each; how many feet of water was it capable of holding?

Ans. 2268 cubic feet.

71. If 100 men, by working 6 hours each day, can, in 27 days, dig 18 cellars, each 40 feet long, 36 feet wide, and 12 feet deep; how many cellars, that are each 24 feet long, 27 feet wide, and 18 feet deep, can 240 men dig in 81 days, by working 8 hours a day?

Ans. 256.

72. A gentleman left his son a fortune, $\frac{1}{5}$ of which he spent in 2 months, $\frac{1}{4}$ of the remainder lasted him 3 months longer, and $\frac{2}{3}$ of what then remained lasted him 5 months longer, when he had only \$895.50 left; how much did his father leave him?

Ans. \$4477.50.

73. A farmer having sheep in two different fields, sold $\frac{1}{4}$ of the number from each field, and had only 102 sheep remaining. Now 12 sheep jumped from the first field into the second; then the number remaining in the first field, was to the number in the second field as 8 to 9; how many sheep were there in each field at first?

Ans. 80 in first field; 56 in second.

74. A and B paid \$120 for 12 acres of pasture for 8 weeks, with an understanding that A should have the grass that was then on the field, and B what grew during the time they were grazing; how many oxen, in equity, can each turn into the pasture, and how much should each pay, providing 4 acres of pasture, together with what grew during the time they were grazing, will keep 12 oxen 6 weeks, and in similar manner, 5 acres will keep 35 oxen 2 weeks?

Ans. $\left\{ \begin{array}{l} \text{A should turn into the field 18 oxen, and pay \$72.} \\ \text{B should turn into the field 12 oxen, and pay \$48.} \end{array} \right.$

ANALYSIS AND SYNTHESIS.

Analysis is the act of separating and comparing all the different parts of any compound, and showing their connection with each other, and thereby exhibiting all its elementary principles.

The converse of Analysis is Synthesis. The meaning and use of these terms will probably be most readily comprehended by reference to their derivation.

They are both pure Greek words. Analysis means *loosing up*. The general reader would here probably expect *loosing down*, as employed in most popular definitions; but we may illustrate the Greek term, *loosing up*, by our own everyday phrase, *tearing up*, which means *rending into shreds*, the English *up* conveying the same idea here as the Greek *ana* in analysis. The Greek synthesis means literally placing together; that is, the component parts being known, the word synthesis indicates the act of combining them into one. We might give many illustrations, but one will suffice, and we choose the one which will be most generally understood. When we analyse a sentence, we loose it *up*, or tear it *up*, into its component parts, and by synthesis we write or compose, *i. e.*, put together the parts, which, by analysis, we have found it to consist of.

When we commence to analyse a *problem* we reason from a given quantity to its unit, and then from this unit to the required quantity; hence, all our deductions are self-evident, and we therefore require *no rule* to solve a problem by analysis.

Although this part of arithmetic is usually called analysis, yet, as it is really both analysis and synthesis, we have given it a title in accordance with the principles now laid down.

EXAMPLE.

1. If 12 pounds of sugar cost \$1.80, what will 7 pounds cost?

SOLUTION.

$$\begin{array}{r} 12 \overline{)1.80} \\ \underline{15} \\ 7 \\ \underline{105} \\ 105 \end{array}$$

If 12 lbs. cost \$1.80, one pound will cost the $\frac{1}{12}$ of \$1.80=15 cents. Now, if 1 lb. cost 15 cents, 7 lbs. will cost 7 times 15 cents—to \$1.05. Therefore, 7 lbs. of sugar will cost \$1.05, if 12 lbs. cost \$1.80.

NOTE.—The work may be somewhat shortened, especially in long questions, by arranging it in the following manner, so as to admit of cancelling, if possible:—

$$\frac{1}{12} \times \frac{180}{1} \times \frac{7}{1} = \frac{105}{1} = \$1.05. \quad \text{Ans.}$$

2. If 5 bushels of pease cost \$5.50, for what can you purchase 19 bushels?
Ans. \$20.90.

3. If 9 men can perform a certain piece of labor in 17 days, how long will it take 3 men to do it? Ans. 51 days.

4. How many pigs, at \$2 each, must be given for 7 sheep, worth \$4 a head? Ans. 14.

5. If \$100 gain \$6 in 12 months, how much would it gain in 40 months? Ans. \$20.

6. If $4\frac{2}{3}$ bushels of apples cost $\$3\frac{1}{9}$, what will be the cost of $7\frac{1}{2}$ bushels?

SOLUTION.

In the first place, $4\frac{2}{3}$ bushels $= \frac{14}{3}$ bushels, and $\$3\frac{1}{9} = \$\frac{28}{9}$. Now, since $\frac{14}{3}$ bushels cost $\$ \frac{28}{9}$, one bushel will cost $\$ \frac{28}{9} \div \frac{14}{3} = \frac{28}{9} \times \frac{3}{14} = \$\frac{2}{3}$, and $7\frac{1}{2}$ or $\frac{15}{2}$ bushels will cost $\frac{15}{2}$ times $\$ \frac{2}{3} = \frac{2}{3} \times \frac{15}{2} = \$5$, the value of $7\frac{1}{2}$ bushels of apples, if $4\frac{2}{3}$ bushels are worth $\$3\frac{1}{9}$.

OPERATION.

$$\frac{\overset{2}{28}}{\underset{9}{9}} \times \frac{\overset{3}{3}}{\underset{14}{14}} \times \frac{\overset{5}{15}}{\underset{2}{2}} = \frac{5}{1} = \$5.$$

7. If $\frac{2}{3}$ of $3\frac{3}{4}$ lbs. of tea cost $\$1\frac{7}{8}$; what will be the cost of $5\frac{1}{2}$ pounds? Ans. $\$4.12\frac{1}{2}$.

8. 100 is $\frac{2}{3}$ of what number? Ans. 150.

9. If $\frac{4}{9}$ of a mine cost \$2800; what is the value of $\frac{2}{3}$ of it? Ans. \$4200.

10. $\frac{2}{3}$ of 24 is $1\frac{2}{3}$ times what number? Ans. 10.

11. $\frac{3}{4}$ of 40 is $\frac{5}{12}$ of how many times $\frac{1}{2}$ of $\frac{4}{5}$ of 20? Ans. 9.

12. A is 16 years old, and his age is $\frac{2}{3}$ times $\frac{2}{3}$ of his father's age; how old is his father? Ans. 36.

13. A and B were playing cards; A lost \$10, which was $\frac{1}{3}$ times $\frac{3}{5}$ as much as B then had; and when they commenced $\frac{2}{3}$ of A's money was equal to $\frac{3}{4}$ of B's; how much had each when they began to play? Ans. A \$50; B \$40.

14. A man willed to his daughter \$560, which was $\frac{1}{3}$ of $\frac{3}{4}$ of what he bequeathed to his son; and 4 times the son's portion was $\frac{2}{3}$ the value of the father's estate; what was the value of the estate? Ans. \$13,440.

15. A gentleman spent $\frac{1}{4}$ of his life in Montreal, $\frac{1}{3}$ of it in Ottawa, and the remainder of it, which was 25 years, in Belleville; what age was he when he died? Ans. 60 years.

16. A owns $\frac{1}{8}$, and B $\frac{1}{12}$ of a ship ; A's part is worth \$650 more than B's ; what is the value of the ship ? Ans. \$15,600.

17. A post stands $\frac{1}{4}$ in the mud, $\frac{1}{3}$ in the water, and 15 feet above the water ; what is the length of the post ? Ans. 36 feet.

18. A grocer bought a firkin of butter containing 56 pounds, for \$11.20, and sold $\frac{3}{4}$ of it for $\$8\frac{2}{5}$; how much did he get a pound ? Ans. 20 cents.

19. The head of a fish is 4 feet long, the tail as long as the head and $\frac{1}{2}$ the length of the body, and the body is as long as the head and tail ; what is the length of the fish ? Ans. 32 feet.

20. A and B have the same income ; A saves $\frac{1}{7}$ of his ; B, by spending \$65 a year more than A, finds himself \$25 in debt at the end of 5 years ; what did B spend each year ? Ans. \$425.

21. A can do a certain piece of work in 8 days, and B can do the same in 6 days ; A commenced and worked alone for 3 days, when B assisted him to complete the job ; how long did it take them to finish the work ?

SOLUTION.

If A can do the work in 8 days, in one day he can do the $\frac{1}{8}$ of it, and if B can do the work in 6 days, in one day he can do the $\frac{1}{6}$ of it, and if they work together, they would do $\frac{1}{8} + \frac{1}{6} = \frac{7}{24}$ of the work in one day. But A works alone for 3 days, and in one day he can do $\frac{1}{8}$ of the work, in 3 days he would do 3 times $\frac{1}{8} = \frac{3}{8}$ of the work, and as the whole work is equal to $\frac{8}{8}$ of itself, there would be $\frac{8}{8} - \frac{3}{8} = \frac{5}{8}$ of the work yet to be completed by A and B, who, according to the conditions of the question, labour together to finish the work. Now A and B working together for one day can do $\frac{7}{24}$ of the entire job, and it will take them as many days to do the balance $\frac{5}{8}$ as $\frac{7}{24}$ is contained in $\frac{5}{8}$, which is equal $\frac{5}{8} \times \frac{24}{7} = 7\frac{1}{7}$ days.

22. A and B can build a boat in 18 days, but if C assists them, they can do it in 8 days ; how long would it take C to do it alone ? Ans. $14\frac{2}{5}$ days.

23. A certain pole was $25\frac{1}{2}$ feet high, and during a storm it was broken, when $\frac{3}{4}$ of what was broken off, equalled $\frac{2}{3}$ of what remained ; how much was broken off, and how much remained ?

Ans. 12 feet broken off, and $13\frac{1}{2}$ remained.

24. There are 3 pipes leading into a certain cistern ; the first will fill it in 15 minutes, the second in 30 minutes, and the third in one hour ; in what time will they all fill it together ?

Ans. 8 min., $34\frac{2}{7}$ sec.

25. A and B start together by G. T. R. from Cobourg to Brockville, a distance of 138 miles. A goes by freight train, at the rate of 12 miles per hour, and B by mixed train, at the rate of 18 miles per hour. C leaves Brockville for Cobourg at the same time by express train, which runs at the rate of 22 miles per hour; how far from Cobourg will A and B each be when C meets them?

Ans. A $48\frac{1}{7}$ miles; B $62\frac{1}{10}$ miles.

26. A cistern has two pipes, one will fill it in 48 minutes, and the other will empty it in 72 minutes; what time will it require to fill the cistern when both are running? Ans. 2 hours, 24 min.

27. If a man spends $\frac{5}{12}$ of his time in working, $\frac{1}{3}$ in sleeping, $\frac{1}{6}$ in eating, and $1\frac{1}{2}$ hours each day in reading; how much time will be left? Ans. 3 hours.

28. A wall, which was to be built 32 feet high, was raised 8 feet by 6 men in 12 days; how many men must be employed to finish the wall in 6 days? Ans. 30 men.

29. A and B can perform a piece of work in $5\frac{5}{11}$ days; B and C in $6\frac{2}{3}$ days; and A and C in 6 days; in what time would each of them perform the work alone, and how long would it take them to do the work together?

Ans. A, 10 days; B, 12 days; C, 15 days; and A, B, and C, together, in 4 days.

30. My tailor informs me that it will take $10\frac{1}{4}$ square yards of cloth to make me a full suit of clothes. The cloth I am about to purchase is $1\frac{7}{8}$ yards wide, and on sponging it will shrink $\frac{1}{20}$ in width and length; how many yards of this cloth must I purchase for my "new suit?" Ans. $6\frac{6}{1083}$ yards.

31. If A can do $\frac{2}{3}$ of a certain piece of work in 4 hours, and B can do $\frac{3}{4}$ of the remainder in 1 hour, and C can finish it in 20 min.; in what time will they do it all working together?

Ans. 1 hour, 30 min.

32. A certain tailor in the City of Hamilton bought 40 yards of broadcloth, $2\frac{1}{4}$ yds wide; but on sponging, it shrunk in length upon every 2 yards, $\frac{1}{6}$ of a yard, and in width, $1\frac{1}{2}$ sixteenths upon every $1\frac{1}{2}$ yards. To line this cloth he bought flannel $1\frac{1}{4}$ yards wide, which, when wet, shrunk $\frac{1}{2}$ the width on every 10 yards in length, and in width it shrunk $\frac{1}{2}$ of a sixteenth of a yard; how many yards of flannel had the tailor to buy to line his broadcloth?

Ans. $71\frac{7}{8}$ yards.

33. If 6 bushels of wheat are equal in value to 9 bushels of barley, and 5 bushels of barley to 7 bushels of oats, and 12 bushels of

oats to 10 bushels of pease, and 13 bushels of pease to $\frac{1}{2}$ ton of hay, and 1 ton of hay to 2 tons of coal, how many tons of coal are equal in value to 80 bushels of wheat?

SOLUTION.

If 6 bushels of wheat are equal in value to 9 bushels of barley, or 9 bushels of barley to 6 bushels of wheat, one bushel of barley would be equal to $\frac{1}{3}$ of 6 bushels of wheat, equal to $\frac{6}{9}$, or $\frac{2}{3}$ of a bushel of wheat, and 5 bushels of barley would be equal to 5 times $\frac{2}{3}$ of a bushel of wheat, equal to $\frac{2}{3} \times 5 = \frac{10}{3} = 3\frac{1}{3}$ bushels of wheat. But 5 bushels of barley are equal to seven bushels of oats; hence, 7 bushels of oats are equal to $3\frac{1}{3}$ bushels of wheat, and one bushel of oats would be equal to $3\frac{1}{3} \div 7 = \frac{10}{21}$ bushels of wheat, and 12 bushels of oats would be equal to 12 times $\frac{10}{21} = \frac{120}{21} = 5\frac{5}{7}$ bushels of wheat. But 12 bushels of oats are equal in value to 10 bushels of pease, hence, 10 bushels of pease are equal to $5\frac{5}{7}$ bushels of wheat, and one bushel of pease would equal $5\frac{5}{7} \div 10 = \frac{4}{7}$ of a bushel of wheat, and 13 bushels of pease would equal $\frac{4}{7} \times 13 = \frac{52}{7} = 7\frac{3}{7}$ bushels of wheat. But 13 bushels of pease equal in value $\frac{1}{2}$ ton of hay, hence, $\frac{1}{2}$ ton of hay equals $7\frac{3}{7}$ bushels of wheat, and one ton would equal $7\frac{3}{7} \times 2 = 14\frac{6}{7}$ bushels of wheat. But one ton of hay equals 2 tons of coal, hence, 2 tons of coal are equal in value to $14\frac{6}{7}$ bushels of wheat, and one ton would equal $14\frac{6}{7} \div 2 = 7\frac{3}{7}$ bushels of wheat. Lastly, if $7\frac{3}{7}$ bushels of wheat be equal in value to one ton of coal, it would take as many tons of coal to equal 80 bushels of wheat, as $7\frac{3}{7}$ is contained in 80, which gives $10\frac{10}{13}$ tons of coal.

NOTE.—This question belongs to that part of arithmetic usually called Conjoined Proportion, or, by some, the “Chain Rule,” which has each antecedent of a compound ratio equal in value to its consequent. We have thought it best not to introduce such questions under a head by themselves, on account of their *theory* being more easily understood when exhibited by Analysis than by Proportion. Questions that do occur like this will most probably relate to Arbitration of Exchange. Although they may all be worked by Compound Proportion as well as by Analysis, yet the most expeditious plan, and the one generally adopted, is by the following

RULE.

Place the antecedents in one column and the consequents in another, on the right, with the sign of equality between them. Divide the continued product of the terms in the column containing the odd term by the continued product of the other column, and the quotient will be the answer.

hours each, would the same lot of men build a wall 20 feet long, 12 feet high, and $2\frac{1}{2}$ feet thick ?

Ans. $23\frac{7}{16}$.

38. If 5 men can perform a piece of work in 12 days of 10 hours each ; how many men will perform a piece of work four times as large, in a fifth part of the time, if they work the same number of hours in a day, supposing that 2 of the second set can do as much work in an hour as 3 of the first set ?

Ans. $66\frac{2}{3}$ men.

NOTE.—Such questions as this, where the answer involves a fraction, may frequently occur, and it may be asked how $\frac{2}{3}$ of a man can do any work. The answer is simply this, that it requires 66 men to do the work, and one man to continue on working $\frac{2}{3}$ of a day more.

39. Suppose that a wolf was observed to devour a sheep in $\frac{7}{8}$ of an hour, and a bear in $\frac{3}{4}$ of an hour ; how long would it take them together to eat what remained of a sheep after the wolf had been eating $\frac{1}{2}$ an hour ?

Ans. $10\frac{5}{13}$ min.

40. Find the fortunes of A, B, C, D, E, and F, by knowing that A is worth \$20, which is $\frac{1}{4}$ as much as B and C are worth, and that C is worth $\frac{1}{3}$ as much as A and B, and also that if 19 times the sum of A, B and C's fortune was divided in the proportion of $\frac{3}{4}$, $\frac{1}{2}$ and $\frac{1}{3}$, it would respectively give $\frac{3}{4}$ of D's, $\frac{1}{2}$ of E's, and $\frac{1}{3}$ of F's fortune.

Ans. A, 20 ; B, 55 ; C, 25 ; and D, E and F, 1200 each.

41. A and B set out from the same place, and in the same direction. A travels uniformly 18 miles per day, and after 9 days turns and goes back as far as B has travelled during those 9 days ; he then turns again, and pursuing his journey, overtakes B $22\frac{1}{2}$ days after the time they first set out. It is required to find the rate at which B uniformly travelled.

Ans. 10 miles per day.

42. A hare starts 40 yards before a greyhound, and is not perceived by him until she has been running 40 seconds, she scuds away at the rate of 10 miles an hour, and the dog pursues her at the rate of 18 miles an hour ; how long will the chase last, and what distance will the hare have run ?

Ans. $60\frac{5}{22}$ sec. ; 490 yards.

43. A can do a certain piece of work in 9 days, and B can do the same in 12 days ; they work together for 3 days, when A is taken sick and leaves, B continues on working alone, and after 2 days he is joined by C, and they finish it together in $1\frac{1}{2}$ days ; how long would C be doing it alone ?

Ans. 12 days.

44. A, in a scuffle, seized on $\frac{2}{3}$ of a parcel of sugar plums ; B caught $\frac{3}{8}$ of it out of his hands, and C laid hold on $\frac{3}{10}$ more ; D ran off with all A had left, except $\frac{1}{7}$ which E afterwards secured slyly for himself ; then A and C jointly set upon B, who, in the conflict, let

fall $\frac{1}{2}$ he had, which were equally picked up by D and E, who lay perdu. B then kicked down C's hat, and to work they all went anew for what it contained ; of which A got $\frac{1}{4}$, B $\frac{1}{3}$, D $\frac{2}{7}$, and C and E equal shares of what was left of that stock. D then struck $\frac{3}{4}$ of what A and B last acquired, out of their hands ; they, with some difficulty, recovered $\frac{5}{8}$ of it in equal shares again, but the other three carried off $\frac{1}{8}$ a piece of the same. Upon this, they called a truce, and agreed that the $\frac{1}{3}$ of the whole left by A at first, should be equally divided among them ; how many plums, after this distribution, had each of the competitors ?

Ans. A had 2863 ; B, 6335 ; C, 10,294, and E, 4950.

PRACTICE.

The rule which is called Practice is nothing else than a particular case of simple proportion, viz., when the first term is unity. Thus : if it is required to find the price of 28 tons of coal, at \$7 a ton—as a question in proportion, it would be, if 1 ton of coal costs \$7, what will 28 tons cost ? and the statement would be $1 : 28 :: 7 : \text{F. P.}$ Here the first term being 1, the question becomes one of simple multiplication, but the answer, \$196, is really the fourth term of an analogy. Again, to find the price of 36 cwt. flour, at £1 10s. 0d. per cwt.—here again, the question stated at length would be, if 1 cwt. of flour cost £1 10s. 0d., what will 36 cwt. cost ? and the statement would be $1 : 36 :: \text{£1 10s. 0d.} : \text{F. P.}$, which thus becomes a question of multiplication, because the first term is unity, and, dividing by 1, would not alter the product of the other two terms. There are various methods of performing the work, which we shall now illustrate by the example given above.

(1.)	(2.)	(3.)
$ \begin{array}{r} \text{£ } 1 \ 10 \ 0 \\ \phantom{\text{£ } 1 \ 10} 36 \\ \hline \text{£ } 54 \ 0 \ 0 \end{array} $	$ \begin{array}{r} 36 \\ 30 \\ \hline 20)108(0 \\ \hline \text{£ } 54 \ 0 \ 0 \end{array} $	$ \begin{array}{r} \text{£ } 36 \\ 1 \\ \hline 10\text{s.} = \frac{1}{2} \text{ of £1}) 36 \\ \phantom{10\text{s.} = \frac{1}{2} \text{ of £1}} 18 \\ \hline \text{£ } 54 \ 0 \ 0 \end{array} $

1st method—multiply £1 10s. 0d. by 36, as in compound multiplication, and the result, £54, is the answer, because it is the product of the second and third terms of the analogy. In the second method the £1 10s. 0d. is reduced to shillings, and 36 multiplied by it, which gives the answer in shillings, and dividing by 20

gives 54, the answer in pounds. By the third method, we note that 36 cwt., at £1 per cwt., will be £36, and since 10s. is the half of a pound, the price at 10s. will be the half of the price at £1, and we write in the usual place for a divisor 10s.= $\frac{1}{2}$ of £1, and we take the half of the price at £1, namely, £18, and then by adding together the price at £1 and the price at 10s., viz., £36 and £18, we get the price at £1 10s. 0d. This is in reality nothing else than

$$\begin{array}{r} 36 \\ 1\frac{1}{2} \\ \hline 36 \\ 16 \\ \hline \pounds 54 \end{array}$$

multiplying by $1\frac{1}{2}$, as in the margin, which is obviously correct, for £1 10s. 0d. is *one* pound and *a-half*. We have chosen an exceedingly simple example for the purpose of illustration, and we now remark that the advantage of any one of these methods above the others will not be apparent till more difficult questions are proposed. The first method is generally the most convenient in calculating dollars and cents, because, as we have already shown, the multiplication of all quanti-

ties expressed decimally can be performed in precisely the same manner as in the case of whole numbers, care being taken to give the decimal point its proper place. The second and third methods, and especially the third, are the best adapted for calculations in pounds, shillings and pence, although the first may often be used with great advantage. The third is called the method of aliquot parts, *i. e.*, a part that is contained in the next higher denomination without remainder ; thus, 10s. above is an aliquot part of £1, viz., 10s. is $\frac{1}{2}$ of £1.

TABLE OF ALIQUOT PARTS.

*Parts of \$1.	Parts of £1.	Parts of 1s.	Parts of a month.
50 cents = $\frac{1}{2}$	10s. = $\frac{1}{2}$	6d. = $\frac{1}{2}$	15 days = $\frac{1}{2}$
33 $\frac{1}{3}$ = $\frac{1}{3}$	6s. 8d. = $\frac{1}{3}$	4d. = $\frac{1}{3}$	10 = $\frac{1}{3}$
25 = $\frac{1}{4}$	5s. = $\frac{1}{4}$	3d. = $\frac{1}{4}$	7 $\frac{1}{2}$ = $\frac{1}{4}$
20 = $\frac{1}{5}$	4s. = $\frac{1}{5}$	2d. = $\frac{1}{5}$	5 = $\frac{1}{5}$
16 $\frac{2}{3}$ = $\frac{1}{6}$	3s. 4d. = $\frac{1}{6}$	1 $\frac{1}{2}$ d. = $\frac{1}{6}$	3 = $\frac{1}{6}$
12 $\frac{1}{2}$ = $\frac{1}{8}$	2s. 6d. = $\frac{1}{8}$	1d. = $\frac{1}{8}$	2 = $\frac{1}{8}$
8 $\frac{1}{3}$ = $\frac{1}{12}$	2s. = $\frac{1}{10}$		1 = $\frac{1}{10}$
6 $\frac{1}{4}$ = $\frac{1}{16}$	1s. 8d. = $\frac{1}{12}$		
5 = $\frac{1}{20}$	1s. 4d. = $\frac{1}{15}$		
4 = $\frac{1}{25}$	1s. 3d. = $\frac{1}{16}$		
2 = $\frac{1}{50}$	1s. = $\frac{1}{20}$		

* In Canada, the hundred weight being 100 lbs., the aliquot parts of the cwt. will be the same as the aliquot parts of the dollar. In Britain, the hundred weight is 112 lbs.

EXERCISES.

Find the prices of the following numbers of articles at the given prices :

(1.)	(2.)	(3.)
187 cwt., at \$5.37½:	1857 lbs., at \$3.87½	4796 tons, at \$4.50
187 5.37½	3.87½	4.50
1309	928½=½ of 1857	
561	12999	239800
935	14856	19184
93½	5571	
		\$21582.00
\$1005.12½	\$7195.87½	

(4.)	(5.)	(6.)
What is the price of 29 score of sheep at \$7.62½ each.	Sold to a cattle dealer 196 head at \$18.75 each.	Sold to a dealer 97 head at \$16.12½ on an average.
29 20	18.75 196	16.12½ 97
580	11250	48½
7.62½	16875	11284
	1875	14508
290		
1160	\$3675.00	\$1564.12½
3480		
4060		
\$4422.50		

7. To find the price of 347 cwt. of coffee at £7.11.6 per cwt.
cwt. 347 × 1 = £347 = price at £1.
7 = No. of pounds

		£2429 = price at £7, obtained by multiplying 347 by 7.
10s. = ½ of £1.	2	173.10.0 = price at 10s. obtained by dividing 347 by 2.
1s. 3d. = ⅓ of 10s.	8	21.13.9 = price at 1s. 3d. obtained by dividing 173.10.0 by 8.
3d. = ⅕ of 1s. 3d.	4	6.9 = price at 3d. obtained by dividing 21.13.9 by 5.
		£2628.10.6 = price at £7.11.6, obtained by adding the four parts.

Here it is evident that if each cwt. cost *one* pound, the whole must be repeated as often as there are units in 7, *i. e.*, 7 times, and hence we multiply 347 by 7, and obtain 2429, which being the *price* of the hundred weights is pounds, *i. e.*, £2429. Again, since 10s. is the $\frac{1}{2}$ of £1, the price at 10s. will be the $\frac{1}{2}$ of the price at £1, *i. e.*, the $\frac{1}{2}$ of £347, which therefore we divide by 2, and place the quotient £173.10.0, under the £2429, and the divisor 2, by which it was obtained, opposite to it. Again, since 1s. 3d. is the $\frac{1}{8}$ of 10s., the price at 1s. 3d. will be the $\frac{1}{8}$ of the price at 10s., we therefore divide £173.10.0 by 8 and place the quotient £21.13.9 below it, and the 8 opposite to it. In the same way we see that as 3d. is the $\frac{1}{5}$ of 1s. 3d., that the price at 3d. will be the $\frac{1}{5}$ of £21.13.9, the price at 1s. 3d., we therefore divide £21.13.9 by 5, and obtain £4.6.9, the price at 3d., and place it under the £21.13.9, and the 5 opposite to it.

We have now the price at 7. 0.0, all which make the whole price 0.10.0 as here indicated. Having thus 0. 1.3 obtained the partial results, we add 0. 0.3 them, and the sum is the price of £7.11.6 347 cwt. at £7.11.6. To keep

before us the aliquot parts used, we place a memorandum of this on the left, as exhibited in the examples. We have written this first example in an expanded form, in order to show every step of the process. The annexed will show in how much smaller space it may be performed :

347 @ £7.11.6			otherwise :		
7			7		
10s.	$\frac{1}{2}$	2429			2429
1s. 3d.	$\frac{1}{8}$	173.10.0	10s.	$\frac{1}{2}$	173.10.0
3d.	$\frac{1}{5}$	21.13.9	1s.	$\frac{1}{10}$	17. 7.0
		4. 6.9	6d.	$\frac{1}{2}$	8.13.6
		£2628.10.6			£2628.10.6

We have taken for granted that those who have learned compound division, do not need to be told that when there was £1 over in dividing £347 by 2, it was to be reduced to shillings, (20) and then divided by 2, giving 10s., and so on. There are other modes which will be illustrated by other examples.

8. If a man has an income of \$12.50 per week ; how much has he per year ?
 Ans. \$650.00.

9. If a clerk has $\$2.12\frac{1}{2}$ salary for every working day in the year ; what is his yearly income ? Ans. $\$665.12\frac{1}{2}$

10. If a tradesman earn 8s. 8d. a day ; how much will he earn in the working days of the year ? Ans. £135.12.8.

11. An Ensign's pay in the British army is 5s. 6d. a day ; how much is that in Leap-year ? Ans. £100.13.0.

12. If an officer's pay is a guinea and a half a day ; how much has he in a common year ? Ans. £574.17.6.

13. What will 479 cwt. of sugar come to at £4.9.6 per cwt. ?

		479
		4
		1916
5s.	$\frac{1}{4}$	119.15.0
4s.	$\frac{1}{5}$	95.16.0
6d.	$\frac{1}{10}$	11.19.6
		£2143.10.6

In this example, as before, we multiply 479 by 4, to get the price at £4 ; then as 9s. 6d. is 5s. + 4s. + 6d., we resolve the shillings and pence into this form, and as 5s. is $\frac{1}{4}$ of £1, we divide 479, the price at £1, by 4, and obtain £119.15.0, the price at 5s. Again, as 4s. is $\frac{1}{5}$ of £1, we divide 479 by 5, and get £95.16.0, the price at 4s.; lastly, as 6d. is $\frac{1}{10}$ of 5s. we divide £119.15.0 by 10, and get £11.19.6, the price at 6d. We might here have taken

6d. as the $\frac{1}{8}$ of 4s., but the division by 10 is easier than the division by 8. We now take the sum of all the partial results.

We would call the learner's special attention to the following directions, as the neglect of it is a fertile source of error. Whenever you take any quantity as an aliquot part of a higher to find the price of the former, *be sure you divide the line which is the price at the rate of that higher denomination.* Thus, in the last example, 4s. is $\frac{1}{5}$ of £1, so we must divide £479, and *not* the £119,15.0, for that would imply that 4s. was $\frac{1}{5}$ of 5s. So also, since 6d. is the $\frac{1}{10}$ of 5s., we divide the £119 15s. 0d. by 10, but, had we taken 6d. as the $\frac{1}{8}$ of 4s., we should then have divided £95 16s. 0d., because that is the price of 4s ; the result would have been the same, however, for $\text{£119 15s. 0d.} \div 10 = \text{£11 19s. 6d.}$, and $\text{£95 16s. 0d.} \div 8 = \text{£11 19s. 6d.}$ also.

METHOD OF COMPLEMENTS.

14.—879^{cwt.}, @ 19s.

1s. = $\frac{1}{20}$	)879	0	0
	43	19	0
	£835	1	0

15.—1793 lbs., @ 18s

10)	1793	0	0
	179	6	0
	£1613	14	0

16.—2781 tons, @ 17s. 6d.

$$\begin{array}{r}
 8)2781 \quad 0 \quad 0 \\
 \underline{347 \quad 12 \quad 6} \\
 \text{£}2433 \quad 7 \quad 6
 \end{array}$$

17.—987 lbs., @ 16s.

$$\begin{array}{r}
 5)987 \quad 0 \quad 0 \\
 \underline{197 \quad 8 \quad 0} \\
 \text{£}789 \quad 12 \quad 0
 \end{array}$$

The principle of these operations may be illustrated by Ex. 14. We observe that if the price were £1, the answer would be £879, but the price is 1s. less than a £1, and as 1s. is $\frac{1}{20}$ of £1, we find £43 19s. 0d., the price at 1s., and subtract it from £879, the price at £1, and we have £835 1s. 0d., the price at 19s. In the same manner in 15, we subtract $\frac{1}{10}$, because 18s. wants 2s. of £1, and 2s. is $\frac{1}{10}$ of £1. So in 16, we subtract $\frac{1}{8}$, and in 17 we subtract $\frac{1}{5}$. This mode is useful in many other operations. It is not of so much service, however, in calculating by dollars and cents as by pounds, shillings and pence. We shall give one example, worked both ways, and let the learner judge for himself.

(18.)

$$\begin{array}{r}
 2479 \text{ @ } \$3.90 \\
 3.90 \\
 \hline
 2231.10 \\
 7437 \\
 \hline
 \$9668.10
 \end{array}$$

$$\begin{array}{r}
 \frac{1}{10} \text{ of } 2479 \text{ @ } \$3.90 \\
 4 \\
 \hline
 9916.00 \\
 247.90 \\
 \hline
 \$9668.10
 \end{array}$$

Here \$4 will be seen to be too large a multiplier by 10 cents, and therefore we subtract $\frac{1}{10}$ of \$2479, viz., \$247.90, and find the same answer as before.

19. To find the rent of 189 acres, 2 roods, 32 rods at \$4.20 per acre. Since the rent of

$$\begin{array}{r}
 2 \text{ roods} = \frac{1}{2} \text{ of } 1 \text{ acre,} \quad 4.20 \\
 \quad \quad \quad 189 \\
 \hline
 20 \text{ rods} = \frac{1}{4} \text{ of } 2 \text{ roods,} \quad 210 \\
 \quad \quad \quad 525 \\
 10 \text{ rods} = \frac{1}{2} \text{ of } 20 \text{ rods,} \quad 2625 \\
 \quad \quad \quad 525 \\
 2 \text{ rods} = \frac{1}{5} \text{ of } 10 \text{ rods,} \quad 3780 \\
 \quad \quad \quad 3360 \\
 \quad \quad \quad 420 \\
 \hline
 \$796.74
 \end{array}$$

1 acre is \$4.20, the half of it, \$2.10, will be the rent of 2 roods, the rent of 20 rods will be .525, the $\frac{1}{4}$ of the rent of 2 roods, the half of that, .2625, will be the rent of 10 rods, and, lastly, .0525 will be the rent of 2 rods, which is the $\frac{1}{5}$ of 10 rods. We then multiply by 189, and set the figures of the

product in the usual order, so that the first figure of the product by 9 shall be under the units of cents, &c., and then adding all the partial results, we find the final answer, \$796.74, the rent of 189 acres, 2 roods and 32 perches.

20. What is the price of 118 acres, 3 roods and 20 rods of cleared land, at \$36.75 per acre? Ans. \$4368.66.

21. What is the price of 286 acres, 1 rood, 24 rods of uncleared land, at \$7.25 per acre? Ans. 2076.40.

22. A has 84 acres, 2 roods, 36 rods of cleared land, worth \$24.60 an acre; B has 248 acres, 3 roods, 24 rods of uncleared land, worth \$4.40 an acre—they exchange, the difference of value to be paid in cash; which has to pay, and how much?

Ans. B \$989.08.

23. What is the price of $675\frac{1}{2}$ cwt. of beef, at $10.86\frac{2}{3}$ per cwt.?

Ans. 7340.43.

24. What is the value of 483 cwt. of coffee, at $\$23.33\frac{1}{3}$ per cwt.?

Ans. \$11270.00.

25. What are 195 lbs. of raisins worth, at 30 cts. per lb.?

Ans. 58.50.

26. What is the value of 514 gallons of oil, at $43\frac{1}{2}$ cts. per gallon?

Ans. \$223.59.

27. What will be the price of 576 yards of broadcloth, at \$3.75 per yard?

Ans. \$2160.00.

28. Find the price of 1625 yards of muslin, at $54\frac{1}{6}$ cts. per yard.

Ans. \$880.21.

29. Find the price of 4265 cwt. of flour, at $\$6.82\frac{1}{2}$ per cwt.

Ans. \$29108.62 $\frac{1}{2}$.

30. What is the price of 7913 cwt. of pearl ashes, at 11.375 per cwt.?

Ans. \$90010.37 $\frac{1}{2}$.

BILLS OF PARCELS.

A BILL OF PARCELS is simply a statement rendered by the seller to the purchaser, showing the articles bought, and the prices of each.

31.

HAMILTON, Jan. 4th, 1866.

MR. ELIAS G. CONKLIN,

Bought of J. BUNTIN & Co.,

12 reams of foolscap paper.....	@	\$3.25
15 dozen School Books.....	@	4.50
23 " Slates	@	1.30
7 " Photograph Albums.....	@	15.00
3 " Bullions Grammar.....	@	7.00
8 " Fifth Reader	@	3.50
5 gallons of Black Ink.....	@	1.10

\$295.90

Received payment,

J. BUNTIN & Co.

32

TORONTO, Jan. 12th, 1866.

MR. JAMES H. BURRITT,

Bought of MORRISON, TAYLOR & Co.,

15 cwt. of cheese.....	@	\$9.00
4 cwt. of flour.....	@	4.25
120 pounds of Bacon.....	@	0.14
7 bushels of corn meal.....	@	0.75
12 firkins of butter.....	@	13.50
20 bushels of dried apples.....	@	2.25
13 " " peaches.....	@	4.00
11 cwt. of buck-wheat flour	@	5.50
15 cwt. maple sugar.....	@	8.00
25 bags of common salt.....	@	1.15
57 barrels of mess pork.....	@	13.00
68 " beef.....	@	9.75
13 bushels of clover seed,.....	@	7.50

\$2143.80.

Received payment by note at 30 days.

FOR MORRISON, TAYLOR & Co.,

A. C. HENRY.

33

HAMILTON, January 2nd, 1866.

MR. M. McCULLOCH,
To JOSEPH LIGHT, Stationer, *Dr.*

For 500 French envelopes.....@	\$3.00 per thousand.
" 12 doz. British American copy books...@	1.15
" 6 " B. B. lead pencils.....@	.50
" 5 gross mourning envelopes.....@	1.05
" 2 reams mourning note paper.....@	3.15
" 4 " tinted note paper.....@	3.15
" 2½ " Foreign note paper.....@	1.40
" 3 " " letter paper.....@	3.00
" 1 doz. First Books.....@	.15
" 5 boxes Gillott's No. 303 pens.....@	.90
" 5 doz. Third Books.....@	1.62½
" 10 quires blank books, half bound.....@	.35
" 2 packs visiting cards.....@	.37½
\$71.98	

NOTE.—Bills should not be signed until settled.

34.

BROCKVILLE, Jan. 5th, 1866.

N. D. GALBREAITH,
To R. FITZSIMMONS & Co., *Dr.*

For 24 lbs. Mackerel	@ 05½c.
" 3 gallons Molasses	@ 45
" 13 lbs. Young Hyson Tea.....@	87½
" 13 lbs. brown Sugar.....@	11
" 15 bushels of Potatoes.....@	45
\$22.23	

CR.

For 10 lbs. Butter.....@	17c.
" 5 doz. Eggs.....@	12½
" 3 gallons Maple Molasses.....@	95
" Note at 20 days, to balance.....	17.05
\$22.23	

R. FITZSIMMONS & Co.

NOTE.—Such a Bill as this would be termed a Barter Bill.

35

KINGSTON, Jan. 2nd, 1866.

JAMES THOMPSON, ESQ.,

To A. JARDINE & Co., Dr.

For 3 doz. Buttons	@	\$0.12	
" 5½ yards of black Broadcloth.....	@	5.50	
" 20 yards Sheeting.....	@	.15	
" 1 chest Y. H. Tea, 83 lbs.....	@	.95	
" 18 yards French Print.....	@	.20	
" 2 skeins of Silk Thread.....	@	.09	
" 5 yards black Silk Velvet.....	@	3.50	
" 20 lbs. Loaf Sugar	@	.18	
" 2 gallons Molasses.....	@	.40	
" 1 bag of common Salt.....	@	1.15	
" 25 lbs. Rice.....	@	.09	
" 3 sacks Coffee, 70 lbs. each.....	@	.12	
	CR.		\$166.74
By Cash.....			50.00
Balance due.....			\$116.74

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ALGONQUIN, Jan. 15th, 1865.

W. FLEMING & Co.,

Bought of J. & A. WRIGHT,

1500 lbs. Canadian Cheese.....	@	\$.09	
300 bushels Fall Wheat.....	@	1.25	
9 brls Pot Ash, net 7056 lbs.....	@	5.75 per cwt.	
150 bushels Spring Wheat.....	@	1.15	
200 " Potatoes.....	@	.45	
600 " Oats.....	@	.37½	
150 " Pease.....	@	.65	
50 " Indian Corn.....	@	.50	
60 " Apples	@	.60	
3 kegs Butter, 110 lbs. each.....	@	.18	
50 bushels Rye.....	@	.70	
40 " Barley	@	.80	
			\$1688.12

Received payment,

J. & A. WRIGHT.

P E R C E N T A G E .

18.—PERCENTAGE is an allowance, or reduction, or estimate of a certain portion of each 100 of the units that enter into any given calculation. The term is a contraction of the Latin expression for one hundred, and means literally *by the hundred*. In calculating dollars and cents, 6 per cent. means 6 dollars for every 100 dollars, or 6 cents for every \$1, or 100 cents. If we are estimating the rate of yearly increase of the population of a rising village, and find that at the end of a certain year it was 100, and at the end of the next it was 106, we say it has increased 6 per cent. *i. e.*, 6 persons have been added to the 100. So, also, if a large city has a population of 100,000 at the end of a certain year, and it is found that it has 106,000 at the end of the following year, we say it has increased 6 per cent., which means that if we count the population by hundreds we shall find that for every 100 at the end of the one year, there are 106 at the end of the next; because one hundred thousands is the same as one thousand hundreds, and we have supposed the increase in every 100 to be 6, the total increase will be one thousand *sixes* or 6,000, giving a total population of 106,000 as above, or an increase at the rate of 6 per cent. A decrease would be estimated in the same manner. Thus, a falling off in the population of 6 persons in the hundred would be denoted by $100 - 6 = 94$, as an increase of 6 in the hundred would be denoted by $100 + 6 = 106$. So, also, in our first example, a deduction of \$6 in \$100 would be $\$100 - 6 = \$94$, and a gain would be $\$100 + \$6 = \$106$.

The portion of 100 so allowed or estimated, is called the *rate per cent.*, as in the examples given, 6 denotes the *rate per cent.*, or the allowance or estimate on every 100. Should the sum on which the estimate is made not reach 100, we can, nevertheless, estimate what is to be allowed on it at the same rate. Thus, if 6 is to be allowed for 100, then 3 must be allowed for 50, and $1\frac{1}{2}$ for 25, &c.

The number on which the percentage is estimated is called the *basis*. Thus, in the example given regarding the population of a city, 100,000 is the basis.

When the basis and percentage are combined into *one*, the result is called the *amount*. If the rate per cent. be an *increase* or *gain*, it is to be added to the basis to get the amount, and if it is a *decrease*, or *loss*, it is to be subtracted from the basis to get the amount. This latter result is sometimes called the *remainder*.

From what has been said, it is plain that percentage is nothing else than taking 100 as a standard unit of measure—(See Art. 1)—and making the rate a fraction of that unit, so that 6 per cent. is $\frac{6}{100} = (\text{Art. 15, V.}) .06$. We may obtain the same result by the rule of proportion. Thus, in our illustrative example of an increase of 6 persons for every 100 on a population of 100,000, the analogy will be 100 persons : 100,000 persons : : 6 (the increase on 100) : 6,000, the increase on 100,000. It is manifest that the same result will be obtained whether we multiply the third by the second, and divide by the first, or whether we divide the third by the first, and multiply the result by the second; or, which is the same thing, multiply the second by the result. Now, we already found that $6 \div 100 = \frac{6}{100} = .06$, the same as before. So also, 7 per cent. of any loss is seven one-hundredths of it, *i. e.*, $\frac{7}{100} = .07$. It should be carefully observed that such decimals represent, *not the rate per cent.*, but *the rate per unit*.

Though this is easily comprehended, yet we know by experience that learners are constantly liable to commit errors by neglecting to place the decimal point correctly. We would therefore direct particular attention to the above caution, which, with the rule already laid down, under the head of decimal fractions, should be sufficient to guide any one who takes even moderate pains.

EXERCISES ON FINDING THE RATE PER UNIT.

At $\frac{1}{4}$ per cent., what is the rate per unit?	Ans. $.00\frac{1}{4}$.
At $\frac{1}{2}$ per cent., what is the rate per unit?	Ans. $.00\frac{1}{2}$.
At 1 per cent., what is the rate per unit?	Ans. .01.
At 2 per cent., what is the rate per unit?	Ans. .02.
At 4 per cent., what is the rate per unit?	Ans. .04.
At $7\frac{1}{4}$ per cent., what is the rate per unit?	Ans. $.07\frac{1}{4}$.
At 10 per cent., what is the rate per unit?	Ans. .10.
At $12\frac{1}{2}$ per cent., what is the rate per unit?	Ans. $.12\frac{1}{2}$.
At 17 per cent., what is the rate per unit?	Ans. .17.
At 25 per cent., what is the rate per unit?	Ans. .25.
At $33\frac{1}{3}$ per cent., what is the rate per unit?	Ans. $.33\frac{1}{3}$.
At $66\frac{2}{3}$ per cent., what is the rate per unit?	Ans. $.66\frac{2}{3}$.
At 75 per cent., what is the rate per unit?	Ans. .75.
At 100 per cent., what is the rate per unit?	Ans. 1.00.
At $112\frac{1}{2}$ per cent., what is the rate per unit?	Ans. $1.12\frac{1}{2}$.
At 150 per cent., what is the rate per unit?	Ans. 1.50.
At 200 per cent., what is the rate per unit?	Ans. 2.00.

I. To find the percentage on any given quantity at a given rate :

On the principles of proportion, we have as 100 : given quantity : : rate : percentage, and as the third term, divided by the first, gives the rate per unit, we have the simple

R U L E :

Multiply the given quantity by the rate per unit, and the product will be the percentage.

E X A M P L E S .

To find how much 6 per cent. is on 720 bushels of wheat, we have $6 \div 100 = .06$, the rate per unit, and $720 \times .06 = 43\frac{1}{5}$ bushels, the percentage.

To find 8 per cent. of \$7963.75, in like manner, we have .08, the rate per unit, and $\$7963.75 \times .08$ gives \$637.10, the percentage.

Instead of *per cent* the mark (%) is now commonly used.

EXERCISES ON THE RULE.

1. What does 6 per cent. of 450 tons of hay amount to ?
Ans. 27.
2. What is 10 per cent. of \$879.62 $\frac{1}{2}$?
Ans. \$87.96.
3. If 12 per cent. of an army of 47,800 men be lost in killed and wounded ; how many remain ?
Ans. 42,064.
4. What is 5 per cent. of 187 bushels of potatoes ?
Ans. 9.35.
5. What is 2 $\frac{1}{2}$ per cent. of a note for \$870 ?
Ans. 21.75.
6. Find 12 $\frac{1}{2}$ per cent. of 97 hogsheads ?
Ans. 12.12 $\frac{1}{2}$.

II. To find what rate per cent. one number is of another given number :—Let us take as an example, to find what per cent. 24 is of 96. Here the basis is 96, and we take 100 as a standard basis, and these are magnitudes of the same kind, and 24 is a certain rate on 96, and we wish to find what rate it is on 100, and by the rule of proportion, we have the statement $96 : 100 :: 24 : F. P. = \frac{24 \times 100}{96} = 25$. Therefore 24 is 25 per cent. of 96.

From this we can deduce the simple

R U L E .

Annex two ciphers to the given percentage, and divide that by the basis, the quotient will be the rate per cent.

7. What per cent. of 150 is 15 ?
Ans. 10.
8. What per cent. of 240 is 36 ?
Ans. 15.

9. What per cent. of 18 is 2? Ans. $11\frac{1}{9}$.
 10. What per cent. of 72 is 48? Ans. $66\frac{2}{3}$.
 11. What per cent. of 576 is 18? Ans. $3\frac{1}{8}$.
 12. What per cent. is 12 of 480? Ans. $2\frac{1}{2}$.
 13. Bought a block of buildings in King street for \$1719, and sold it at a gain of 18 per cent.; what was the gain?
Ans. \$309.42.
 14. Vested \$325 in an oil well speculation, and lost 8 per cent.; what was the loss? Ans. \$26.00.
 15. In 1841 the population of Toronto was about 15,000, it is now about 50,000; what is the rate of increase? Ans. $233\frac{1}{3}$.
 16. An estate worth \$4,500 was sold; A bought 30 per cent. of it; B, 25 per cent.; C, 20 per cent.; and D purchased the remainder; what per cent. of the whole was D's share? Ans. 25.
 17. If a man walk at the rate of 4 miles an hour; what per cent. is that of a journey of 32 miles? Ans. $12\frac{1}{2}$.
 18. What is the percentage on \$1370 at $2\frac{3}{4}$ per cent.? Ans. $37.67\frac{1}{2}$.

III. Given, a number, and the rate per cent. which it is of another number, to find that other number, .400 is 40 per cent. of a certain number, to find that number. As $40 : 100 :: 400 : F. P. = \frac{400 \times 100}{40} = 1,000$. Hence we derive the

R U L E .

Annex two ciphers to the given number, and divide by the rate per cent.

E X E R C I S E S .

1. A bankrupt can pay \$2600, which is 80 per cent of his debts; how much does he owe? Ans. \$3250.
 2. A clerk pays \$8 a month for rent, which is 16 per cent. of his salary; what is his yearly salary? Ans. \$600.
 3. In a manufacturing district in England, 40,000 persons died of cholera in 1832, this was 25 per cent. of the population; what was the population? Ans. 160,000.
 4. Bought a certain number of bags of flour, and sold 124 of them, which is $12\frac{1}{2}$ per cent. of the whole. Required, the number of bags purchased. Ans. 992.
 5. In a shipwreck 480 tons are lost, and this amount is 15 per cent. of the whole cargo. Find the cargo. Ans. 3200 tons.

6. A firm lost \$1770 by the failure of another firm; the loss was 30 per cent. of their capital; what was their capital? Ans. \$5900.

IV. To find the basis when the amount and rate are given :— Suppose a man buys a piece of land for a certain sum, and by selling it for \$300, gains 25 per cent.; what did he pay for it at first?— Here it is plain that for every dollar of the cost, 25 cents are gained by the sale, *i. e.*, 125 cents for every 100, which gives us the analogy, 125 : 100 :: 300 : F. P.; or, dividing the two terms by 100, 1.25 : 1.00 :: 300 : F. P., which by the rules for the multiplication and division of decimals, gives $\frac{300.00}{1.25} = \$240$, the original cost.

Again, suppose the farm had been sold at a loss of 25 per cent. This being a loss, we subtract 25 from 100, and say, as 75 : 100 :: 300 : F. P. $= \frac{300.00}{.75} = \$400$, the prime cost in this case.

Hence we derive the

R U L E .

Divide the given amount by one increased or diminished by the given rate per unit, according as the question implies increase or decrease, gain or loss.

E X E R C I S E S .

1. Given the amount \$198, and the rate of increase 20 per cent. to find the number yielding that percentage. Ans. \$165.

2. A field yields 840 bushels of wheat, which is 250 per cent. on the seed; how many bushels of seed were sown? Ans. 240 bushels.

3. At 5 per cent. gain; what is the basis if the amount be \$126? Ans. \$120.

4. At 10 per cent. loss; what is the basis, the amount being \$328.5? Ans. \$365.

5. A ship is sold for \$12045, which is a gain of $\frac{3}{8}$ per cent. on the sum originally paid for it; for how much was it bought at first? Ans. \$12000.

6. A gambler lost 10 per cent. of his money by a venture, and had \$279 left; how much had he at first, and how much did he lose? Ans. He lost \$31, and had \$310 at first.

7. A grocer bought a lot of flour, and having lost 20 per cent. of the whole, had 160 bags remaining; how many bags did he buy? Ans. 200.

8. A merchant lost 12 per cent. of his capital by a bankruptcy, and had still \$2200 left; what was his whole capital? Ans. \$2500.

9. Sold a sheep for \$5, and gained 25 per cent. ; what did I pay for it ? Ans. \$4.

10. Lost \$12000 on an investment, which was 30 per cent. of the whole ; what was the investment ? Ans. \$40000.

INTEREST.

From a transition common in language, the word interest has been inappropriately applied to the *sum paid* for the use of money, but its original and true meaning is simply the *use* of money. To illustrate this, we will suppose that A borrows of B \$100 for one year, and at the end of the year, when A wishes to settle the account, he gives B \$107. Were we to ask the question of almost any person except an accountant, whether A or B received the interest, we should undoubtedly receive for an answer that B received it. But such is not the case. A having had the *use* of that money for one year, paid B \$7 for that *use* or *interest* ; hence A received the *interest* or *use* of that money, and B received \$7 in cash for the same. It is only by considering this subject in its true light that accountants are able to determine upon the proper debits and credits that arise from a transaction where interest is involved. If an individual borrows money, he *receives* the use of that money, and when he pays for that use or interest, he places the sum so paid to that side of his "interest account" which represents interest received, and if he lends money, he *has parted with* the use of that money, and when he receives value for that use or interest, he places the sum so received to that side of his "interest account" which represents interest delivered.

We think that this explanation is sufficiently clear to illustrate the difference between *interest* and the *value received* or *paid* for it.

It will also be noticed that we have given many of the exercises in the usual form, *e. g.*, we say what is the interest on \$100 for one year, instead of saying *what must be paid for the interest* of \$100 for one year, but we have done this more in accordance with custom than from any intention to deviate from the true meaning of the word interest.

Interest is reckoned on a scale of so many units on every \$100 for one year, and hence it is called so much *per cent. per annum*, from the Latin *per centum*, by the hundred, and *per annum*, by the year. Thus, \$6 a year for every \$100, is called *six per cent. per*

annum. The term is also extended to designate the return accruing from any investment, such as shares in a joint stock company.

To show the object and use of such transactions, we may suppose a case or two.

A person feels himself cramped or embarrassed in his circumstances and operations, and he applies to some friendly party that lends him \$100 for a year, on the condition that at the expiration of the year he is to receive \$106, that is, the \$100 lent, and \$6 more as a return for the use of the \$100; or, if the borrower gets \$600, he pays at the end of the stipulated time not only the \$600, but also \$36 (\$6 for each \$100) in return for the use of the \$600. By this means the borrower gets clear of his difficulty, and maintains his credit at a small sacrifice. Again, a merchant may find that there is an opportunity for a speculation by which a good sum may be realized, but he has not capital sufficient, and accordingly he borrows a sum sufficient for the purpose, and pays, say 6 per cent. for it. We shall now suppose that the speculation yields him 24 per cent., then it is plain that after paying 6 per cent. for the money borrowed, he is still a gainer of 18 per cent. on that money; that is, for every \$100 that he borrowed he *clears* \$18. We shall cite one case more of common occurrence. A mercantile house fails; another house is in danger of being involved in the disaster by having extensive transactions with the former, but by effecting a loan to meet present emergencies, maintains its credit, and goes on with the business. In such a case, a small sacrifice in the shape of interest is of no account compared with the damage of a failure, and so in numberless other cases.

The sum on which interest is paid is called the *principal*.

The sum paid for the use of money is called the *interest*.

The sum paid on each \$100 is called the *rate*.

The sum of the principal and interest is called the *amount*.

When interest is charged on the principal only, it is called *simple interest*.

When interest is charged on the amount, it is called *compound interest*.

SIMPLE INTEREST.

As simple interest, when calculated for one year, differs in no way from a percentage on a given sum, we have only four things to consider, viz., the principal, the rate, (100 being the basis,) the inter-

est, and the time, any three of which being known, the fourth can be found. The finding of the interest includes by far the greatest number of cases.

We shall first show the general principle, and from it deduce an easy practical rule.

Let it be required to find the interest on \$468 for one year, at 6 per cent.

As 100 is taken as the *basis principal* in relation to which all calculations are made, it is plain that 100 will have the same ratio to any given principal that the rate, which is the interest on 100, has to the interest on the given principal. Hence, in the question proposed, we have as $\$100 : \$468 :: \$6 : \text{interest} = \$468 \times \frac{6}{100} = \$468 \times .06 = \$28.08$. Now .06 is the rate *per unit*, and from this we can deduce rules for all cases.

CASE I.

To find the interest of any sum of money for one year, at any given rate per cent.

RULE.

Multiply the principal by the rate per unit.

EXERCISES.

1. What is the interest on \$15, for 1 year, at 3 per cent. ?
Ans. \$0.45.
2. What is the interest on \$35, for 1 year, at 5 per cent. ?
Ans. \$1.75.
3. What is the interest on \$100, for 1 year, at 7 per cent. ?
Ans. \$7.00.
4. What is the interest on \$2.25, for 1 year, at 8 per cent. ?
Ans. \$0.18.
5. What is the interest on \$6.40, for 1 year, at $8\frac{1}{2}$ per cent. ?
Ans. \$0.54.
6. What is the interest on \$250, for 1 year, at $9\frac{1}{2}$ per cent. ?
Ans. \$23.75.
7. What is the interest on \$760.40, for 1 year, at $7\frac{1}{2}$ per cent. ?
Ans. \$57.03.
8. What is the interest on \$964.50, for 1 year, at $6\frac{1}{2}$ per cent. ?
Ans. \$62.69.
9. What is the interest on \$568.75, for 1 year, at $7\frac{1}{4}$ per cent. ?
Ans. \$41.23.

CASE II.

To find the interest of any sum of money, for any number of years, at a given rate per cent.

RULE.

Find the interest for one year, and multiply by the number of years.

EXERCISES.

10. What is the interest of \$4.60, for 3 years, at 6 per cent. ?

Ans. \$0.83.

11. What is the interest of \$570, for 5 years, at $7\frac{1}{2}$ per cent. ?

Ans. \$213.75.

12. What is the interest of \$460.50, for 3 years, at $6\frac{1}{4}$ per cent. ?

Ans. \$86.34.

13. What is the interest of \$17.40, for 3 years, at $8\frac{1}{3}$ per cent. ?

Ans. \$4.35.

14. What is the interest of \$321.05, for 8 years, at $5\frac{3}{4}$ per cent. ?

Ans. \$147.68.

15. What is the interest of \$1650.45, for 2 years, at 9 per cent. ?

Ans. \$297.08.

16. What is the interest of \$964.75, for 4 years, at 10 per cent. ?

Ans. \$385.90.

17. What is the interest of \$1674.50, for 3 years, at $10\frac{1}{2}$ per cent. ?

Ans. \$527.47.

18. What is the interest of \$640.80, for 5 years, at $4\frac{3}{4}$ per cent. ?

Ans. \$152.19.

19. What is the interest of \$965.50, for 7 years, at $5\frac{1}{2}$ per cent. ?

Ans. \$371.72.

20. What is the interest of \$2460.20, for 4 years, at 7 per cent. ?

Ans. \$688.86.

CASE III.

To find the interest on any sum of money for any number of months, at a given rate per cent.

RULE.

Find the interest for one year, and take aliquot parts for the months ; or,

Find the interest for one year, divide by 12, and multiply by the number of months.

EXERCISES.

21. What is the interest on \$684.20, for 4 months, at 6 per cent.?
Ans. \$13.68.
22. What is the interest on \$760.50, for 5 months, at 7 per cent.?
Ans. \$22.18.
23. What is the interest on \$899.99, for 2 months, at 8 per cent.?
Ans. \$12.00.
24. What is the interest on \$964.50, for 4 months, at 9 per cent.?
Ans. \$28.94.
25. What is the interest on \$1500, for 7 months, at 10 per cent.?
Ans. \$87.50.
26. What is the interest on \$1560, for 11 months, at $7\frac{1}{2}$ per cent.?
Ans. \$107.25.
27. What is the interest on \$1575.54, for 8 months, at $6\frac{1}{4}$ per cent.?
Ans. \$65.65.
28. What is the interest on \$1728.28, for 9 months, at $8\frac{1}{2}$ per cent.?
Ans. \$110.18.
29. What is the interest on \$268.25, for 13 months, at 7 per cent.?
Ans. \$20.34.
30. What is the interest on \$1569.45, for 1 year, 3 months, at 8 per cent.?
Ans. \$156.95.
31. What is the interest on \$642.99, for 1 year, 5 months, at 10 per cent.?
Ans. \$91.09.
32. What is the interest on \$560.45, for 1 year, 6 months, at $9\frac{1}{2}$ per cent.?
Ans. \$79.86.
33. What is the interest on \$48.50, for 3 years, 9 months, at $10\frac{1}{2}$ per cent.?
Ans. \$19.10.
34. What is the interest on \$560.80, for 2 years, 8 months, at $11\frac{3}{4}$ per cent.?
Ans. \$175.72.
35. What is the interest on \$2360.40, for 19 months, at 12 per cent.?
Ans. \$448.48.

CASE IV.

To find the interest on any sum of money, for any number of months and days, at a given rate per cent.

RULE.

Find the interest for the months, and take aliquot parts for the days, reckoning the month as consisting of 30 days.

EXAMPLE.

36. What is the interest on \$875.50, for 8 months, 18 days, at 11 per cent.?

SOLUTION.

Principal.....	\$875.50
Rate per unit.....	.11
Interest for 1 year.....	96.3050
Interest for 6 months ; or, $\frac{1}{2}$ of interest for 1 year.....	48.1525
Interest for 2 months ; or, $\frac{1}{3}$ of interest for 6 months.....	16.0508
Interest for 15 days ; or, $\frac{1}{4}$ of interest for 2 months.....	4.0127
Interest for 3 days ; or, $\frac{1}{5}$ of interest for 15 days.....	.8025
Interest for 8 months, 18 days.....	\$69.0185

We find the interest for 1 year to be \$96.305, and as 6 months are the $\frac{1}{2}$ of 1 year, the interest for 6 months will be the $\frac{1}{2}$ of the interest for 1 year ; likewise the interest for 2 months will be the $\frac{1}{3}$ of the interest for 6 months, and as 15 days are the $\frac{1}{4}$ of 2 months or 60 days, the interest for 15 days will be the $\frac{1}{4}$ of the interest for 2 months, and likewise the interest for 3 days, will be the $\frac{1}{5}$ of the interest for 15 days. Adding the interest for the *months* and *days* together, we obtain \$69.02, the sum to be paid for the use of \$875.50, for 8 months, 18 days, at 11 per cent.

EXERCISES.

37. What is the interest on \$468.75, for 4 months, 15 days, at 7 per cent. ?
Ans. \$12.30.
38. What is the interest on \$1654.40, for 3 months, 8 days, at 5 per cent. ?
Ans. \$22.52.
39. What is the interest on \$345.65, for 11 months, 25 days, at 6 per cent. ?
Ans. \$20.45.
40. What is the interest on \$74.85, for 5 months, 22 days, at 9 per cent. ?
Ans. \$3.22.
41. What is the interest on \$673.75, for 8 months, 19 days, at $7\frac{1}{2}$ per cent. ?
Ans. \$36.35.
42. What is the interest on \$57.45, for 1 year, 2 months, 12 days, at 6 per cent. ?
Ans. \$4.14.
43. What is the interest on \$2647, for 1 year, 5 months, 18 days, at $6\frac{1}{4}$ per cent. ?
Ans. \$242.64.
44. What is the interest on \$268.40, for 2 years, 1 month, 1 day, at 8 per cent. ?
Ans. \$44.79.
45. What is the interest on \$2345.50, for 3 years, 7 months, 20 days, at 10 per cent. ?
Ans. \$853.50.

46. What is the interest on \$4268.45, for 4 years, 11 months, 11 days, at $11\frac{3}{4}$ per cent. ? Ans. \$2481.24.

47. What is the interest of \$642.20, for 2 years, 7 months, 24 days, at 12 per cent. ? Ans. \$201.65.

48. What is the interest of \$64.50, for 2 years, 11 months, 2 days, at 7 per cent. ? Ans. \$13.19.

49. What is the amount of \$746.25, for 1 year, 10 months, 12 days, at 5 per cent. ? Ans. \$815.90.

50. What is the interest of \$680, for 4 years, 1 month, 15 days, at 6 per cent. ? Ans. \$168.30.

CASE V.

To find the interest on any sum of money, for any number of days, at a given rate per cent.*

RULE.

Find the interest for one year, and say, as one year (365 days,) is to the given number of days, so is the interest for one year to the interest required ; or,

Having found the interest for one year, multiply it by the given number of days, and divide by 365.

EXERCISES.

51. What is the interest on \$464, for 15 days, at 6 per cent. ? Ans. \$1.14.

52. What is the interest on \$364, for 12 days, at 7 per cent. ? Ans. 84 cents.

53. What is the interest on \$56.82, for 14 days, at 8 per cent. ? Ans. 17 cents.

* To find how many years elapse between any two dates, we have only to subtract the earlier from the later date. Thus, the number of years from 1814 to 1865 is 51 years. To find months, we must reckon from the given date in the first named month, to the same date in each successive month. Thus, five months from the 10th of March brings us on to the 10th of August. To find days, we require to count how many days each month contains, for to consider every month as consisting of 30 days, in the calculation of interest, is not strictly correct, although for portions of a single month it causes no serious error. Thus, the correct time from March 2nd to June 14th, would be 104 days, viz., 29 for March, 30 for April, 31 for May, and 14 for June. A very convenient plan for reckoning time between two given dates is to count the number of months and odd days that intervene. Thus, from June 14th to November 20th, would be 5 months and 6 days.

54. What is the interest on \$75.50, for 18 days, at $8\frac{1}{2}$ per cent. ?
Ans. 32 cents.
55. What is the interest on \$125.25, for 20 days, at 5 per cent. ?
Ans. 34 cents.
56. What is the interest on \$150.40, for 33 days, at 6 per cent. ?
Ans. 82 cents.
57. What is the interest on \$56.48, for 45 days, at $6\frac{1}{2}$ per cent. ?
Ans. 45 cents.
58. What is the interest on \$75.75, for 65 days, at 7 per cent. ?
Ans. 94 cents.
59. What is the interest on \$268.40, for 70 days, at $7\frac{1}{2}$ per cent. ?
Ans. \$3.86.
60. What is the interest on \$464.45, for 80 days, at 8 per cent. ?
Ans. \$8.14.
61. What is the interest on \$15.84, for 120 days, at 9 per cent. ?
Ans. 47 cents.
62. What is the interest on \$240, for 135 days, at $9\frac{1}{2}$ per cent. ?
Ans. \$8.43.
63. What is the interest on \$2460, for 145 days, at 10 per cent. ?
Ans. \$97.73.
64. What is the interest on \$1568, for 170 days, at 11 per cent. ?
Ans. \$80.33.
65. What is the interest on \$2688, for 235 days, at $11\frac{3}{4}$ per cent. ?
Ans. \$203.35.
66. What is the amount of \$364.80, for 320 days, at $11\frac{1}{2}$ per cent. ?
Ans. \$401.58.

CASE VI.

To find the interest on any sum of money, for any time, at 6 per cent.

Since .06 would be the rate per unit, or the interest of \$1 for 1 year, it follows that the interest for *one month* would be the $\frac{1}{12}$ of .06, or $\frac{6}{12}$ of a cent, equal to $\frac{1}{2}$ cent or .005, and for 2 months it would equal $\frac{1}{2}$ cent, or $.005 \times 2 = .01$. Therefore, when interest is at the rate of 6 per cent., the interest of \$1, for every 2 months, is *one cent*. Again, if the interest of \$1, for *one month*, or 30 days, is $\frac{1}{2}$ cent or .005, it follows that the interest for 6 days will be the $\frac{1}{5}$ of .005 or .001. Therefore, when interest is at the rate of 6 per cent., the interest of \$1 for every 6 days is *one mill*. Hence the

R U L E .

Find the interest of \$1 for the given time by reckoning 6 cents for every year, 1 cent for every 2 months, and 1 mill for every 6 days; then multiply the given principal by the number denoting that interest, and the product will be the interest required.

NOTE.—This method can be adopted for any rate per cent. by first finding the interest at 6 per cent., then adding to, or subtracting from the interest so found, such a part or parts of it, as the given rate exceeds, or is less than 6 per cent.

This method, although adopted by some, is not exactly correct as the year is considered as consisting of 360 days, instead of 365; so that the interest, obtained in this manner, is too large by $\frac{5}{365}$ or $\frac{1}{73}$, which for every \$73 interest, is \$1 too much, and must therefore be subtracted if the exact amount be required.

E X A M P L E .

67. What is the interest of \$24, for 4 months, 8 days, at 6 per cent.?

S O L U T I O N .

The interest of \$1, for 4 months, is.....	.02
The interest of \$1, for 8 days, is.....	.001 $\frac{1}{3}$
Hence the interest of \$1, for 4 months, 8 days, is.....	.021 $\frac{1}{3}$

Now, if the interest of \$1, for the given time, is .021 $\frac{1}{3}$, the interest of \$24 will be 24 times .021 $\frac{1}{3}$, which is \$.512.

E X E R C I S E S .

68. What is the interest on \$171, for 24 days, at 6 per cent.?
Ans. 68 cents.
69. What is the interest on \$112, for 118 days, at 6 per cent.?
Ans. \$2.20.
70. What is the interest on \$11, for 112 days, at 6 per cent.?
Ans. 21 cents.
71. What is the interest on 50 cents, for 360 days, at 6 per cent.?
Ans. 3 cents.
72. What is the interest on \$75.00, for 236 days, at 6 per cent.?
Ans. \$2.95.
73. What is the interest on \$111.50, for 54 days, at 6 per cent.?
Ans. \$1.00.
74. What is the interest on \$15.50, for 314 days, at 6 per cent.?
Ans. 81 cents.

75. What is the interest on \$174.25, for 42 days, at 6 per cent.?
Ans. \$1.22.
76. What is the interest on \$10, for 1 month, 18 days, at 6 per cent.
Ans. 8 cents.
77. What is the interest on \$154, for 3 months, at 6 per cent.?
Ans. \$2.31.
78. What is the interest on \$172, for 2 months, 15 days, at 6 per cent.?
Ans. \$2.15.
79. What is the interest on \$25, for 4 months, at 6 per cent.?
Ans. 50 cents.
80. What is the interest on \$36, for 1 year, 3 months, 11 days, at 7 per cent.?
Ans. \$3.23.
81. What is the interest on \$500, for 160 days, at 6 per cent.?
Ans. \$13.33.
82. What is the interest on \$92.30, for 78 days, at 5 per cent.?
Ans. \$1.00.
83. What is the interest on \$125, for 3 years, 5 months, 15 days, at 10 per cent.
Ans. \$43.23.
84. What is the amount of \$200, for 9 months, 27 days, at 6 per cent.?
Ans. \$209.90.
85. What is the interest on \$125.75, for 5 months, 17 days, at 7 per cent.?
Ans. \$4.08.
86. What is the interest on \$84.50, for 1 month, 20 days, at 5 per cent.?
Ans. 59 cents.
87. What is the amount of \$45, for 1 year, 1 month, 1 day, at 8 per cent.?
Ans. \$48.91.
88. What is the interest on \$175, for 7 months, 6 days, at $5\frac{1}{2}$ per cent.?
Ans. \$5.78.
89. What is the interest on \$225, for 3 months, 3 days, at 9 per cent.?
Ans. \$5.23.
90. What is the interest on \$212.60, for 9 months, 8 days, at $8\frac{1}{2}$ per cent.?
Ans. \$13.95.

CASE VII.

To find the interest on any sum of money, in pounds, shillings, and pence, for any time, at a given rate per cent.

RULE.

Multiply the principal by the rate per cent., and divide by 100.

EXAMPLE.

91. What is the interest of £47 15s. 9d., for 1 year, 9 months, 15 days, at 6 per cent. ?

SOLUTION.

	£	s.	d.	£	s.	d.
Interest for 1 year.....	2	17	4	47	15	9
Interest for 6 mos., or $\frac{1}{2}$ of int. for 1 year,	1	8	8			6
Interest for 3 mos., or $\frac{1}{2}$ of int. for 6 mos.,	0	14	4			
Interest for 15 days, or $\frac{1}{6}$ of int. for 3 mos.,	0	2	$4\frac{1}{2}$	2)	86	14 6
					20	
Interest for 1 year, 9 months, 15 days....	£5	2	$8\frac{1}{2}$			
				17)	34	
					12	
				4)	14	

92. What is the interest of £25, for 1 year, 9 months, at 5 per cent. ?
Ans. £2 3s. 9d.

93. What is the interest of £75 12s. 6d., for 7 months, 12 days, at 8 per cent. ?
Ans. £3 14s. $7\frac{1}{2}$ d.

94. What is the amount of £64 10s. 3d., for 3 months, 3 days, at 7 per cent. ?
Ans. £65 13s. 7d.

95. What is the interest of £35 4s. 8d., for 6 months, at 10 per cent. ?
Ans. £1 15s. $2\frac{3}{4}$ d.

96. What is the amount of £18 12s., for 10 months and 3 days, at 6 per cent. ?
Ans. £19 10s. $9\frac{3}{4}$ d.

CASE VIII.

To find the PRINCIPAL, the interest, the time, and the rate per cent. being given.

EXAMPLE.

97. What principal will produce \$4.50 interest in 1 year, 3 months, at 6 per cent. ?

SOLUTION.

If a principal of \$1 is put on interest for 1 year, 3 months, at 6 per cent., it will produce .075 interest. Now, if in this example, .075 be the interest on \$1, the number of dollars required to produce \$4.50, will be represented by the number of times that .075 is contained in \$4.50, which is 60 times. Therefore, \$60 will produce \$4.50 interest in 1 year, 3 months, at 6 per cent. Hence the

R U L E .

Divide the given interest by the interest of \$1 for the given time, at the given rate per cent.

E X E R C I S E S .

98. What principal will produce 77 cents interest in 3 months, 9 days, at 7 per cent. ? Ans. \$40.

99. What principal will produce \$10.71 interest in 8 months, 12 days, at $7\frac{1}{2}$ per cent. ? Ans. \$204.

100. What principal will produce \$31.50 interest in 4 years, at $3\frac{1}{2}$ per cent. ? Ans. \$225.

101. What sum of money will produce \$79.30 interest in 2 years, 6 months, 15 days, at $6\frac{1}{2}$ per cent. ? Ans. \$480.

102. What sum of money is sufficient to produce \$290 interest in 2 years and 6 months, at $7\frac{1}{4}$ per cent. ? Ans. \$1600.

C A S E I X .

To find the RATE PER CENT., the principal, the interest, and the time being given.

E X A M P L E .

103: If \$3 be the interest of \$60 for 1 year, what is the rate per cent. ?

S O L U T I O N .

If the interest of \$60 for 1 year, at 1 per cent, is .60, the required *rate per cent.* will be represented by the number of times that .60 is contained in 3.00, which is 5 times. Therefore, if \$3 is the interest of \$60 for 1 year, the rate per cent. is 5. Hence the

R U L E .

Divide the given interest by the interest of the given principal at 1 per cent. for the given time.

E X E R C I S E S .

104. If the interest of \$40, for 2 years, 9 months, 12 days, is \$13.36 ; what is the rate per cent. ? Ans. 12.

105. If I borrow \$75 for 2 months, and pay \$1 interest ; what is the rate per cent. ? Ans. 8.

106. If I give \$2.25 for the use of \$30 for 9 months; what rate per cent. am I paying? Ans. 10.

107. At what rate per cent. will \$150 amount to \$165.75, in 1 year, 4 months, 24 days? Ans. $7\frac{1}{2}$.

108. At what rate per cent. must \$1, or any sum of money, be on interest to double itself in 12 years? Ans. $8\frac{1}{3}$.

109. At what rate per cent. must \$425 be lent to gain \$11.73 in 3 months, 18 days? Ans. $9\frac{1}{5}$.

110. At what rate per cent. will any sum of money amount to three times itself in 25 years? Ans. 8.

111. If I give \$14 for the interest of \$125 for 1 year, 7 months, 6 days; what rate per cent am I paying? Ans. 7.

CASE X.

To find the TIME, the principal, the interest, and the rate per cent. being given.

EXAMPLE.

112. How long must \$75 be at interest, at 8 per cent., to gain \$12?

SOLUTION.

The interest for \$75, for 1 year, at 8 per cent., is \$6. Now, if \$75 require to be on interest for 1 year to produce \$6, it is evident that the number of years required to produce \$12 interest, will be represented by the number of times that 6 is contained in 12, which is 2. Therefore, \$75 will have to be at interest for 2 years to gain \$12. Hence the

RULE.

Divide the given interest by the interest of the principal for one year, at the given rate per cent.

EXERCISES.

113. In what time will \$12 produce \$2.88 interest, at 8 per cent? Ans. 3 years.

114. In what time will \$25 produce 50 cents interest, at 6 per cent.? Ans. 4 months.

115. In what time will \$40 produce 75 cents interest, at $6\frac{1}{4}$ per cent.? Ans. 3 months, 18 days.

117. In what time will any sum of money quadruple itself, at 9 per cent. ? Ans. 33 years, 4 months.

119. Borrowed, January 1, 1865, \$60, at 6 per cent, to be paid as soon as the interest amounted to one-half the principal. When is it due? Ans. May 1, 1873.

120. A merchant borrowed a certain sum of money on January 2, 1856, at 9 per cent., agreeing to settle the account when the interest equalled the principal. When should he pay the same?
 Ans. Feb. 11, 1867.

For showing in what time any sum of money will double itself, at any rate per cent., from one to twenty, simple interest.

Per cent.	Years.	Per cent.	Years.	Per cent.	Years.	Per cent.	Years.
1	100	6	$16\frac{2}{3}$	11	$9\frac{1}{11}$	16	$6\frac{1}{4}$
2	50	7	$14\frac{2}{7}$	12	$7\frac{1}{2}$	17	$5\frac{1}{17}$
3	$33\frac{1}{3}$	8	$12\frac{1}{2}$	13	$7\frac{9}{13}$	18	$5\frac{5}{9}$
4	25	9	$11\frac{1}{9}$	14	$7\frac{1}{7}$	19	$5\frac{5}{19}$
5	20	10	10	15	$6\frac{2}{5}$	20	5

Ans. \$13.49.

Ans. 3.76.

Ans. \$444.16.

Ans. 51 cents.

Ans. \$57.81.

* This and the following exercises (marked with a *) are to be worked by Case VI.

126. What must be paid for the interest of \$45 for 72 days, at 9 per cent. ?* Ans. 81 cents.

127. What is the interest of \$240 from January 1, 1866, to June 4, 1866, at 7 per cent. ? Ans. \$7.14.

128. What will \$140.40 amount to from August 29, 1865, to November 29, 1866, at $6\frac{1}{2}$ per cent. ? Ans. \$151.83.

129. What principal will give \$4.40 interest in 1 year, 4 months, 15 days, at 8 per cent. ? Ans. \$40.

130. In what time will \$40 amount to \$44.40, at 8 per cent. ? Ans., 1 yr., 4 mos., 15 days.

131. At what rate per cent. will \$40 produce in 1 yr., 4 mos., 15 days, \$4.40 interest ? Ans. 8.

132. What must be paid for the interest of \$145.50 for 240 days, at $9\frac{1}{2}$ per cent. ?* Ans. \$9.22.

133. What will \$160 amount to in 175 days, at 6 per cent. ?* Ans. \$164.67.

134. At what rate per cent. must any sum of money be on interest to quadruple itself in 33 years and 4 months ? Ans. 9.

135. In what time will any sum of money double itself, at 10 per cent. ? Ans. 10 years.

136. What is the interest of \$30 for 30 days, at 6 per cent. ?* Ans. 15 cents.

137. What is the interest on \$460 from January 2, 1866, to November 15, 1866, at $7\frac{1}{2}$ per cent. ?* Ans. \$30.

138. What will \$25 amount to from December 24, 1865, to January 1, 1867, at 6 per cent. ? Ans. \$26.53.

139. What sum of money will amount to \$6400 in 4 years and 8 months, at 6 per cent. ? Ans. \$5000.

140. In what time will \$480, at $4\frac{1}{2}$ per cent., produce \$81.60 interest ? Ans. 3 years, 9 months, 10 days.

NEGOTIABLE INSTRUMENTS.

Bills of Exchange and Promissory Notes constitute, in their different shapes, the commercial medium for the payment of money.

A *Bill of Exchange* is a written order for the payment of a certain sum of money unconditionally.

A *Promissory Note* is a written promise to pay a certain sum of money unconditionally.

Cheques are only a description of Bills of Exchange. A *Cheque* is a written order addressed to a bank, or banker, and directing them

to pay on presentment, to a person named in the cheque (or bearer, or order), a certain sum of money.

Bank Notes are simple promissory notes, payable on demand.

FORM AND REQUISITES.

A bill or note is called *inland*, when both made and payable in one country ; and *foreign*, when made or payable abroad.

The following is a form of an inland bill :

\$500.

TORONTO, January 1, 1866.

At sight, (or on demand, or at ——— days after sight, or at ——— days after date), pay to Messrs. A. B. Smith & Co., or order, (or bearer), Five Hundred Dollars.

MUSGROVE & WRIGHT.

To Messrs. JONES & BROWN, Montreal.

The following may be taken as an example of a promissory note :

\$500.

TORONTO, January 1, 1866.

Three months after date we promise to pay to Messrs. A. B. Smith & Co., or order, Five Hundred Dollars, value received.

MUSGROVE & WRIGHT.

A bill of exchange amounts to an undertaking on the part of the drawer to the payee, and every subsequent holder, that the drawee will accept the bill when requested, and pay it when it becomes due.

The maker of a promissory note undertakes to the payee, and every subsequent holder, that he will pay the note when due.

All bills and notes made, drawn or accepted in Canada, are subject to a duty, collected by means of stamps. On every such instrument, if the amount do not exceed \$25, a duty of *one cent* is imposed. From \$25 up to \$50, the duty is *two cents* ; from \$50 to \$100 it is *three cents* ; and after that, it is *three cents* for every hundred dollars, or fraction of a hundred dollars. In the case of foreign bills, in sets of two, the duty is *two cents* for each of the set, on the \$100 ; in sets of three or more, the duty is only *one cent* for each of the set on the \$100.

Foreign bills are usually drawn in *sets* ; that is, copies of the bills are made on separate pieces of paper, each part containing a condition that it shall continue payable only so long as the others remain unpaid. The object of this is to diminish the chances of losing a bill ; for if one part should fail to reach its destination, one of the others would be likely to do so.

PARTIES TO BILLS AND NOTES.

The person who draws a bill of exchange is called the *drawer*; he, to whom it is addressed, the *drawee*, and, when he accepts it, the *acceptor*; he in whose favor it is made, the *payee*.

The person who signs a promissory note is called the *maker*; the person to whom the promise is made, the *payee*.

The moment a promissory note is *indorsed* by the payee, by his writing his name on the back of it, he is called the *indorser*. The person to whom it is indorsed is the *indorsee*.

When a bill is accepted, or a note is made by several persons who are not in Partnership, the question whether they are bound jointly, or jointly and separately, depends upon the wording of the document. If a note begin thus:—"I promise," and be signed by several persons, it is several, as well as joint.

The name of the maker or drawer must be inserted or subscribed by himself, or his agent. There must be no uncertainty about the maker or drawer. For example, a note may not be signed "John Smith, or else Robert Jones."

TRANSFER OF BILLS AND NOTES.

A bill or note may be payable to a particular individual, or to a particular individual or his *order*, or generally to *bearer*. When a bill or note is made payable simply to an individual, it is not negotiable. If it be payable to an individual or *order*, he may transfer his right to another, by endorsing his name upon it. If it be payable to an individual or *bearer*, it may be transferred by mere delivery, without any indorsement. If a blank be left for the payee's name, any *bona fide* holder may insert his own.

An indorsement is said to be *in blank* when it does not mention the name of the party in whose favor it is made. A bill or note, when indorsed in blank, is transferable by delivery.

A full or special indorsement is one which mentions the name of the party in whose favor it is made; and it has to be endorsed in blank by the latter before it can be rendered negotiable.

A *restrictive* indorsement puts an end entirely to any further negotiability of a note or bill. For instance, if a note be indorsed thus:—"Pay the contents to John Smith only,"—it would be no longer transferable.

Negotiable paper in the hands of an innocent holder, without notice of anything wrong, is good, although the person from whom

he obtained it may have come by it as a thief or finder. There is an exception to this when a person takes a bill or note *after* it is due. He then is in no better position than the person from whom it was received, and could not recover if it had been fraudulently obtained by the latter.

PRESENTMENT AND ACCEPTANCE.

A bill should be always presented for acceptance; and bills and notes for payment, when payable at or after sight, or at some particular place. A note payable on demand need not be presented in order to charge the maker. Presentment for payment must be made at a reasonable hour of the day upon which an instrument becomes payable. An acceptance of a bill may be made without any particular form of words, and even without the signature of the acceptor. An acceptance may be conditional, and in that case is not of any force until the condition is performed. When a bill or note specifies a particular place for payment, it should be presented there; although the maker or acceptor would not be discharged by a failure to present there, unless the words "and not otherwise or elsewhere" had been added.

Presentment for acceptance must be made within a reasonable time. No delay upon the part of the holder should extend over any period not rendered clearly necessary by the circumstances of the case.

PROTEST AND NOTICE.

When a bill or note is dishonoured, either by non-acceptance or non-payment, it should be protested by a notary public. Notice of such protest should be sent to every party on the instrument to whom the holder desires to have recourse. Notice means more than mere knowledge, and must be a formal communication of the dishonour of the bill. This notice of protest may be given by being deposited in the post office nearest to the place of presentment during the day the protest is made, or the next lawful day. From the date of protest interest commences to run upon the instrument, although there are no words in it about interest. Besides interest, and the expenses of protest and notice, the holder is entitled, when foreign bills are protested, to damages varying from four to ten per cent. of the principal amount specified in the bill.

DAYS OF GRACE.

The time limited by the words of a bill or note for its payment is extended in this country by *days of grace*; so that a bill is not really payable till three days after the day upon which it purports to fall due. When a bill is made payable so many days after the the happening of a particular event, for instance, after sight, the day on which that event happens is excluded. Accordingly, a bill payable ten days after sight, and presented on the 1st August, would purport to be due on the 11th, but adding the days of grace, would in reality not be payable till the 14th.

PARTIAL PAYMENTS.

Partial payments, as the term indicates, are the part payments of promissory notes, bonds, or other obligations.

When these payments are made the creditor specifies in writing, on the *back* of the note, or other instrument, the sum paid, and the time when it is paid, and acknowledges it by signing his name.

The method we generally adopt in Canada for casting interest upon bonds, notes, or other obligations, upon which partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the time when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal.

R U L E :

Find the amount of the principal to the time of the first payment; subtract the payment from the amount, and then find the amount of the remainder to the time of the second payment; deduct the payment as before; and so on to the time of settlement.

But if any payment is less than the interest then due, find the amount of the sum due to the time when the payments, added together, shall be equal, at least, to the interest already due; then find the balance, and proceed as before.

E X A M P L E .

1. On the 4th of January, 1865, a note was given for \$800, payable on demand, with interest at 6 per cent. The following payments were receipted on the back of the note :

February 7th, 1865, received.....	\$150
April 16th, " " 	100
Sept., 30th, " " 	180
January 4th, 1866. " 	170
March 24th, " " 	100
June 12th, " " 	50

Settled July 1st, 1867. How much was due ?

S O L U T I O N :

Face of the note, or principal.....	\$800.00
Interest on the same to February 7th, 1865 (1 month, 3 days).....	4.40
Amount due at time of 1st payment.....	804.40
First payment to be taken from this amount.....	150.00
Balance remaining due February 7th, 1865.....	654.40
Interest on the same from February 7th, 1865, to April 16th, 1865.....	7.525
Amount due at time of 2nd payment.....	661.925
Second payment to be taken from this amount.....	100.000
Balance remaining due April 16th, 1865.....	561.925
Interest on the same from April 16th, 1865, to September 30th, 1865.....	15.359
Amount due at time of 3rd payment.....	577.284
Third payment to be taken from this amount.....	180.000
Balance remaining due Sept. 30th, 1865.....	397.284
Interest on the same from Sept. 30th, 1865, to January 4th, 1866	6.290
Amount due at time of 4th payment.....	403.574
Fourth payment to be taken from this amount.....	170.000
Balance remaining due January 4th, 1866.....	233.574

Interest on the same from Jan. 4th, 1866, to March 24th, 1866	3.114
Amount due at time of 5th payment.....	236.688
Fifth payment to be taken from this amount.....	100.000
Balance remaining due, March 24th, 1866.....	136.688
Interest on the same from March 24th, 1866, to June 12th, 1866.....	1.799
Amount due at time of 6th payment.....	138.487
Sixth payment to be taken from this amount.....	50.000
Balance remaining due June 12th, 1866.....	88.487
Interest on the same from June 12th, 1866, to July 1st, 1867	5.589
Amount due on settlement.....	94.076

2. \$1600.

PERTH, C. W., February 16th, 1865.

On demand, I promise to pay Jacob Anderson, or order, one thousand six hundred dollars, with interest, at 7 per cent.

JOHN FORTUNE JR.

There was paid on this note,

April 19th, 1865.....	\$460
July 22nd " 	150
August 25th, 1866.....	50
Sept. 12th, " 	100
Dec. 24th. " 	700

How much was due December 31st, 1866 ?

SOLUTION.

Face of the note or principal.....	\$1600.00
Interest on the same from Feb. 16th, 1865, to April 19th, 1865.....	19.60
Amount due at time of 1st payment.....	1619.60
First payment to be taken from this amount.....	460.00
Balance remaining due, April 19th. 1865.....	1159.60

Interest on the same from April 19th, 1865, to July 22nd, 1865	20.969
Amount due at time of 2nd payment.....	1180.569
Second payment to be taken from this amount.....	150.000
Balance remaining due, July 22nd, 1865.....	1030.569
Interest on the same from July 22nd, 1865, to Aug. 25th, 1866, less than 3rd payment,*	
Interest on the same from July 22nd, 1865, to Sept. 12th, 1866.....	82.359
Amount due at time of 4th payment	1112.928
Third and fourth payments to be taken from this amount,	150.000
Balance remaining due Sept. 12th, 1866.....	962.928
Interest on the same from Sept. 12th, 1866, to Dec. 24th, 1866	19.098
Amount due at time of last payment.....	982.026
Last payment to be taken from this amount.....	700.000
Balance remaining due Dec. 24th, 1866.....	282.026
Interest on the same from Dec. 24th, 1866, to Dec. 31st, 1866	.382
Amount due at time of settlement, Dec. 31st, 1866.....	\$282.408

3. \$350.

OTTAWA, May 1st, 1864.

On demand I promise to pay William Brown, or order, three hundred and fifty dollars, with interest, at 6 per cent.

JAMES WESTON.

There was paid on this note,

December 25th, 1864.....	\$50
June 30th, 1865.....	5

* The interest on \$1030.569, from July 22nd, 1865, to August 25th, 1866, is \$78.752, and the payment made at this date, is only \$50, not enough to pay the interest, so if we proceeded, as in the former case, to add the interest to the principal, and subtract the payment from the amount obtained, we would be taking interest, until the next payment, on the excess of the interest, \$78.752, over the payment, \$50, which would be in effect interest upon interest, or compound interest which the law does not allow.

August 22nd,	1866.....	15
June 4th,	1867.....	100

How much was due April 5th, 1868? Ans. \$251.67.

4. \$609.65. BRANTFORD, June 8th, 1861.

Six months after date, we jointly and severally promise to pay John Anderson, or order, six hundred and nine $\frac{65}{100}$ dollars, at the Royal Canadian Bank in Toronto, with interest at 6 per cent. after maturity.

SAMUEL GRAHAM.
T. B. BEARMAN.

There was paid on this note,

October 4th, 1862.....	\$25.00
March 15th, 1863.....	16.25
August 24th, 1864.....	36.56

What was due December 19th, 1865? Ans. 679.27.

5. \$874.95. KINGSTON, May 9th, 1863.

Three months after date, I promise to pay Harmon Cummings, or order, eight hundred and seventy-four $\frac{95}{100}$ dollars, with interest after maturity at 6 per cent.

THOMAS GOODPAY.

There was paid on this note,

April 12th, 1864.....	\$56.30
July 14th, 1865.....	24.80
Sept. 18th, 1866.....	240.60

What was due February 9th, 1868? Ans. \$773.07.

When the interest accruing on a note is to be paid annually adopt the following

R U L E . *

Compute the interest on the principal to the time of settlement, and on each year's interest after it is due, then add the sum of the

* When notes, bonds, or other obligations, are given, "with interest payable annually," the interest is due at the end of each year, and may be collected, but if not collected at that time, the interest due draws only *simple interest*, and the original principal must not be increased by any addition of yearly interest. If nothing has been paid until maturity on a note drawing annual interest, the amount due consists of the principal, the total annual interest, or the simple interest, and the simple interest on each item of annual interest from the time it became due until paid.

interests on the annual interests to the amount of the principal, and from this amount take the payments, and the interest on each, from the time they were paid to the time of settlement, the remainder will be the amount due.

6. \$500.

PRESCOTT, May 1st, 1864.

One year after date, for value received, I promise to pay Musgrove & Wright, or order, Five Hundred Dollars, at their office, in the city of Toronto, with interest at 6 per cent., payable annually.

JAMES MANNING.

There was paid on this note :

May 4th, 1865..... \$150

Dec. 18th, " 300

How much was due June 1st, 1866 ?

SOLUTION.

Face of note, or principal.....	\$500.00
Interest on the same from May 1st 1864, to June 1st, 1866	62.50
Amount of the principal at time of settlement.....	562.50
First year's interest on principal.....	\$30
Interest on the same from May 1st, 1865, to June 1st, 1866	\$1.95
Second year's interest on principal.....	\$30
Interest on the same from May 1st, 1866, to June 1st, 1866.....	.15
Amount of interest upon <i>annual interest</i>	2.10
Total amount of principal.....	\$564.60
First payment, May 4th, 1865.....	\$150.00
Interest on the same from May 4th, 1865, to June 1st, 1866.....	9.70
Second payment, December 18th, 1865.....	300.00
Interest on the same from December 18th, 1865, to June 1st, 1866	8.20
Payments and interest on the same.....	467.90
Amount due June 1st, 1866	\$96.70

7. \$700.

BELLEVILLE, January 2nd, 1863.

Eighteen months after date, I promise to pay to the order of H. C. Wright, Seven Hundred Dollars, for value received, with interest at 6 per cent., payable annually.

THOS. A. BRYCE.

There was paid on this note:

January 15th, 1864..... \$350

July 2nd, 1864..... 300

What amount was due January 2nd, 1865? Ans. \$107.22.

8. \$950.

DUNDAS, C. W., Jan. 3rd, 1863.

Two years after date, I promise to pay T. C. Musgrove, or order, Nine Hundred and Fifty Dollars, with interest at 9 per cent., payable annually, value received.

JAMES S. PARMENTER.

The following payments were receipted on the back of this note:

February 1st, 1864, received..... \$500

May 14th, " " 100

January 12th, 1865, " 300

What was due May 6th, 1865? Ans. 188.94.

9. \$250.

OSHAWA, January 2nd, 1863.

Three years from date, for value received, I promise to pay Michael Wright, or order, Two Hundred and Fifty Dollars, with interest, payable annually, at 6 per cent.

CALVIN W. PEARSONS,

At Bank of Montreal, Brockville.

What was the amount of this note at maturity? Ans. \$297.70.

CONNECTICUT RULE.

The Supreme Court of the State of Connecticut has adopted the following

RULE.

Compute the interest on the principal to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above, and in like manner from one payment to another, till all the

payments are absorbed, provided the time between one payment and another be one year or more.

If any payments be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest, added as above.

If any payments be made, of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.

NOTE.—If a year extends beyond the time of settlement, find the amount of the remaining principal to the time of settlement; find also the amount of the payment or payments, if any, from the time they were paid to the time of settlement, and subtract their sum from the amount of the principal.

EXAMPLE.

10. \$900.

TORONTO, June 1st, 1862.

On demand we promise to pay J. R. Smith & Co., or order, nine hundred dollars, for value received, with interest from date, at 6 per cent.

MUSGROVE & WRIGHT.

On the back of this note were receipted the following payments:

June 16th, 1863, received.....	\$200
August 1st, 1864, “	160
Nov. 16th, 1864, “	75
Feby. 1st, 1866 “	220

What amount was due August 1st, 1866?

SOLUTION.

Face of note or principal.....	\$900.00
Interest on the same from June 1st, 1862, to June 16th, 1863.....	56.25
Amount of principal and interest, June 16th, 1863.....	956.25
First payment to be taken from this amount.....	200.00
Balance due.....	756.25
Interest on the same from June 16th, 1863, to August 1st, 1864.....	51.046
Amount due August 1st, 1864	807.296

SOLUTION.

Principal or face of note.....	\$400.00
Interest on the same from Jan. 1st, 1865, to Dec. 28th, 1865	23.80
Amount of principal at settlement.....	\$423.80
First payment.....	\$100.00
Interest on the same from Feb. 4th, 1865, to Dec. 28th, 1865.....	5.40
Second payment.....	75.00
Interest on the same from May 16th, 1865, to Dec. 28th, 1865.....	2.77½
Third payment.....	100.00
Interest on the same from August 28th, 1865, to Dec. 25th, 1865.....	2.00
Fourth payment.....	80.00
Interest on the same from Nov. 25th, 1865, to Dec. 28th, 1865.....	.44
Amount of payments to be taken from <i>amount</i> of principal.....	365.61½
Balance due, December 28th, 1865.....	\$58.18½

12. \$500. HAMILTON, January 1st, 1865.

Three months after date, I promise to pay James Manning, or order, five hundred dollars, for value received, at the Royal Canadian Bank in Toronto.

CYRUS KING.

Mr. King paid on this note, July 1st, 1865, \$200.

What was due April 1st, 1866, the rate of interest being 7 per cent.?

Ans. \$324.50.

13. \$240. SMITH'S FALLS, May 4th, 1865.

On demand, I promise to pay A. K. Frost & Co., or order, two hundred and forty dollars, for value received, with interest at 6 per cent.

DAVID FLOOK.

The following payments were receipted on the back of this note:

September 10th, 1865, received.....	\$60
January 16th, 1866, "	90

What was due at the time of settlement, which was May 4th, 1866?

Ans. \$100.44.

14. \$340.

NEWMARKET, June 16th, 1864.

Three months after date, I promise to pay Thomas Culverwell, or order, three hundred and forty dollars, with interest, at 6 per cent.

WILLIAM MUSGROVE.

On this note were receipted the following payments :

October 14th, 1864, received..... \$86

February, 12th, 1865, " 40

What was due at time of settlement, Aug. 10, 1865 ? Ans. \$232.06.

COMPOUND INTEREST.

When interest is unpaid at the end of a year, it may, by special agreement, be added to the principal, and in its turn bear interest, and so on from year to year. When added to the principal in this way, it is said to be *compounded*.

It is not against the law in Canada to take compound interest; but it can never be collected unless it has been specially agreed upon beforehand, or unless it is the custom of a house, and known to the customer to that effect.

EXAMPLE.

1. What is the compound interest of \$60, for 4 years, at 7 per cent. ?

SOLUTION.

Principal.....	\$60.00
Interest on the same for one year.....	4.20
New principal for 2nd year.....	64.20
Interest on the same for one year.....	4.494
New principal for 3rd year	68.694
Interest on the same for one year.....	4.808
New principal for 4th year.....	73.502
Interest on the same for one year.....	5.145
Amount for 4 years.....	78.647
Principal to be taken from same.....	60.000
Compound interest for 4 years.....	\$18.647

The method of finding compound interest is usually much shortened by the following table, which shows the amount of \$1 or £1 for any number of years not exceeding 50, at 3, $3\frac{1}{2}$, 4, 5, 6 and 7 per cent. The amount of \$1 or £1 thus obtained, being multiplied by the given principal, will give the required amount, from which, if the principal be taken, the remainder will be the compound interest :

TABLE,

SHOWING THE AMOUNT OF ONE DOLLAR AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS
NOT EXCEEDING FIFTY.

No.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.030 000	1.035 000	1.040 000	1.050 000	1.060 000	1.070 000
2	1.060 900	1.071 225	1.081 600	1.102 500	1.123 600	1.144 900
3	1.092 727	1.108 718	1.124 864	1.157 625	1.191 016	1.225 043
4	1.125 509	1.147 523	1.169 859	1.215 506	1.262 477	1.310 796
5	1.159 274	1.187 686	1.216 653	1.276 282	1.338 226	1.402 552
6	1.194 052	1.229 255	1.265 319	1.340 096	1.418 519	1.500 730
7	1.229 874	1.272 279	1.315 932	1.407 100	1.503 630	1.605 781
8	1.266 770	1.316 809	1.368 569	1.477 455	1.593 848	1.718 186
9	1.304 773	1.362 897	1.423 312	1.551 328	1.689 479	1.838 459
10	1.343 916	1.410 599	1.480 244	1.628 895	1.790 848	1.967 151
11	1.384 234	1.459 970	1.539 454	1.710 339	1.898 299	2.104 852
12	1.425 761	1.511 069	1.601 032	1.795 856	2.012 196	2.252 192
13	1.468 534	1.563 956	1.665 074	1.885 649	2.132 928	2.409 845
14	1.512 590	1.618 694	1.731 676	1.979 932	2.260 904	2.578 534
15	1.557 967	1.675 349	1.800 944	2.078 928	2.396 558	2.759 032
16	1.604 706	1.733 986	1.872 981	2.182 875	2.540 352	2.952 164
17	1.652 848	1.794 675	1.947 901	2.292 018	2.692 773	3.158 815
18	1.702 433	1.857 489	2.025 817	2.406 619	2.854 339	3.379 932
19	1.753 506	1.922 501	2.106 849	2.526 950	3.025 600	3.616 526
20	1.806 111	1.989 789	2.191 123	2.653 298	3.207 135	3.869 684
21	1.860 295	2.059 431	2.278 768	2.785 963	3.399 564	4.140 562
22	1.916 103	2.131 512	2.369 919	2.925 261	3.603 537	4.430 402
23	1.973 587	2.206 114	2.464 716	3.071 524	3.819 750	4.740 530
24	2.032 794	2.283 328	2.563 304	3.225 100	4.048 935	5.072 367
25	2.093 778	2.363 245	2.665 836	3.386 355	4.291 871	5.427 433
26	2.156 591	2.445 959	2.772 470	3.555 673	4.549 383	5.807 353
27	2.221 289	2.531 567	2.883 369	3.733 456	4.822 346	6.213 868
28	2.287 928	2.620 177	2.998 703	3.920 129	5.111 687	6.648 838
29	2.356 566	2.711 878	3.118 651	4.116 136	5.418 388	7.114 257
30	2.427 262	2.806 794	3.243 398	4.321 942	5.743 491	7.612 255
31	2.500 080	2.905 031	3.373 133	4.538 039	6.088 101	8.145 113
32	2.575 083	3.006 708	3.508 059	4.764 941	6.453 387	8.715 271
33	2.652 335	3.111 942	3.648 381	5.003 189	6.840 590	9.325 340
34	2.731 905	3.220 860	3.794 316	5.253 348	7.251 025	9.978 114
35	2.813 862	3.333 590	3.946 089	5.516 015	7.686 087	10.676 581
36	2.890 278	3.450 266	4.103 933	5.791 816	8.147 252	11.423 942
37	2.985 227	3.571 025	4.268 090	6.081 407	8.636 087	12.223 618
38	3.074 783	3.696 011	4.438 813	6.385 477	9.154 252	13.079 271
39	3.167 027	3.825 372	4.616 366	6.704 751	9.703 507	13.994 820
40	3.262 038	3.959 260	4.801 021	7.039 989	10.285 718	14.974 458
41	3.359 899	4.097 834	4.993 061	7.391 988	10.902 861	16.022 670
42	3.460 696	4.241 258	5.192 784	7.761 588	11.557 033	17.144 257
43	3.564 517	4.389 702	5.400 495	8.149 667	12.250 455	18.344 355
44	3.671 452	4.543 342	5.616 515	8.557 150	12.985 482	19.628 460
45	3.781 596	4.702 358	5.841 176	8.985 003	13.764 611	21.002 452
46	3.895 044	4.866 941	6.074 823	9.434 258	14.590 487	22.472 623
47	4.011 895	5.037 284	6.317 816	9.905 971	15.465 917	24.045 707
48	4.132 252	5.213 589	6.570 528	10.401 270	16.393 872	25.728 907
49	4.256 219	5.396 065	6.833 349	10.921 333	17.377 504	27.529 930
50	4.383 906	5.584 927	7.106 683	11.467 400	18.420 154	29.457 025

NOTE.—If each of the numbers in the table be diminished by 1, the remainder will denote the interest of \$1, instead of its amount.

EXERCISES.

2. What is the compound interest on \$75, for 2 years, at 7 per cent. ?
Ans. \$10.87.

3. What will \$50 amount to in 3 years, at 6 per cent., compound interest ?
Ans. \$59.55.

4. What is the compound interest on \$600, for 2 years, at 6 per cent., payable half-yearly ?
Ans. \$75.31.

5. What will \$320 amount to in $2\frac{1}{2}$ years, at 7 per cent., compound interest ?
Ans. \$379.19.

6. What is the compound interest of \$150, for 3 years, at 9 per cent. ?
Ans. \$44.25.

7. What is the compound interest on \$1,000, for 2 years, at $3\frac{1}{2}$ per cent, payable quarterly ?
Ans. \$72.18.

8. What will \$460 amount to in 3 years, 4 months, 10 days, at 6 per cent., compound interest ?
Ans. \$559.74.

9. What is the compound interest on \$1860, for 8 years, at 7 per cent. ?
Ans. \$1335.83.

10. What will be the compound interest on \$75.20, for 20 years, at $3\frac{1}{2}$ per cent. ?
Ans. \$74.43.

11. How much more will \$500 amount to at compound than simple interest, for 20 years, 3 months, 15 days, at 7 per cent. ?
Ans. \$764.14.

12. What sum will \$50, deposited in a savings bank, amount to at compound interest, for 21 years, at 3 per cent, payable half-yearly ?
Ans. \$173.03.

13. If a note of \$60.60, dated October 25th, 1856, with the interest payable yearly, at 6 per cent., be paid October 25th, 1860 ; what will it amount to at compound interest ?
Ans. \$76.51.

14. What remains due on the following note, April 1st, 1863, at 7 per cent. compound interest ?

\$1,000.

PERTH, C. W., January 1, 1858.

For value received, I promise to pay A. B. Smith & Co., or order, one thousand dollars on demand, with interest at 7 per cent.

J. D. FOSTER.

On the back of this note were receipted the following payments :

June 10, 1858, received.....	\$70
Sept. 25, 1859, “	80
July 4, 1860, “	100

Nov. 11, 1861,	"		30
June 5, 1862,	"		50
			Ans. \$1022.34.

DISCOUNT AND PRESENT WORTH.

Discount being of the same nature as interest, is, strictly speaking, the use of money before it is due. The term is applied, however, to a deduction of so much per cent. from the face of a bill, or the deducting of interest from a note before any interest has accrued. This is the practice followed in our Banks, and is therefore called Bank discount, in order to distinguish it from true discount.

The method of computing bank discount differs in no way from that of computing simple interest, but the method of finding *true discount* is quite different, *e. g.*, a debt of \$107, due one year hence, is considered to be worth \$100 now, for the reason that \$100 let out at interest now, at 7 per cent., would amount to \$107 at the end of a year.

In calculating interest, the sum on which interest is to be paid is known, but in computing discount we have to find *what sum* must be placed at interest so that *that sum*, together with its interest, will amount to the given principal. The sum thus found is called the "Present Worth."

We have already seen that \$1.00 is the present worth of \$1.07 due one year hence, at 7 per cent., therefore, to get the present worth of any sum due one year hence, at 7 per cent., it is only necessary to find how many times \$1.07 is contained in the given sum, and we have the present worth; hence

To find the present worth of any sum, and the discount for any time, at any rate per cent., we have the following

R U L E .

Divide the given sum by the amount of \$1 for the given time and rate, and the quotient will be the present worth.

From the given sum subtract the present worth, and the remainder will be the discount.

E X E R C I S E S .

1. What is the present worth of \$224, due 2 years hence, at 6 per cent. ?
Ans. \$200.

be paid December 19th, 1863; \$161.15, to be paid July 16th, 1864; \$112.50, to be paid June 23rd, 1862; \$96.81, to be paid April 19th, 1866, allowing discount at 6 per cent.; how much cash should Manning receive as an equivalent, January 1st, 1862?

Ans. \$653.40.

16. I buy a bill of goods amounting to \$2500 on 6 months' credit, and can get 5 per cent. off by paying cash; how much would I gain by paying the bill now, provided I have to borrow the money, and pay 6 per cent. a year for it?

Ans. \$53.75.

BANKING.

Money has a tendency to concentrate. We thus hear of the money centres of a country, which are the cities where the largest amount of available capital has accumulated. The money which represents this capital is to be found in largest quantities in the Banks of these places. There are many reasons why this should be the case. In the first place, let us consider these institutions as

BANKS OF DEPOSIT.

One strong inducement to place money in a bank is, that it is likely to be more safely kept for the owner than in any private hands. Then the funds are more easily available by means of cheques upon the banker, than in the shape of coin. When cheques are drawn payable to the order of a person, they serve, after indorsement, as receipts. It would be a serious loss of time, as well as a fertile source of mistake, to be always counting out, in small currency, the large amounts that a business man receives and pays. In countries where silver and baser metals constitute the currency, payments and exchanges are very much facilitated by the system of bank deposits and cheques.

In keeping a deposit at a particular bank, it is clear that the depositor makes a continual loan to the institution. For this, business men generally expect an equivalent in the shape of temporary loans; or, "accommodation," as they call it. This is only given by

BANKS OF DISCOUNT,

Which are combined, in Canada, with banks of deposit, issue and exchange. These temporary loans are usually effected by means of endorsed notes or bills, which are cashed by the banks, and the

interest upon them is paid in advance by being discounted, or counted off the face of the paper. A wholesale merchant when he makes a sale, very often takes a note at three or four months for part of his money. He is willing to pay a trifle to get this paper cashed, so he hands it to his banker, who places the amount to his credit, less the interest in advance for the time it has to run.

BANKS OF ISSUE.

All the Canadian banks, with the exception of Savings banks, belong to this class. They are at liberty to issue their notes payable in specie on demand, at the place where they bear date. The amount of notes in circulation at any one time must not exceed the aggregate amount of the paid-up capital stock of the bank, and the specie and government securities on hand. A bank cannot, any more than a private individual, make or issue a note for any less sum than one dollar; and there is generally a restriction in the charter against issuing more than a certain proportion of notes of the lower denominations.

It will be evident that a bank can have a much larger circulation or capital in use, than the original capital as paid in by stockholders. Indeed, part of the notes that they circulate represent government securities, which are paying them interest all the time. To those who do not understand this, it is a matter of surprise that banks can lend their money on notes that they discount at 7 per cent., and still pay working expenses, and declare dividends of 8 per cent. or more. The reason is, that they can lend so much more than they have to pay dividends upon. Besides the large circulation, the deposits of customers are a source of profit, for while interest is seldom paid upon deposits, the money is not allowed to remain idle by the banks, but a portion, at any rate, is loaned out and made productive.

Another source of profit arises from our banks being

BANKS OF EXCHANGE.

In this capacity they are the medium of making payments at distant places through drafts or bills of exchange, and without the actual transmission of money. Under the head of "Exchange," in another place, this branch of the working of banking institutions is fully explained.

PASS BOOKS.

A book, called a *pass book*, is delivered by the bankers to the customer, in which the bankers are described as the debtors, and the

customer as the creditor, in the account. On the debtor side are entered all sums paid to, or received by the bankers on account of the customer, and on the credit side, all sums paid to him, or on his account. For the purpose of having the book made up by the bankers from their own books of account, the customer returns it to them from time to time; and the proper entries being made by them, they hand it again to the customer, who then examines it, and if there appears any error or omission, it is his business to send it back to be rectified; if he does not, his silence is regarded as an admission that the entries are correct.

BANK CORRESPONDENTS.

In the ordinary course of a banking business, it is often necessary to engage the services of other banking houses carrying on business at a distance, who are called *correspondents*. The principal legal point to be noticed in this connection is, that when a bank employs a correspondent, and a loss ensues from the conduct of such agent, the bank is directly responsible to the customer for the loss. This does not prevent the bank having recourse against their correspondent to make good the loss.

BANK DISCOUNT.

The Bank Discount of a note is the simple interest on the sum for which it is given from the time it is *discounted* to the time it becomes due, including three days of grace.

Suppose, for example, in getting a note of \$200 discounted at a bank I am charged \$12 for discount, which being deducted, I receive but \$188, so that I pay interest on \$12 which I did not receive. From this it is clear that I am paying a higher rate of interest in discounting a note at a bank, than I would pay were I to borrow money at the same rate. As bank discount is the same as interest, we derive the following

RULE.

Find the interest on the sum specified in the note at the given rate, and for the given time, including three days of grace, and this will be the BANK DISCOUNT.

Subtract the discount from the face of the note, and the remainder will be the PROCEEDS OR PRESENT WORTH.

EXERCISES.

1. What is the bank discount on a note, given for 60 days, for \$350, at 6 per cent ? * Ans. \$3.62.

2. What is the bank discount on a note of \$495, for 2 months, at 5 per cent. ? Ans. \$4.33.

3. What is the present value of a note of \$7840 discounted at a bank, for 4 months and 15 days, at 6 per cent. ? Ans, \$7659.68.

4. How much money should be received on a note for \$125, payable at the end of 1 year, 3 months, and 15 days, if discounted at a bank at 8 per cent. ? Ans. \$112.

5. A note, dated December 3rd, 1860, for \$160.40, and having 6 months to run, was discounted at a bank, April 3rd, 1861, at 6 per cent. ; how long had it to run, and what were the proceeds ?

Ans. 64 days ; proceeds \$158.71.

6. On the first day of January, 1866, I received a note for \$240, at 60 days, and on the 12th of the same month had it discounted at a bank at 7 per cent ; how much did I realize upon it.

Ans. \$237.61.

7. A merchant sold 240 bales of cotton, each weighing 280 pounds, for $12\frac{1}{2}$ cents per pound, which cost him, the same day, 10 cents per pound ; he received in payment a good note, for 4 months' time, which he discounted immediately at a bank at 7 per cent. ; what will be his profits ? Ans. \$1479.10.

It is sometimes necessary to know the amount for which a note must be given, in order that it shall produce a given sum when discounted at a bank.

EXAMPLE.

1. Suppose we require to obtain \$236.22 from a bank, and that we are to give our note, due in two months ; for what amount must we draw the note, supposing that money is worth 9 per cent. ?

SOLUTION.

From the nature of this example, we can readily perceive that such a sum must be put on the face of the note, that when dis-

* Throughout all the exercises, unless otherwise specified, the year is to be considered as consisting of 365 days. Since it is customary in business when a fraction of a cent occurs in any result to reject it, if less than half a cent, and if not less, to call it a cent, we have adopted this principle throughout the book. •

counted the proceeds will be exactly \$236.22. If we were to take a *one dollar note* and discount it at a bank for the given time, and at the given rate, the proceeds would be .98425. Hence, for *every dollar* we put upon the face of the note we receive .98425, and to receive \$236.22 we would have to put as many dollars on the face of the note as are represented by the number of times that .98425 is contained in \$236.22, which is 240. Therefore, we must put \$240 on the face of a note due at the end of two months to produce \$236.22 when discounted at a bank at 9 per cent. From this we deduce the following

R U L E .

Deduct the bank discount on \$1, for the given time and rate, from \$1, and divide the desired amount by the remainder. The quotient will be the face of the note required.

2. For what sum must a note be given, having 4 months to run, that shall produce \$1950, if discounted at a bank at 7 per cent. ?

Ans. \$1997.78.

3. What must be the face of a note, so that when discounted for 5 months and 21 days, at 7 per cent., it will produce \$57.97, cash ?

Ans. \$60.

4. Suppose your note for 6 months is discounted at a bank at 6 per cent., and \$484.75 placed to your credit, what must have been the face of the note ?

Ans. \$500.

5. A merchant bought a quantity of goods for \$600. For what sum must he write his note, to be discounted at a bank for 6 months, at 6 per cent. ?

Ans. \$618.88.

6. A farmer bought a farm for \$5000 cash, and having only one-half of the sum on hand, he wishes to obtain the balance from the bank. For what sum must he give his note, to be discounted for 9 months, at 6 per cent. ?

Ans. \$2619.17.

7. If a merchant wishes to obtain \$550 of a bank, for what sum must he give his note, payable in 60 days, allowing it to be discounted at $\frac{1}{2}$ per cent. per month ?

Ans. \$555.75.

From the many dealings business men have, in regard to discount and interest, it is frequently required to know what rate of interest corresponds to a given rate of bank discount.

E X A M P L E .

1. What rate of interest is paid when a note, payable in 362 days, is discounted at 10 per cent. ?

SOLUTION.

If we discount \$1 for the given time, and at the given rate, the proceeds will be .90, or 90 cents. Hence, the discount being 10 cents, we are paying 10 cents for the use of 90 cents. Now, if we pay 10 cents for the use of 90, for the use of 1 cent we must pay $\frac{1}{90}$ of 10 cents, or $\frac{1}{9}$ of a cent, and for \$1, or 100 cents, we must pay 100 times $\frac{1}{9}$ of a cent, or $\frac{100}{9} = 11\frac{1}{9}$, and for \$100, \$11 $\frac{1}{9}$, or 11 $\frac{1}{9}$ per cent. Therefore, to find the rate of interest corresponding to a given rate of bank discount, we deduce the following

RULE.

Divide the given rate per cent., expressed decimally, or the rate per unit, by the number denoting the proceeds of \$1 for the given time and rate. The quotient will be the rate of interest required.

EXERCISES.

2. What rate of interest is paid when a note, payable in 60 days, is discounted at 7 per cent. ?

Ans. $7\frac{21}{247}$.

3. What rate of interest is paid when a note, payable in 3 months, is discounted at 6 per cent. ?

Ans. $6\frac{186}{1969}$.

4. A note, payable in 6 months, is discounted at 1 per cent. a month; what rate of interest is paid ?

Ans. $12\frac{244}{313}$.

5. What rate of interest is paid, when a note of \$200, payable in 70 days, is discounted at $\frac{3}{4}$ per cent. a month ?

Ans. $9\frac{81}{491}$.

6. When a note of \$45, payable in 65 days, is discounted at 7 per cent., to what rate of interest does the bank discount correspond ?

Ans. $7\frac{833}{9006}$.

7. A bank, by discounting a note at 6 per cent., receives for its money a discount equivalent to 6 $\frac{1}{2}$ per cent. interest; how long must the note have been discounted before it was due ?

Ans. 1 yr., 3 mos., 12d.

COMMISSION.

COMMISSION is the term applied to money paid to an agent to remunerate him for his trouble in buying, selling, valuing, or for forwarding merchandise or other property.

The goods sent to a commission merchant or agent, to be sold on account and risk of another, are termed a *consignment*.

The person to whom these goods are consigned is called the *consignee* or *correspondent*.

The term shipment is sometimes used instead of consignment.

EXAMPLE.

A commission merchant sells for me goods worth \$1200, and charges 4 per cent.; what have I to pay him?

SOLUTION.

4 per cent. of \$1200 is equal to $\$1200 \times .04 = \$48$. Hence I would have to pay \$48, and from this we deduce the following

RULE.

Find the percentage on the given sum at the given rate, which will be the commission.

EXERCISES.

1. Consigned to A. K. Boomer, Esq., Toronto, by the Montreal foundry, agricultural implements which are sold for \$1875.75; what is the agent's commission at $2\frac{1}{2}$ per cent.? Ans. \$46.89.

2. Bought in Montreal 12 chests of tea, containing on an average 64 lbs. each, the price was $\$1.12\frac{1}{2}$ per lb., and my employer allowed me $1\frac{3}{4}$ per cent.; how much did I make? Ans. \$15.12.

3. My London correspondent has bought for me 2768 lbs. of bacon, at $12\frac{1}{2}$ cts. a pound; what is his commission at $3\frac{1}{4}$ per cent.? Ans. \$11.25.

4. Bought a carriage and pair of horses for W. K., Esq., Lachine Rapids; paid for the horses \$240, and charged $4\frac{1}{2}$ per cent., and paid for the carriage \$160, and charged $1\frac{1}{2}$ per cent.; how much did I earn? Ans. \$13.20.

5. A commission agent in a Southern State bought cotton worth \$2284 for an English manufacturer, and charged $5\frac{1}{2}$ per cent.; what is his commission? Ans. \$125.62.

6. On another occasion the manufacturer gave the commission merchant \$165.78, for purchasing for him cotton worth \$3684; what was the rate per cent.? Ans. $4\frac{1}{2}$.

7. An English commission merchant buys for a Quebec house, £576 10s. 0d. worth of provisions, and charges $4\frac{1}{2}$ per cent.; what is his commission? Ans. £25 18s. $10\frac{1}{5}$ d..

8. A Toronto provision merchant instructs a Belfast (Ireland) commission merchant to purchase for him £534 4s. 0d. worth of

3. Remitted to my correspondent to Ottawa \$266.76, to pay for lumber which he purchased for me, and to pay his own commission at 4 per cent. ; what was the price of the lumber, and what the commission ? Ans. \$256.50, and \$10.26.

4. John Jones, Newmarket, commissions W. Orr, Port Hope, to procure for him a quantity of fine flour, and remits \$917.61 ; how much flour can he have, after allowing $4\frac{3}{4}$ per cent., and what will the commission amount to ? Ans. \$876, and \$41.61.

5. John Stalker, London, commissions J. Fleming, Toronto, to purchase for him as much butter as he can procure for the balance between \$779.52, and his own commission at $1\frac{1}{2}$ per cent. ; how many pounds butter did he get at 25 cents per lb. ; what the whole price, and what was the commission ? Ans. 3072 lbs., \$768, and \$11.52.

6. Dr. Gallipot is about to remove to England, and sends to a London cabinet maker \$4005.45 towards getting his house furnished, he is charged $3\frac{1}{2}$ per cent. over and above the price of the furniture, for time and labour ; what does the furniture cost ? Ans. \$3870.

7. Graham Bros., of Londonderry, send to R. White, Kingston, bacon and hams worth \$1560, they charge $5\frac{1}{2}$ per cent. commission, and the charge for lading is \$75.15 ; how much does R. White owe them ? Ans. \$1720.95.

8. P. Robson, commission merchant, Montreal, buys for T. Black & Co., London, C. W., groceries, the price of which, together with their commission at 4 per cent., comes to \$475.02 ; what was the price of the goods, and what was the amount of the commission ? Ans. \$456.75, and \$18.27.

BROKERAGE.

BROKERAGE is a percentage paid to an agent for negotiating bills, collecting accounts, exchanging money, buying and selling shares and stocks, and all similar transactions. Such an agent is called a Broker. A smaller percentage is usually allowed to a broker than to a commission merchant, because the work he has to do requires less time and labour. Like commission, brokerage is merely a particular case of percentage, and hence the

R U L E .

To find the brokerage on any sum, find the percentage on the given sum at the given rate, which will be the brokerage.

EXERCISES.

1. A broker in Hamilton has bought for me \$1275 worth of R. R. stock; what will be the brokerage at $2\frac{1}{8}$ per cent.? Ans. \$27.09.
2. I pay a collector of accounts 2 per cent. for collecting \$118.50; how much does it cost me? Ans. \$2.37.
3. I pay a broker $1\frac{7}{8}$ per cent. for selling \$2716.75 government stock; how much do I give him? Ans. \$50.94 nearly.
4. Advised R. P., broker, to collect two bills amounting to \$897, he has collected $\frac{2}{3}$ of it, and I have given him $1\frac{1}{2}$ per cent. on the amount collected; how much have I paid him? Ans. \$8.97.
5. A. B. sent me \$756 to purchase flour for him. I have charged $2\frac{1}{4}$ per cent. commission on the whole sum, and purchased flour with the remainder; what is my commission, and how much do I vest in flour for A. B. Ans. \$738.99, and \$17.01.
6. The school taxes on all the sections of a county amount to \$1180, and collectors get $2\frac{5}{8}$ per cent.; how much remains available for school purposes? Ans. \$1149.03.
7. Remitted to a broker in London, \$798 to buy G. W. R. shares, deducting his charge for brokerage at $\frac{7}{8}$ per cent.; how much did he invest for me? Ans. \$791.02.
8. I am charged $\frac{1}{4}$ per cent. by a broker in Chatham, for negotiating a draft for \$750; what are the proceeds coming to me? Ans. \$748.12 $\frac{1}{2}$.
9. Bought G. W. R. shares to the amount of \$578, and paid my broker $2\frac{1}{4}$ per cent.; how much did I give him? Ans. \$13.01.
10. Gave D. F. $3\frac{1}{3}$ per cent. for collecting accounts for me to the amount of \$639; how much did I give him? Ans. \$21.30.

To find the sum that can be invested when the given amount includes both the brokerage and the investment.

For example, if I wish a broker to invest for me \$700, and his charge is 2 per cent., I must obviously remit to him \$714, as \$14 is 2 per cent. on \$700; conversely if I send him \$714, and instruct him to invest for me that sum, *minus* his own percentage, he will have to calculate how much he will have remaining to invest after deducting his own charge. Now, since his percentage is \$2 on every \$100, he should get from me \$102 for every \$100 he is to invest, and therefore the sum he can invest will be the 102nd part of what I remit, *i. e.*, $\$714 \div 1.02 = \$700$. Hence the

R U L E .

Divide the given amount by one, increased by the given rate per unit of brokerage, and the quotient will be the sum to be invested; subtract this from the given amount, and the remainder will be the brokerage.

E X E R C I S E S .

1. A broker receives \$574, with instructions to invest what remains after deducting brokerage at $2\frac{1}{2}$ per cent., in R. R. shares; how much has he to invest? Ans. \$560.

2. The assessment on a certain district, together with the percentage for collection at $2\frac{1}{4}$ per cent., is \$1717.80; what is the amount of the assessment, and what the expense of collection? Ans. \$1680, and \$37.80.

3. A tax amounting to \$3276.52, including collector's fees at 4 per cent., is levied on a certain town; what is the amount of the tax, and how much is the collector entitled to? Ans. \$3150.50, and \$126.02.

4. A gentleman once invested in U. S. government bonds, a certain sum which, with the broker's fee at $1\frac{3}{4}$ per cent., amounted to \$18,315; what was the amount of the investment? Ans. \$18,000.

5. A Montreal broker negotiates a draft for \$1218 for a Hamilton merchant, at $1\frac{1}{2}$ per cent.; what are the proceeds? Ans. \$1199.73.

6. A broker, after deducting his charge at $1\frac{1}{4}$ per cent., invests the balance of \$2450.25 for his employer in bank stock; how much does he invest? Ans. \$2420.

7. My broker invests for me in oil well shares, at \$83 each, what remains after deducting his fee at $\frac{1}{2}$ per cent. from \$8341.50; how much does he invest. and how many shares does he purchase? Ans. \$8300, and 100 shares.

8. A broker's charge is \$285, at $1\frac{1}{2}$ per cent., on a certain sum invested; what is the sum?—(Sec Percentage, Case II.) Ans. \$19000.

9. A broker sells stocks for me, and the sum which is realized, together with the brokerage at 4 per cent., amounts to \$910; what is the sum procured, and what the brokerage? Ans. \$875 and \$35.

MIXED EXERCISES ON COMMISSION AND BROKERAGE.

1. My Liverpool correspondent charges me $1\frac{1}{2}$ per cent. commission on goods which he has purchased for me worth \$32,000, and $\frac{3}{8}$ per cent. for harbour dues paid by him; how much have I to remit to him? Ans. \$32,600.

2. A broker in Montreal has negotiated bills for me to the amount of \$1287.50, and charges $1\frac{5}{8}$ per cent.; how much do I owe him? Ans. \$20.92.

3. A collector receives \$20 for collecting \$900; at what per cent. is he paid? Ans. $2\frac{2}{9}$.

4. One afternoon, the train being late, and the banks closed before I reached Toronto, I wanted gold for Canadian bills to the amount of \$256; I applied to a broker, who charged me $\frac{1}{4}$ per cent.; how much did the lateness of the cars cost me? Ans. 64 cents.

5. A broker invests for me \$1750, and I pay him for his trouble \$43.75, at what rate per cent. do I pay him? Ans. $2\frac{1}{2}$.

6. An Auctioneer valued the furniture of a deceased gentleman, and charging 4 per cent., he was paid \$53.86; what was the value of the furniture? Ans. \$1346.50.

7. Attorney Screw was instructed to pay the widow Flaunt \$1500 yearly, he was to have \$48.75 for himself; what was the rate per cent. of his remuneration? Ans. $3\frac{1}{4}$.

8. A certain district pays \$800 school taxes, the collector gets \$38 for collecting; what percentage does he get? Ans. $4\frac{3}{4}$.

9. Sold by my broker at Portland, my shares in the G. T. R. for \$1780; what brokerage must I pay at $1\frac{3}{4}$ per cent.?

Ans. \$31.15.

10. An accountant is entrusted to make schedules of the debts and assets of a bankrupt; he charges only $2\frac{1}{2}$ per cent. on the debts, on the principle that he will have little trouble in getting the accounts due by the bankrupt sent in; but as he knows very well that he will have trouble in getting correct statements sent in of accounts due to the bankrupt, he stipulates for $5\frac{1}{2}$ per cent. on these; how much does he get altogether, the debts being \$2786, and the assets \$618?

Ans. \$103.64.

INSURANCE.

INSURANCE is an engagement by which one party is bound, in consideration of receiving a certain sum, to indemnify another for something in case it should in any way be lost. The party undertaking the risk is seldom, if ever, an individual, but a joint stock company, represented by an agent or agents, and doing business under the title of an "*Insurance Company*," or "*Assurance Company*," such as the "*Royal Insurance Company*," the "*Mutual Insurance Company*."

Some companies are formed on the principle that each individual shareholder is insured, and shares in the profits, and bears his portion of the losses. Such a company is usually called a *Mutual Insurance Company*.

The sum paid to the party taking the risk is called the *Premium of Insurance*, or simply the *Premium*.

The document binding the parties to the contract, is called the *Policy of Insurance*, or simply the *Policy*.

The party that undertakes to indemnify is called the *Insurer*, or *underwriter* after he has written his name at the foot of the policy.

The person or party guaranteed is called the *Insured*.

As there are many different kinds of things that may be at stake or risked, so there are different kinds of insurance which may be classified under three heads.

Fire Insurance, including all cases on land where property is exposed to the risk of being destroyed by fire, such as dwelling houses, stores and factories.

Marine Insurance.—This includes all insurances on ships and cargoes. Such an insurance may be made on the ship alone, and in that case it is sometimes called *hull insurance*, and sometimes *bottomry*, the ship's bottom representing the whole ship, just as we say fifty sail for fifty ships. The insurance may be made on the cargo alone, and is then usually called *Cargo Insurance*. It may be made on both ship and cargo, in which case the general term *Marine Insurance* will be applicable. This kind, as the name implies, insures against all accidents by sea.

Life Insurance.—This is an agreement between two parties, that in case the one insured should die within a certain stated time, the other shall, in consideration of having received a stipulated sum annually, pay to the lawful heir of the deceased, or some one men-

tioned in his will, or some other party entitled thereto, the amount recorded in the policy.

For instance, a man may, on the occasion of his marriage, insure his life for a certain sum, so that should he die within a certain time, his widow or children shall be paid that sum by the other party. Again, a father may insure the life of his child, so that in case of the child's death within a specified time, he shall be paid the sum agreed upon, or that the child, if it lives to a certain age, shall be entitled to that sum. One person may insure the life of another. Supposing that A owes B a certain sum, there is the risk that A may die before he is able to pay B; another party engages, for a certain yearly sum, to pay B in case A should fail to do so during his life time.

In some instances, insurances are effected to gain a support in case of sickness. Such a contract is called a *Health Insurance*. Insurances are now also effected for compensation in case of railway accidents. These we may call *Railway Accident Insurances*.

A policy is often transferred from one party to another, especially as collateral security for debt or some analogous obligation. If the payments agreed upon are not regularly kept up, the policy lapses, that is, becomes null and void, so that the holder of it forfeits not only his claim to the sum insured, but also the instalments previously paid. In many companies a person can insure in such a way as to be entitled to have a share of the profits.

The date at which the system of insurance began cannot be clearly ascertained; but, whatever its date, its origin seems to have been protection against the perils of the sea. We know that it was practised, in a certain way, by the ancient Greeks and Romans. If a Roman merchant sent a cargo to a distant port, he made a contract with some one engaged in such business, that he would advance a certain sum, to be repaid with interest, if the vessel reached her destination in safety, but should the vessel or cargo, or both be lost, the lender was to bear the loss. This was termed *respondentia*, (a *respondence*) a term corresponding pretty nearly to the English word *repayment*. It was lawful to charge interest in such cases, above the legal interest in ordinary cases, on account of the greatness of the risk. The lender of the money usually sent an agent of his own on board the vessel to look after the cargo, and receive the repayment on the safe delivery of the goods. This agent corresponded pretty nearly to our more modern *supercargo*. As the art of navigation advanced, and the securities afforded by law became

more stringent, and also facilities of communication increased, this system gradually gave way, and has eventually been supplanted by communications by post, and telegraphic messages to agents at the ports of destination.

With regard to the equitableness of insurances, and their utility in promoting commercial enterprise, we may remark that they make the interest of every merchant, the interest of every other. To show this, we may compare an *insurance office* to a *club*. Suppose the merchants of Canada to form a club, and establish a fund, out of which every member, if a loser, was to be indemnified, it is plain that no loss would fall on the individual, except his share as a member of the club. Even so the insurance system causes that each speculator, by insuring his own stake, contributes so much to the funds of a company, which is bound to indemnify each loser. On the other hand, the insurer or insuring company, gains in this way, that the profits accruing from cases where no loss is sustained, far exceed the cases where loss is sustained, and the trifling expense of insuring is of no moment to the insured, in comparison with the damage of a disastrous voyage, or consuming conflagration. By the insurance system, loss is virtually distributed over a large community, and therefore falls heavily on no individual, from which we draw our conclusion, that it is equivalent to a *mutual mercantile indemnification club*.

We must now show the rules of the *club*, and principles on which its calculations are made.

The principal thing to be taken into account, in all insurances, is the amount of risk. For example, a store, where nothing but iron is kept, would be considered *safe*; a factory, where fire is used, would be accounted *hazardous*, and one where inflammable substances are used would be designated *extra hazardous*, and the rates would be higher in proportion to the increased risks. As, however, the degrees of risk are so very varied, only a rough scale can be made, and hence the estimate is nothing more than a calculation of probabilities. In life insurances, the rates are regulated chiefly by the age, and general health of the individual, and also by the general health of the family relations. Connected with this is the calculation of the average length of human life.

Almost all the calculations in insurance come under two heads. FIRST, to find the premium of insurance on a given amount, and at a given rate; and, SECONDLY, to find how much must be insured at a

given rate, so that in case of loss, both the principal and premium may be recovered.

As the premium is reckoned as so much by the hundred, insurance is merely a particular case of percentage. Hence to find the premium of insurance on any given amount, at a given rate per cent., we deduce the following

R U L E .

*Multiply the given amount by the rate per unit.**

E X A M P L E S .

1. To find the cost of insuring a block of buildings valued at \$2688, at 6 per cent. ? Here we have .06 for the rate per unit, and $\$2688 \times .06 = \$161.28$, the answer.

2. What will be the cost of insuring a cargo worth \$3679, at 3 per cent. ? The rate per unit is .03, and $\$3679 \times .03 = \$110.37$, the answer.

3. A gentleman employed a broker to insure his residence and outhouses, valued at \$2760, the rate being 8 per cent., and the broker's charge $1\frac{1}{2}$ per cent. ; how much had he to pay ? The cost of insurance is $\$2760 \times .08 = \$220.80$, and the brokerage \$41.40, which added to \$220.80, will give \$262.20, the answer.

E X E R C I S E S .

What will be the premium of insurance on goods worth \$1280, at $5\frac{1}{2}$ per cent. ? Ans. \$70.40.

2. A ship and cargo, valued at \$85,000, is insured at $2\frac{1}{4}$ per cent. ; what is the premium ? Ans. \$1912.50.

3. A ship worth \$35,000, is insured at $1\frac{1}{2}$ per cent., and her cargo, worth \$55,000, at $2\frac{1}{2}$ per cent. ; what is the whole cost ? Ans. \$1900.00.

4. What will be the cost of insuring a building valued at \$58,000, at $2\frac{1}{2}$ per cent. ? Ans. \$1450.00.

* It is plain that the rate can be found, if the amount and premium are given, and the amount can be found if the rate and premium are given. In the case of insuring property, a professional surveyor is often employed to value it, and likewise in the case of life insurance, a medical certificate is required, and in each case the fee must be paid by the person insured. As 100, the basis of percentage, is a constant quantity, when any two of the other quantities are given, the third can be found.

5. What must I pay to insure a house valued at \$898.50, at $\frac{3}{4}$ per cent. ?
Ans. \$673.88.

6. A village store was valued at \$1180 ; the proprietor insured it for six years ; the rate for the first year was $3\frac{1}{4}$ per cent., with a reduction of $\frac{1}{4}$ each succeeding year ; the stock maintained an average value of \$1568, and was insured each of the six years, at $2\frac{1}{4}$ per cent. ; how much did the proprietor pay for insurance during the six years ?
Ans. \$397.53.

7. A store and yard were valued at \$1280, and insured at $1\frac{1}{8}$ per cent. ; the policy and surveyor's fee came to \$2.25 ; what was the whole cost of insuring ?
Ans. \$16.65.

8. W. Smith, Port Hope, requests R. Tomlinson, Toronto, to insure for him a building valued at \$976 ; R. Tomlinson effects the insurance at $4\frac{2}{3}$ per cent., and charges $\frac{2}{3}$ per cent commission ; how much has W. Smith to remit to R. Tomlinson, the latter having paid the premium ?
Ans. \$46.36.

9. The cost of insuring a factory, valued at \$25,000, is \$125 ; what is the rate per cent. ?
Ans. $\frac{1}{2}$.

10. A $1\frac{1}{4}$ per cent. insuring my dwelling house cost me \$50 ; what is the value of the house ?
Ans. \$4000.00.

To find how much must be insured for, so that in case of loss, both principal and premium may be recovered.

Here it is obvious that the sum insured for must exceed the value of the property in the same ratio that 100 exceeds the rate.

EXAMPLE.

To find what sum must be insured for on property worth \$600, at 4 per cent., to secure both property and premium, we have as \$100—4=\$96 : \$100 :: \$600 : F. P. $=\frac{600 \times 100}{96} = \$625$, the sum required. Taking the rate per unit we find $\frac{100-4}{100} = \frac{96}{100} = .96$. This gives the

RULE.

Divide the value of the property by 1, diminished by the rate per unit, and the quotient will be the sum required.

EXAMPLES.

1. A foundry is valued at \$874 : for what sum at 8 per cent. must it be insured to secure both the value of the property and the premium ? One *minus* the rate or $1.00 - .08 = .92$, and $\$874 \div .92 = \$950$, the answer.

The premises of a gunsmith, who sells gunpowder, are valued at \$2618.85: for how much, at 15 per cent. must they be insured in order to recover the value of the property and also the premium of insurance? Subtract .15, the rate per unit, from 1, and the remainder is .85 and $\$2618.85 \div .85$ gives \$3081, the sum required.

EXERCISES.

1. A chemist's laboratory and appurtenances are valued at \$26,250, for what sum should he insure them at $6\frac{1}{4}$ per cent., to secure both property and premium? Ans. \$28,000.

2. A Liverpool merchant sent goods worth \$1,186, by the steamer *Hibernian* to Quebec. He insured them from Liverpool to Londonderry at $1\frac{1}{4}$ per cent., and from Londonderry to Quebec at $2\frac{1}{2}$ per cent., and in both cases so as to secure the premium as well as the cargo, how much did the whole cost him? Ans. \$45.42.

3. A person owned a flour mill, valued at \$1846.05, which he insured at $1\frac{1}{4}$ per cent. He also owned a flax mill, valued at \$846.30, which he insured at $2\frac{1}{2}$ per cent., and in both cases at such a sum as to secure both property and premium. Which cost him most, and how much more?

Ans. The flour mill cost him \$1.67. more than the other.

4. Tidman & Co., Montreal, order a quantity of pork from T. S. Coates & Son, Belfast, which amounts to \$2423.10. They insure it to Liverpool at $\frac{1}{2}$ per cent., and from Liverpool to Portland at 3 per cent., and in all cases so as to secure the price and the premium both. How much does the whole insurance come to? Ans. \$87.12.

5. In order to secure both the value of goods shipped and the premium, at $1\frac{3}{4}$ per cent., an insurance is effected on \$1526.72. What is the value of the goods? Ans. \$1500.00.

6. The Mechanics' Institute is valued at \$18,000: it is insured at $1\frac{1}{4}$ per cent., so that in case of fire, the property and premium may both be recovered. For how much is it insured? Ans. \$18,227.85.

7. How much must be insured on a cargo worth \$40,000, at $\frac{1}{2}$ per cent., to secure both the value of the cargo and the cost of insurance? Ans. \$40,201.00.

8. The Rossin House, King-street, Toronto, is valued at, say, \$150,000, and is insured at $1\frac{3}{4}$ per cent, so that in case of another conflagration, both the value of the property and the premium of insurance may be recovered. For how much must it be insured?

Ans. \$152,671.76, nearly.

9. A jail and court-house, adjoining chemical works, and therefore deemed hazardous, will not be insured under $2\frac{1}{2}$ per cent. How much will secure both property and premium, the valuation being \$17,550.00?

Ans. \$18,000.00.

10. A cotton mill is insured for \$12,000, at 4 per cent., to secure both premium and property. What is the value of the property?

Ans. \$12,500.00.

11. What sum must be insured on a vessel and cargo valued at \$40,000, at $5\frac{1}{2}$ per cent., in order to secure both the premium and property?

Ans. \$42,328.04.

12. How much must be insured on property worth \$70,000, at $4\frac{1}{2}$ per cent., to secure both premium and property, a commission of $\frac{3}{4}$ per cent. having been charged?

Ans. \$73,848.17.

LIFE INSURANCE.

A LIFE INSURANCE may be effected either for a term of years or for the whole period of life. The former is called a *Temporary Insurance*, and binds the insurer to pay the amount to the legal heir or legatee or creditor, if the insured should die within the specified time. The latter is called a *Life Insurance*, because it is demandable at death, no matter how long the insured may live.

The rate per annum that the insured is to pay is reckoned from tables constructed on a calculation of the average duration of life beyond different ages. This calculation is made from statistical returns called *BILLS OF MORTALITY*, and the result is called *THE EXPECTATION OF LIFE*.

The annual premium is fixed at such a rate as would, at the end of the expectation of life, amount to the sum insured. From tables of the expectation of life other tables are constructed, showing the premium on \$100 for one year, calculated on the supposition that it is to be paid annually in advance.

LIFE INSURANCE TABLE.

Age next Birthday.	1 year.	7 years.	For Life.	Age next Birthday.	1 year.	7 years.	For Life.
15	.83	.85	1.44	38	1.19	1.28	2.75
16	.84	.86	1.47	39	1.22	1.31	2.85
17	.85	.87	1.51	40	1.24	1.36	2.95
18	.86	.88	1.54	41	1.27	1.41	3.07
19	.87	.90	1.58	42	1.31	1.47	3.19
20	.88	.91	1.62	43	1.35	1.54	3.32
21	.89	.92	1.66	44	1.40	1.62	3.45
22	.90	.93	1.70	45	1.47	1.71	3.60
23	.91	.95	1.74	46	1.54	1.80	3.75
24	.92	.96	1.79	47	1.62	1.90	3.92
25	.93	.98	1.84	48	1.71	2.02	4.09
26	.95	.99	1.89	49	1.81	2.14	4.27
27	.96	1.01	1.94	50	1.91	2.28	4.46
28	.98	1.03	2.00	51	2.03	2.42	4.67
29	.99	1.05	2.06	52	2.15	2.59	4.89
30	1.01	1.07	2.12	53	2.29	2.76	5.12
31	1.03	1.09	2.18	54	2.44	2.95	5.36
32	1.05	1.11	2.25	55	2.60	3.15	5.62
33	1.07	1.14	2.32	56	2.78	3.38	5.89
34	1.09	1.16	2.40	57	2.96	3.62	6.19
35	1.11	1.19	2.48	58	3.17	3.87	6.50
36	1.14	1.21	2.56	59	3.39	4.17	6.83
37	1.16	1.24	2.65	60	3.64	4.50	7.18

EXAMPLES.

Supposing a young man, on coming of age, wishes to effect an insurance for \$3000 *for the whole period of his life*. To find the annual premium which he must pay, we look for 21 in the left hand column, and opposite that, in the column headed FOR LIFE, we find the number 1.66, which is the premium for one year on \$100, and $\frac{1.66}{100} = .0166$ is the premium on \$1 for 1 year, and hence $\$3000 \times .0166 = \$49.80$, is the whole annual premium.

If the insurance is to last *for seven years only*, we find under that heading .92, and $\frac{.92}{100} = .0092$, and $\$3000 \times .0092 = \$27.60$, the annual premium.

If the insurance is to be *for one year only*, we find .89 under that head, and $\$3000 \times .0089 = \$26.70$, the premium.

From these explanations we can now derive a rule for finding the annual premium, when the age of the individual and the sum to be insured for are known.

R U L E .

Find the age in the left hand column of the table, and opposite this in the vertical column for the given period will be found the premium on \$100 for one year, and this divided by 100 will give the premium on \$1 for one year, and the given sum multiplied by this will be the whole annual premium.

E X E R C I S E S .

1. What will be the annual premium for insuring a person's life, who is 18 years old, for \$1000 for 7 years? Ans. \$8.80.

2. What amount of annual premium must be paid by A. B. Smith, who wishes to insure his life for 7 years for \$2000, his age being 25 years? Ans. \$19.60.

3. John Jones, 35 years of age, wishes to effect an insurance for life for \$1500. What amount of annual premium must he pay? Ans. \$37.20.

4. A gentleman in Toronto, 32 years of age, being about to start for Australia, and wishing to provide for his family in case of his death, obtains an insurance for seven years for \$3000. What amount of annual premium must he pay? Ans. \$33.30.

5. Amos Fairplay, 48 years of age, being bound on a dangerous voyage, and wishing to provide for the support of his widowed mother, in case of accident to himself, insures his life for 1 year for \$2500. What amount of premium must he pay? Ans. \$42.75.

6. A gentleman, 50 years of age, gets his life insured for \$3000, by paying an annual premium of \$4.46 on each \$100 insured; if he should die at the age of 75 years, how much less will be the amount of insurance than the payments, allowing the latter to be without interest? Ans. \$345.

7. A gentleman, 45 years of age, gets his life insured for \$5000, for which he pays an annual premium of \$180, and dies at the age of 50 years. Suppose we reckon simple interest at 7 per cent. on his payments, what is gained by the insurance? Ans. \$3911.

PROFIT AND LOSS.

IN the language of arithmetic, the expression *Profit and Loss* is usually applied to something gained or something lost in mercantile transactions, and the most important rule relating to it directs how to find at what increased rate above the cost price goods must be sold to produce a fair remuneration for time, labour and expenditure; or, in case of loss by unforeseen circumstances, to estimate the amount of that loss as a guide in future transactions.

There are other cases, however, which we shall consider in detail.

CASE I.

When the prime cost and selling price are known, to find the gain or loss.

RULE.

Find, by the rule of practice, the price at the difference between the prime cost and selling price, which will be the gain or loss according as the selling price is greater or less than the prime cost; or, Find the price at each rate, and take the difference.

EXAMPLES.

To find what is gained by selling 4 cwt. of sugar, which cost $12\frac{1}{2}$ cents per lb., at 15 cents per lb.

Here the difference between the two prices is $2\frac{1}{2}$ cents per lb., and 400 lbs., at $2\frac{1}{2}$ cents per lb., will give \$10. Also, 400 lbs. at 15 cents per lb. = \$60, and at $12\frac{1}{2}$ cents = \$50, and \$60 — \$50 = \$10.

Again, if 120 lbs. of tobacco be bought at 92 cts. per lb., and, being damaged, is sold at 75 cents per lb., the loss will be a loss of 17 cents in the pound, and 120 lbs., at 17 cents per lb., is \$20.40; or, 120 lbs., at 92 cents, will come to \$110.40, and at 75 cents, to \$90, and \$110.40 — \$90 = \$20.40.

EXERCISES.

1. If 224 lbs. of tea be bought at 60 cents per lb., and sold at 95 cents per lb.; how much is gained? Ans. \$78.40.

2. A greaser bought 24 barrels of flour, at \$5.80 per barrel, and sold 12 barrels of it at \$6.10 per barrel, 9 barrels at \$6.20 per barrel, and the rest at \$6.25; how much did he gain? Ans. \$8.55.

3. If a person is obliged to sell 216 yards of flannel, which cost him \$86.40, at $37\frac{1}{2}$ cents per yard; how much does he lose?

Ans. \$5.40.

4. If a dealer buys 78 bushels of potatoes, at $62\frac{1}{2}$ cents per bushel, and retails them at $87\frac{1}{2}$ cents per bushel; how much does he gain? Ans. \$19.50.

5. A wine merchant bought 374 gallons of wine, at \$3.20 per gallon, and sold it at \$3.35 per gallon; how much did he gain? Ans. \$56.10.

CASE II.

To find at what price any article must be sold, to gain a certain rate per cent., the cost price, and the gain or loss per cent. being known.

RULE.

Multiply the cost price by 1 plus the gain, or 1 minus the loss.

EXAMPLE.

If a quantity of linen be bought for 75 cents a yard; at what price must it be sold to gain 16 per cent.?

Since 16 per cent. is 16 cents for every dollar, each dollar in the cost price would bring \$1.16 in the selling price, so that we have $\$1.16 \times .75 = .86$, or 86 cents.

EXERCISES.

1. Railroad shares being purchased for \$2500, and sold at a gain of 20 per cent.; for what amount were they sold? Ans. \$3000.

2. A property having been bought for \$2000 was sold at a gain of 10 per cent. For what was it sold? Ans. \$2200.

3. A horse was bought for \$50, but, proving lame, was sold at a loss of 15 per cent. At what price was he sold? Ans. \$42.50.

4. Bought a horse for \$897 and sold it at a loss of 11 per cent; for what sum was it sold? Ans. \$798.33.

5. A merchant buys dry goods for \$1562, and sells them at a profit of 22 per cent. For what were they sold? Ans. \$1905.64.

CASE III.

To find the cost when the selling price and the gain per cent. are known.

RULE.

Divide the selling price by 1 plus the gain, or 1 minus the loss.

To find what was the first cost of a quantity of flour which produced 8 per cent. profit by being sold for \$127.44.

Since the gain is 8 per cent. of the cost, it follows that each dollar laid out has brought in a return of \$1.08, and therefore the cost must have been as many dollars as the number of times that 1.08 is contained in 127.44, which is 118, and therefore the first cost must have been \$118.

EXERCISES.

1. If flaxseed is sold at \$17.40 per bushel, and 13 per cent. lost, what was the first cost? Ans. \$20.00.
2. A dealer bought 116 hogs for \$580, and sold them at a gain of 25 per cent.; at what price did he sell each on an average? Ans. \$4.
3. If 13 sheep be sold for \$52.65, and 25 per cent. gained on the first cost, how much was paid for each at first? Ans. \$3.24.
4. If $16\frac{3}{4}$ per cent. be lost on the sale of linen at \$1.25, what was the first cost? Ans. \$1.50.
5. If a quantity of glass be sold for \$4, and 10 per cent. gained, for what sum was it bought? Ans. \$3.64, nearly.

CASE IV.

To find the gain or loss per cent. when the first cost and selling price are known.

RULE.

Divide the gain or loss by the first cost.

EXAMPLE.

If a web of linen be bought for \$20 and sold for \$25, what is the gain per cent?

Here \$5 are gained on \$20, and \$20 is $\frac{1}{5}$ of \$100, therefore \$25 will be gained on \$100, *i. e.*, 25 per cent.

EXERCISES.

1. If a quantity of goods be bought for \$318.50, and sold for \$299.39, how much per cent. is lost? Ans. 6 per cent.
2. If two houses are bought, the one for \$150 and the other for \$250; and the first sold again for \$100 and the latter for \$350, what per cent. is gained on the whole? Ans. $12\frac{1}{2}$.
- A grocer buys butter at 24 cents per lb. and sells it at 30 cents per lb., what does he gain per cent? Ans. 25.

4. If a cattle dealer buys 20 cows, at an average price of \$20, and pays 50 cents for the freight of each per railroad, what per cent. does he gain by selling them at \$25.62 $\frac{1}{2}$ each? Ans. 25.

5. A tobacconist bought a quantity of tobacco for \$75, but a part of it being lost, he sold the remainder for \$60: what per cent. did he lose? Ans. 20.

CASE V.

Given the gain or loss per cent. resulting from the sale of goods at *one* price, to find the gain or loss per cent. by selling the same at *another* price.

RULE.

Find by case III. the first cost, and then by case IV. the gain or loss per cent. on that cost at the second selling price.

EXAMPLE.

If a farmer sells his hogs at \$5 each, and realizes 25 per cent.; what per cent. would he realize by selling them at \$7 each.

We find by case III., that the cost was \$4, and then by case IV. what the gain per cent. would be on the second supposition, that is $\$3 \div 4 = .75$, or 75 per cent.

EXERCISES.

1. If a grocer sells rum at 90 cents per bottle, and gains 20 per cent.; what per cent. would he gain by selling it at \$1.00 per bottle? Ans. 33 $\frac{1}{3}$.

2. If a hatter sells hats at \$1.25 each, and loses 25 per cent.; what per cent. would he lose by selling them at \$1.60 each? Ans. 4.

3. If a storekeeper sells cloth at \$1.25, and loses 15 per cent.; would he gain or lose, and how much, by selling at \$1.65? Ans. He would gain 12 per cent. nearly.

4. A milliner sold bonnets at \$1.25, and thereby lost 25 per cent.; would she have gained or lost by selling them at \$1.40? Ans. She would have lost 16 per cent.

5. A merchant sold a lot of goods for \$480, and lost 20 per cent.; would he have gained or lost by selling them for \$720, and how much? Ans. He would have gained 20 per cent.

6. A quantity of grain was sold for \$90, which was 10 per cent. less than the cost; what would have been the gain per cent. if it had been sold for \$150? Ans. 50.

7. A grocer sold tea at 45 cents per pound, and thereby gained $12\frac{1}{2}$ per cent. ; what would he have gained per cent. if he had sold the tea at 54 cents per pound ?

Ans. 35.

8. A farmer sold corn at 65 cents per bushel, and gained 5 per cent. ; what per cent. would he have gained if he had sold the corn at 70 cents per bushel ?

Ans. $13\frac{1}{3}$.

MISCELLANEOUS EXERCISES.

1. If I buy goods amounting to \$465, and sell them at a gain of 15 per cent. ; what are my profits ?

2. Suppose I buy $400\frac{1}{3}$ barrels of flour, at \$16.75 a barrel, and sell it at an advance of $\frac{3}{8}$ per cent. ; how much do I gain ?

Ans. \$25.14.

3. If I buy 220 bushels of wheat, at \$1.15 per bushel, and wish to gain 15 per cent. in selling it ; what must I ask a bushel ?

4. A grocer bought molasses for 24 cents a gallon, which he sold for 30 cents ; what was his gain per cent. ?

Ans. 25.

5. A man bought a horse for \$150, and a chaise for \$250, and sold the chaise for \$350, and the horse for 100 ; what was his gain per cent. ?

Ans. $12\frac{1}{2}$.

6. A gentleman sold a horse for \$180, and thereby gained 20 per cent. ; how much did the horse cost him ?

Ans. \$150.

7. In one year the principal and interest of a certain note amounted to \$810, at 8 per cent. ; what was the face of the note ?

Ans. \$750.

8. A carpenter built a house for \$990, which was 10 per cent. less than what it was worth ; how much should he have received for it so as to have made 40 per cent. ?

Ans. \$1540.

9. A broker bought stocks at \$96 per share, and sold them at \$102 per share ; what was his gain per cent. ?

Ans. $6\frac{1}{4}$.

10. A merchant sold sugar at $6\frac{1}{2}$ cents a pound, which was 10 per cent. less than it cost him ; what was the cost price ?

Ans. $7\frac{2}{3}$ cents per pound.

11. A merchant sold broadcloth at \$4.75 per yard, and gained $12\frac{1}{2}$ per cent. ; what would he have gained per cent. if he had sold it at \$5.25 per yard ?

Ans. $24\frac{1}{3}\frac{3}{8}$.

12. I sold a horse for \$75, and by so doing, I lost 25 per cent. ; whereas, I ought to have gained 30 per cent. ; how much was he sold for under his real value ?

Ans. \$55.

13. A watch which cost me \$30 I have sold for \$35, on a credit of 8 months; what did I gain by my bargain, allowing money to be worth 6 per cent. ?

Ans. \$3.65.

14. Bought 84 yards of broadcloth, at \$5.00 per yard; what must be my asking price in order to fall 10 per cent., and still make 10 per cent. on the cost ?

Ans. \$6.11 $\frac{1}{9}$.

15. A farmer sold land at 5 cents per foot, and gained 25 per cent. more than it cost him; what would have been his gain or loss per cent. if he had sold it at 3 $\frac{1}{2}$ cents per foot ?

Ans. 12 $\frac{1}{2}$ per cent. loss.

16. What must I ask per yard for cloth that cost \$3.52, so that I may fall 8 per cent., and still make 15 per cent., allowing 12 per cent. of sales to be in bad debts ?

Ans. \$5.

17. A merchant sold two bales of cotton at \$240 each; for one he received 60 per cent. more than its cost, and for the other 60 per cent. less than its cost. Did he gain or lose by the operation, and how much ?

Ans. loss \$270.

18. Bought 2688 yards of cloth at \$2.16 per yard, and sold one-fourth of it at \$2.54 per yard; one-third of it at \$2.75 per yard, and the remainder at \$2.90 per yard. Find the whole gain, and the gain per cent.

Ans. \$1612.80 and 27 $\frac{883}{1121}$ per cent.

19. A flour merchant bought the following lots:—

118 barrels at.....	\$9.25 per barrel.
212 " 	9.50 "
315 " 	9.12 $\frac{1}{2}$ "
400 " 	10.00 "

The expenses amounted to \$29.50, besides insurance at $\frac{1}{30}$ per cent. At what price must he sell it per barrel to gain 15 per cent. ?

Ans., \$11.05.

20. Bought 100 sheep at \$5 each; having resold them at once and received a note at six months for the amount; having got the note discounted at the Royal Canadian Bank, at six per cent., I found I had gained 20 per cent. by the transaction. What was the selling price of each sheep ?

Ans., \$6.19.

STORAGE.

When a charge is made for the accommodation of having goods kept in store, it is called *storage*.

Accounts of storage contain the entries showing when the goods were received and when delivered, with the number, the description of the articles, the sum charged on each for a certain time, and the total amount charged for storage, which is generally determined by an average reckoned for some specified time, usually one month (30 days).

EXAMPLES.

1. What will be the cost of storing wheat at 3 cents per bushel per month, which was received and delivered as follows:—Received, August 3rd, 1865, 800 bushels; August 12th, 600 bushels. Delivered, August 9th, 250 bushels; September 12th, 350 bushels; September 15th, 400 bushels, and October 1st, the balance.

SOLUTION.

1865.	Bush.	Days.	Bush.
August 3. Received	800	×	6 = 4800 in store for one day.
“ 9. Delivered.....	250		
Balance.	550	×	3 = 1650 in store for one day.
“ 12. Received	600		
Balance	1150	×	31 = 35650 in store for one day.
Sept. 12. Delivered.....	350		
Balance.....	800	×	3 = 2400 in store for one day.
“ 15. Delivered.....	400		
Balance.....	400	×	16 = 6400 in store for one day.
Oct. 1. Delivered.....	400		
Total.....			50900 in store for one day.

50,900 bushels in store for *one day* would be the same as $50900 \div 30 = 1696\frac{2}{3}$ bushels in store for *one month* of 30 days, and the storage of 1697 bushels for one month, at 3 cents per month, would equal $1697 \times .03 = \$50.91$.

It is customary, in business, when the number of articles upon which storage is to be charged, as found, contains a fraction less

than a half, to reject the fraction ; but if it is *more than a half*, to regard it as an entire article.

From the solution of the foregoing example, we deduce the following

R U L E .

Multiply the number of bushels, barrels, or other articles, by the number of days they are in store, and divide the sum of the products by 30, or the number of days in any term agreed upon. The quotient will give the number of bushels, barrels, or other articles on which storage is to be charged for that term.

2. What will be the cost of storing salt at 3 cents a barrel per month, which was put in store and taken out as follows: Put in, January 2, 1866, 450 barrels; January 3, 75 barrels; January 18, 300 barrels; January 27, 200 barrels; February 2, 75 barrels. Taken out, January 10, 60 barrels; January 30, 150 barrels; February 10, 190 barrels; February 20, 300 barrels; March 1, 250 barrels; and on March 12, the balance, 150 barrels.

Ans., \$39.44.

3. Received and delivered, on account of T. C. Musgrove, sundry bales of cotton, as follows: Received January 1, 1866, 2310 bales; January 16, 120 bales; February 1, 300 bales. Delivered February 12, 1000 bales; March 1, 600 bales; April 3, 400 bales; April 10, 312 bales; May 10, 200 bales. Required the number of bales remaining in store on June 1, and the cost of storage up to that date, at the rate of 5 cents a bale per month.

Ans., 218 bales in store; \$321.18 cost of storage.

4. W. T. Leeming & Co., Commission Merchants, Montreal, in account with A. B. Smith & Co., Toronto, for storage of salt and gunpowder, which was received and delivered as follows:

Received January 18, 1866, 400 kegs of gunpowder and 50 barrels of salt; January 25, 250 barrels of salt; February 4, 150 barrels of salt and 50 kegs of gunpowder; February 15, 100 kegs of gunpowder; March 5, 64 kegs of gunpowder; April 15, 50 kegs of gunpowder and 75 barrels of salt. Delivered, February 25, 15 kegs of gunpowder, and 40 barrels of salt; March 10, 150 kegs of gunpowder and 285 barrels of salt; April 20, 200 kegs of gunpowder; April 25, 150 barrels of salt and 200 kegs of gunpowder. Required, the number of barrels of salt and kegs of gunpowder in

store May 1, and the bill of storage up to that date. The rate of storage for salt being 3 cents a barrel per month, and for gunpowder 12 cents a keg per month.

Ans. In store, 50 barrels of salt and 99 kegs of gunpowder; bill of storage, \$206.01.

GENERAL AVERAGE.

THIS is the term used to denote the contribution of all persons interested in a ship, freight, or cargo, towards the loss or damage incurred by any particular part of a ship, or cargo, for the preservation of the rest. This sacrifice of property is called *jettison*, from the goods being cast into the sea to save the vessel; although not only property destroyed in that way is the subject of general average, but also any damages or expenses voluntarily incurred for the good of all. For example, the expense of unloading the cargo that the ship may be repaired; masts or sails cut away and abandoned to save the ship.

The only articles exempt from contribution are provisions, wearing apparel of passengers, and wages of the seamen.

The owners contribute according to the clear value of the ship and freight at the end of the voyage, after deducting the wages of the crew and other expenses.

Goods that have been subject to jettison, and are lost, are valued, when the average is calculated at the place of the ship's destination, at the price they could have sold for there; but when the average is to be ascertained at the port of lading, the invoice price is the standard of value.

In making an account of the articles which are to contribute, the property lost or sacrificed must be included, and its owners must suffer the same proportionate loss as the rest. The losses to the different parties interested in the vessel, freight, and cargo, are paid by their insurers.

When repairs have to be made to a ship—new sails, masts, or rigging, for example,—one-third of the expense is deducted on account of *melioration*, or the improved condition of the ship by these repairs. When the ship is new, and on her first voyage, the full amount of the expense of repairs is allowed in computation of the loss.

EXAMPLE.

On the 26th June, 1865, the steamer *Canada* left Quebec for Liverpool, with a cargo as follows:—Shipped by T. A. Collins, \$7480; R. Evans & Co., \$5365; H. C. Wright, \$9218; W. Manning & Co., \$11,428; E. Carpenter, \$7559. In the Gulf of St. Lawrence a heavy gale was experienced, during which cargo to the value of \$3498 was thrown overboard; of this \$1123.40 belonged to R. Evans & Co., and the balance to E. Carpenter. The necessary repairs of the steamer cost \$876, and the expenses in port, while getting repaired, were \$253. The steamer was valued at \$100,000; gross freight, \$4310. The seamen's wages were \$860. What was the loss per cent., and what was the loss of each contributory interest?

SOLUTION.

*Loss for general benefit.**Contributory interests.*

Cargo thrown overboard, \$3498	Value of steamer.....\$100,000
Repairs to steamer less $\frac{1}{3}$ 584	Invoice price of cargo.... 41,050
Expenses in port..... 253	Fr'ght less seamen's wages 3,450
Total loss.....\$4335	Total contrib. int..\$144,500

$\$4335 \div 144,500 = .03$ loss per unit, or 3 per cent.

$\$100,000 \times .03 = \$3000.00$, steamer's share of loss.

$7,480 \times .03 = 224.40$, T. A. Collins' share of loss.

$5,365 \times .03 = 160.95$, R. Evans & Co.'s share of loss.

$9,218 \times .03 = 276.54$, H. C. Wright's share of loss.

$11,428 \times .03 = 342.84$, W. Manning & Co.'s share of loss.

$7,559 \times .03 = 226.77$, E. Carpenter's share of loss.

$3.450 \times .03 = 103.50$, Freight's share of loss.

\$4335.00, Total loss.

$\$3000.00 - 837.00 = \$2163.00$, balance payable by steamer.

$1123.40 - 160.95 = 962.45$, balance receivable by R. Evans & Co.

$2374.60 - 226.77 = 2147.83$, balance receivable by E. Carpenter.

NOTE.—It is evident that since the steamer lost \$837, (\$584 by repairs, and \$253 by expenses,)—that the net amount required from the steamer will be $\$3000 - 837 = \$2163$. R. Evans & Co. having lost by merchandise being thrown overboard \$1123.46, a sum greater than their share of the general loss, so that there must be due them $\$1123.40 - 160.95 = \$962.45$; so also the amount of E. Carpenter's share of the general loss must be deducted from his individual loss in order to find the balance due him.

R U L E .

Find the rate per unit of loss, by which multiply the value of each contributory interest, and the product will be the share of loss to be sustained by each.

E X E R C I S E S .

1. The steamship *Nova Scotian*, on her trip from Quebec to Liverpool, was crippled in a storm, in consequence of which the captain had to throw overboard a portion of the cargo, amounting in value to \$4465.50, and the necessary repairs of the vessel cost \$423. The contributory interests were as follows :—Vessel, \$30,000 ; gross freight, \$6225 ; cargo shipped by J. Jones & Co., \$3650 ; by Henry Anderson, \$6500 ; by George Millan, \$2000 ; by J. Foster & Son, \$550 ; by Brown Brothers, \$5450 ; and by Wilson & Carter, \$8500. Of the cargo thrown overboard, there belonged to Henry Anderson the value of \$3000, and to Brown Brothers the remainder, \$1465.50. The cost of detention in port in consequence of repairs, was \$116.50 ; seamen's wages, \$2075. How ought the loss to be shared among the contributory interests ?

Ans. 8 per cent.

2. The steamer *Spartan* left Toronto for Montreal, June 30th, loaded with 7210 bushels of spring wheat, shipped by J. M. Musgrove, and invoiced at 95 cents per bushel ; 4815 bushels of corn, shipped by Thomas A. Bryce & Co., and invoiced at 60 cents per bushel ; 2180 barrels of flour, shipped by A. B. Smith & Co., and invoiced at \$5.50 per barrel. When near Montreal, the steamer ran upon a rock, and the captain found it necessary to throw overboard 1600 bushels of wheat, 1280 bushels of corn, and 720 barrels of flour. On estimating the proportionate loss, it was allowed that the wheat would have sold in Montreal at an advance of 10 per cent., the corn at an advance of 15 per cent., and the flour for \$5 per barrel. The contributory interests were :—Steamer, \$95,000 ; cargo, \$; gross freight, \$2361.20. The cost of repairs to steamer was \$2198.15 ; cost arising from detention during repairs, \$318 ; seamen's wages, \$252.50. How much of the loss had each contributory interest to bear ?

Ans. $7\frac{1}{2}$ per cent.

3. The propeller *Edith* left Hamilton for Kingston with 7600 bushels of wheat, valued at \$1.25 per bushel, shipped by Dunn, Lloyd & Co., and insured in the Queen Insurance Company at $1\frac{3}{4}$ per cent. ; 9300 bushels of corn, valued at 75 cents per bushel,

shipped by J. W. Roe, and insured in the Ætna Insurance Company at $1\frac{3}{4}$ per cent.; 14,800 bushels of oats, valued at $37\frac{1}{2}$ cents per bushel, shipped by Morris, Wright & Co., and insured in the Provincial Insurance Company at $1\frac{1}{2}$ per cent.; 1,800 barrels of flour, valued at \$5.25 per barrel, shipped by Smith & Worth, and insured in the Beaver Insurance Company at $1\frac{1}{4}$ per cent. When near Kingston, a collision with the steamer *Spartan* occurred, and it was found necessary to throw overboard the flour, 4600 bushels of oats, and 3150 bushels of wheat. The propeller was valued at \$45,000, and insured in the Beaver Insurance Company for \$12,000, at 2 per cent., and in the Royal for \$25,000, at $2\frac{1}{4}$ per cent. The gross freight was \$4950; seamen's wages, \$340, and repairs to the boat, \$3953.75; what was the loss sustained by each of the contributory interests, the propeller being on her first trip?

TAXES AND CUSTOMS DUTIES.

A *tax* is a money payment levied upon the subjects of a State, or the members of any community, for the support of the government.

A tax is either levied upon the property or the persons of individuals. When levied upon the person, it is called a *poll tax*.

It may be either *direct* or *indirect*. When direct, it is levied from the individuals, or the property in the hands of the ultimate owners. When indirect, it is in the nature of a *customs* or *excise duty*, which is levied upon imports, or manufactures, before they reach the consumer, although in the end they are paid by the latter.

Customs duties are paid by the importer of goods at the port of entry, where a *custom-house* is stationed, with government employees called *custom-house officers*, to collect these dues.

Excise duties are those levied upon articles manufactured in the country.

An *invoice* is a complete list of the particulars and prices of goods sent from one place to another.

A *specific duty* is a certain sum paid on a ton, hundred weight, yard, gallon, &c., without regard to the cost of the article.

An *ad valorem* duty is a percentage levied on the actual cost, or fair market value of the goods in the country from which they are imported.

Gross weight is the weight of goods, upon which a specific duty is to be levied, before any allowances are deducted.

Net weight is the weight of the goods after all allowances are deducted.

Among the allowances made are the following:—

Breakage—an allowance on fluids contained in bottles or breakable vessels.

Draft—the allowance for waste.

Leakage—an allowance for waste by leaking.

Tare and *tret* are the deductions made for the weight of the case or barrel which contains the goods.

When goods, upon which duty is payable, are exported to Canada from any foreign country, through the United States, under bonds, they are only valued for duty as if they were imported direct into Canada from the place of export. An Order in Council has extended this provision to *free* as well as dutiable goods.

As customs duties are payable in gold, or its equivalent, upon goods imported from the United States, it is evident, that an injustice would be done if the amount mentioned in the American invoice were taken to be the dutiable value of the goods. Yet, as the value of American currency, when compared with the gold standard, varies almost every day, it would be a source of great confusion and irregularity to allow each custom-house officer to affix his own value to the currency. To avoid this, notices are published every week in the *Canada Gazette* as to the rate of discount to be allowed on American invoices, which is fixed in accordance with the price of gold, as represented by exchange. This method is at once fair and productive of uniformity.

EXAMPLES.

To find the specific duty on any quantity of goods.

Suppose a Montreal Provision Merchant imports from Ireland 59 casks of butter, each weighing 68 lbs., and that 12 lbs. tare is allowed on each cask, and 2 cents per lb. duty on the net weight.

We find the gross is..... $59 \times 68 = 4012$ lbs.

“ tare is $59 \times 12 = 708$ lbs.

Hence the net weight is.....3304 lbs.

The duty is 2 cents per lb..... 2

The duty, therefore, is.....\$66.08

To find the *ad valorem* duty on any quantity of goods.

Suppose a Hamilton dry goods merchant to import from France 436 yards of silk, at \$1.75 per yard, and that 35 per cent. duty is charged on them.

Here we have first the whole price by the rule of Practice to be \$763, then the rest of the operation is a direct case of percentage, and therefore we multiply \$763, by .35, which gives \$267.05, the amount of duty on the whole.

Hence we have the following

RULE FOR SPECIFIC DUTY.

Subtract the tare, or other allowance, and multiply the remainder by the rate of duty per box, gallon, &c.

RULE FOR AD VALOREM DUTY.

Multiply the amount of the invoice by the rate per unit.

EXERCISES.

1. Find the specific duty on 5120 lbs. of sugar, the tare being 14 per cent., and the duty $2\frac{3}{4}$ cents per lb. Ans. \$121.09.

2. What is the *ad valorem* duty on a quantity of silks, the amount of the invoice being \$95,800, and the duty $62\frac{1}{2}$ per cent? Ans. \$59,875.

3. At 30 per cent., what is the *ad valorem* duty on an importation of china worth \$1260? Ans. \$378.

4. What is the specific duty at 10 cents per lb. on 45 chests of tea, each weighing 120 lbs., the tare being 10 per cent? Ans. \$486.

5. What is the *ad valorem* duty on a shipment of fruit, invoiced at \$4560, the duty being 40 per cent? Ans. \$1824.

6. What is the specific duty on 950 bags of coffee, each weighing 200 lbs., the duty being 2 cents per lb. and the tare 2 per cent? Ans. \$3724.

7. What is the *ad valorem* duty on 20 casks of wine, each containing 75 gallons, at 18 cents a gallon? Ans. \$270.

8. A. B. shipped from Montreal 24 pipes of molasses, each containing 96 gallons; 2 per cent. was deducted for leakage, and 12 cents duty per gallon charged on the remainder; how much was the duty? Ans. \$270.95.

9. What is the ad valorem duty on a shipment of cutlery from Sheffield, England, to Montreal, the invoice amounting to \$840, and the duty charged 25 per cent. ?

Ans. \$210.

10. What is the duty on 11,900 lbs. of pepper at $6\frac{1}{2}$ cents per lb., the duty being charged at $3\frac{1}{2}$ per cent., and the tare being 5 per cent. ?

Ans. \$25.72.

11. Peter Smith & Co., London, import from Cadiz, 80 baskets of port wine, at 70 francs per basket; 42 baskets of sherry wine, at 35 francs per basket; 60 casks of champagne, containing 31 gallons each, at 4 francs per gallon. The waste of the wine in the casks was reckoned at a gallon each cask, and the allowance for breakage in the baskets was 5 per cent.; what was the duty at 30 per cent., $18\frac{3}{5}$ cents being taken as equal to 1 franc ?

Ans. \$776.54.

12. Mitchell and Graham, Glasgow, import from Quebec, per ship *Walter Scott*, 24 boxes of sugar, 400 lbs. each, at 5 cents; 40 hogsheads of molasses, containing 63 gallons each, at 30 cents per gallon; 260 boxes of oranges, at \$2 per box, and 410 boxes of cigars, at \$7 a box; the tare on the sugar was 10 per cent., and the leakage on the molasses 2 per cent.; the duty on the sugar and molasses was 24 per cent.; on the oranges 8 per cent., and on the cigars 30 per cent.; what was the whole duty ?

Ans. \$1184.09.

13. What duty would a merchant in Toronto have to pay on merchandise purchased in New York city to the amount of \$1834.60, American currency, the rate of duty being 25 per cent., and a discount of $31\frac{1}{2}$ per cent. being allowed at the custom-house on the invoice price ?

Ans. \$314.18.

14. John McMaster & Co., of Collingwood, bought of A. Smith, of Buffalo, N. Y., goods invoiced at \$5430.50, which should have passed through the custom-house during the first week in May, when the discount on American invoices was $43\frac{1}{4}$ per cent., but they were not passed until the fourth week in May, when the discount was $36\frac{3}{4}$ per cent. The duty in both cases being 20 per cent.; what was the loss sustained by McMaster & Co. on account of their goods being delayed ?

Ans. \$70.60.

In the above case, at what per cent. higher would McMaster & Co. require to mark their goods, above cost, in order to make a clear gain of 25 per cent. ?

STOCKS AND BONDS.

The capital of a company is called its stock, and is usually divided into portions or shares, which are subscribed for by those who intend becoming members, or stockholders in the company. The whole sum is very seldom paid in at once on each share, but is only paid in by instalments or *calls*, according as the money is required for the purposes of the undertaking. It very often happens that the whole amount of the stock is never called in; for example, if the shares are \$100, the first call may be for \$20, then the next for \$10, and so on, as the necessities of the company demand. It is quite possible that more money may be required after the original shares have all been paid up. To raise this sum the company frequently is compelled to dispose of *preference shares*, upon which a certain rate of interest is guaranteed out of the first profits.

The interest or profits paid upon stock are called *dividends*, because they are declared by the board to be the amount that the shareholders are entitled to have divided amongst them.

When \$100 stock sells for \$100 cash, it is said to be at *par*; when it can be placed on the market for more than \$100 cash, it is at a *premium*; and when it will only sell for less than \$100 cash, it is at a *discount*.

At the meetings of shareholders the election of officers and other questions are usually decided by vote. It is often provided that the right to vote shall not be increased in exact proportion to the number of shares held by any one person. For example, all persons holding, say four shares and under, may cast a vote for each share; those holding ten shares may give seven votes; those holding twenty may give ten votes, and so on, with a provision that no shareholder shall have more than thirty votes. The vote may generally be given by a proxy, who must be a qualified stockholder.

Governments, municipal corporations, and incorporated companies, contract loans at some fixed rate of interest, and give their *bonds* to the lenders as security for the repayment. The interest is usually payable half-yearly, and the time for payment of the principal is, in most cases, specified in the bond, although in the case of the National debt of England, the time for the payment of the principal is in the option of the government. The rate of interest upon this debt is only three per cent., and is equivalent to a perpe-

tual annuity. These stocks are called the *Funds*, or *Consols*, a contraction for "consolidated annuities." As the interest on them is fixed, the value of the principal will rise and fall according to the abundance or scarcity of money, and its value, from time to time. When capital is abundant and there are few channels for profitable investment, the rate of interest will be low, and the funds will accordingly rise. On the other hand, when there is a demand for capital, and interest is high, the funds will fall. When the rate of interest is fixed, the value of the principal must vary, and when the amount of the principal is fixed, the rate of interest upon it must vary, in order to make the stock marketable at all times.

The interest upon bonds, or debentures, as they are sometimes called, is often expressed to be payable upon the presentment of *coupons* attached to the bonds originally, and cut off as they are required for use.

Certificates of stock are given to shareholders by companies, to show what stock they are entitled to upon the books.

CASE I.

The premium or discount being known, to find the market value of any amount of stock.

EXAMPLES.

If G. W. R. shares are at 7 per cent. *premium*, to find the value of 30 shares of \$100.

Here it is plain that each \$100 will bring \$107, and that each \$1 will bring \$1.07, and as the par value is \$3000, the advanced value will be 3000 times 1.07, which gives \$3210, the market value, and $\$3210 - \$3000 = \$210$, the gain.

Again, if the same are sold at a *discount* of 7 per cent., it is plain that each \$100 would bring only \$93, and therefore each \$1 would bring only \$0.93, and therefore as the par value is \$3000, the depreciated value will be 3000 times .93, which gives \$2790, and therefore the loss would be $\$3000 - 2790 = 210$.

From this we derive the

RULE.

Multiply the par value by 1 plus or minus the rate per unit, according as the shares are at a premium or a discount.

EXERCISES.

1. What is the market value of \$450 stock, at $8\frac{1}{2}$ per cent. discount? Ans. \$411.75.

2. What is the value of 29 shares of \$50 each, when the shares are 11 per cent. below par? Ans. \$1290.50.

3. A man purchased 60 shares of \$5 each, from an oil well company, when the shares were at a discount of 8 per cent., and sold them when they were at a premium of 10 per cent; how much did he gain? Ans. \$54.

4. A man purchased \$10,000 stock when it was at an advance of 8 per cent., and sold when it was at a discount of 8 per cent.; how much did he lose? Ans. \$1600.

5. If a man buys 15 shares of \$100 each, when the shares are at a premium of 5 per cent., and sells when they have advanced to 12 per cent., how much does he gain? Ans. \$105.

CASE II.

To find how much stock a given sum will purchase at a given premium or discount.

Let it be required to find how much stock can be purchased for \$21,600 when at a *premium* of 8 per cent.

In this case it will require \$108 to purchase \$100 stock, and therefore \$1.08 to purchase \$1 stock, and hence the amount that can be purchased for \$21600 will be represented by the number of times that \$1.08 is contained in 21600, which gives \$20000.

Again: Let it be required to find how much stock can be purchased for \$5520, when at a discount of 8 per cent. When stocks are 8 per cent. below par, \$92 will purchase \$100 stock, and therefore \$0.92 will purchase \$1, and hence the amount that can be purchased for \$5520 will be represented by the number of times that .92 is contained in 5520, which gives \$6000 stock.

Hence we derive the

RULE.

Divide the given sum by 1 plus or minus the rate per unit, according as the shares are at a premium or a discount.

EXERCISES.

6. When stocks are at a premium of 12 per cent., how much can be purchased for \$8064? Ans. \$7200.

7. When stocks are at a discount of 9 per cent., how much can be bought for \$3640 ? Ans. \$4000.

8. When G. T. R. stock is at 18 per cent. below par, how much can be bought for \$42,640. Ans. \$52000.

9. When G. W. R. stock is at a premium of 9 per cent., how much will \$4578 purchase ? Ans. \$4200.

10. When government stock is selling at $92\frac{1}{2}$, what amount of stock will \$28,675 purchase, and to what will it amount with brokerage at $\frac{1}{4}$ per cent. ? Ans. \$31077.50.

CASE III.

The premium or discount being known, to find the par value.

To find the par value of \$1,296, when stock is at a premium of 8 per cent.

At 8 per cent. premium, each \$1 brings \$1.08, hence the par value will be represented by the number of times 1.08 is contained in 1296, which gives \$1200 for the par value.

To find the par value of \$1104, when stock is at a discount of 8 per cent.

Each \$1 will bring \$0.92, and therefore the par value will be represented by the number of times that .92 is contained in 1104, which gives \$1200, the par value. Hence the

RULE.

Divide the market value by 1 plus or minus the rate per unit, according as the stocks are selling above or below par.

EXERCISES.

11. What is the par value of \$24420, when stock is 11 per cent. above par ? Ans. \$22000.

12. What is the par value of \$10800, when stocks are at a discount of 4 per cent. ? Ans. \$11250.

13. When government stocks are at 6 per cent. premium; how much will \$20246 purchase at par value ? Ans. \$19100.

14. The shares in a canal company are at 15 per cent. discount; how many shares of \$100 will \$11390 purchase ? Ans. 134.

15. The shares of a British gas company were selling in 1848, at a discount of 12 per cent.; a speculator purchased a certain number of shares for £792; the value of the shares suddenly rose to par; how many shares did he purchase, and how much did he gain ?

Ans. 9 shares; £108 gain.

CASE IV.

To find to what rate of interest a given dividend corresponds.

If a person receives a dividend of 12 per cent. on an investment made at 20 per cent. above par, the corresponding interest may be calculated thus :

As the stock was bought at 20 per cent., or .20 above par, \$1.20 of market value corresponds to \$1 of par value, and as every \$1 of par value corresponds to 12 per cent. interest, or .12, it follows that the per cent. which was invested will be represented by the number of times that 1.20 is contained in .12, which is .10 or 10 per cent. Hence the

RULE.

*Divide the rate per unit of dividend by 1 plus or minus the rate per cent. premium or discount, according as the stocks are above or below par.**

EXERCISES.

16. If a dividend of 10 per cent. be declared on stock vested at 25 per cent. advance ; what is the corresponding interest ?

Ans. 8 per cent.

17. If a dividend of 4 per cent. be declared on stock invested at 12 per cent. below par, what is the corresponding interest ?

Ans. $4\frac{6}{11}$.

18. If money invested at 24 per cent. yields a dividend of 15 per cent., what is the rate of interest ?

Ans. $12\frac{2}{3}$.

19. If railroad stock is invested at 18 per cent. above par, and a dividend of 6 per cent. be declared, what is the rate of interest ?

Ans. $5\frac{5}{9}$.

20. If bank stock be invested at 15 per cent. below par, and a dividend of 10 per cent. declared, what is the rate of interest ?

Ans. $11\frac{1}{7}$.

MISCELLANEOUS EXERCISES.

1. What must be paid for 20 shares of railway stock, at 5 per cent. premium, the shares being \$100 each ?

Ans. \$2100.

* To find at what price stock paying a given rate per cent. dividend can be purchased, so that the money invested shall produce a given rate of interest, divide the rate per unit of dividend by the rate per unit of interest.

2. What is the par value of bank stock worth \$8740, at a premium of 15 per cent. ? Ans. \$7600.

3. Railway stock was bought at $15\frac{3}{4}$ below par, for \$1895.62 $\frac{1}{2}$; how many shares were there, each share being \$150 ?

Ans. 15 shares.

4. If 6 per cent. stock yields 8 per cent. on an investment, at what per cent. discount was it bought ? Ans. 25.

5. If bank stock which pays 11 per cent. dividend, is 10 per cent. above par, what is the corresponding rate of interest on any investment ? Ans. 10.

6. When 4 per cent. stocks were at $17\frac{7}{8}$ discount, A bought \$1000; how much did he pay, and how much did he gain by selling when stock had risen to $86\frac{1}{4}$? Ans. \$821.25, and \$41.25.

7. What will \$850 bank stock cost at a discount of $9\frac{3}{8}$ per cent., $\frac{1}{8}$ per cent. being charged for brokerage ? Ans. \$771.38.

8. On the data of the last example, how much would be lost by selling out at $10\frac{1}{2}$ per cent. ? Ans. \$10.03.

9. What income should I get by laying out \$1620 in the purchase of 3 per cent. stock at 81 ? Ans. \$60.

10. What sum must be invested in the 4 per cent. stocks at 84, to yield an income of \$280 ? Ans. \$5880.

11. What rate of interest will a person receive by investing in the $4\frac{1}{2}$ per cent. stocks at 90 ? Ans. 5 per cent.

12. A person transfers his capital from the $3\frac{1}{2}$ per cent. stocks at 77, to the 4 per cent. at 89; what is the increase or decrease per cent. in his income ? Ans. Decrease 25.

13. A person sells out of the 3 per cent. stock at 96, and invests his money in railway 5 per cent. stock at par; how much per cent. is his income increased ? Ans. 60.

14. What must be the market value of $5\frac{1}{2}$ per cent. stock, so that after deducting an income tax of 2 cents on the dollar, it may produce 5 per cent. interest ? Ans. $107\frac{4}{5}$.

15. A gentleman invested \$7560 in the $3\frac{1}{2}$ per cent. stocks at $94\frac{1}{2}$, and on their rising to 95 sold out, and purchased G. T. R. 4 per cent. stock at par; what increase did he make in his annual income ? Ans. \$24.

16. How much more may a person increase his annual income by lending \$3800, at 6 per cent., than by purchasing railway 5 per cent. stock at 95 ? Ans. \$28.

17. A person sells \$4200 railway stock which pays 6 per cent. at 115, and invests one-third of the proceeds in the 3 per cent. consols at $80\frac{1}{2}$, and the balance in savings' bank stock, which pays 9 per cent. at par; what is the decrease or increase of his annual income?

Ans. Increase \$97.80.

18. A person having \$10,000 consols, sells \$5000 at $94\frac{7}{8}$, and on their rising to $98\frac{5}{8}$ he sells \$5000 more; on their again rising he buys back the whole at 96; how much does he gain? Ans. \$75.

19. The sum of \$4004 was laid out in purchasing 3 per cent. stocks at $89\frac{3}{8}$, and a whole year's dividend having been received upon it, it was sold out, the whole increase of capital being \$302.40; at what price was it sold out? Ans. $93\frac{1}{8}$.

20. Suppose a person to have been an original subscriber for 500 shares of \$50 each, in the Royal Canadian Bank, payable by instalments, as follows:— $\frac{1}{8}$ in three months, which he sold for $5\frac{1}{4}$ per cent. advance; $\frac{2}{8}$ in 6 months, which brought him $6\frac{3}{8}$ per cent. advance, and the balance in nine months, which he was compelled to sell at $8\frac{3}{4}$ per cent. discount; what did he gain by the whole transaction? Ans. \$808.33.

PARTNERSHIP.

Partnership has been defined to be the result of a contract, under which two or more persons agree to combine property, or labour, for the purpose of a common undertaking, and the acquisition of a common profit.

A *dormant*, or *sleeping* partner, is one who shares in the concern, but does not appear to the world as such.

A *nominal* partner is one who lends his name and credit to a firm, without having any real interest in the profits.

All the partners may contribute equally to the business; or the capital may be contributed by some or one, and the skill and labour by the other. Or, unequal proportions may be furnished by each.

The contract need not be in writing, but all parties to be bound must assent to it, and it is usually contained in an instrument called "*Articles of Partnership*." A dissolution can take place at any time by mutual consent.

A partnership *at will* is one to which there is no limited time affixed for its continuance, and the whole firm may be dissolved

by any of its members at a moment's notice. A document is, however, generally drawn up and signed upon a dissolution, called a *settlement*, which contains a statement of the mode of adjustment of the accounts, and the apportionment of profits or losses.

EXAMPLE.

Two persons, A. and B., enter into partnership. A. invests \$300 and B. \$400. They gain during one year \$210; what is each man's share of the profit?

SOLUTION BY PROPORTION.

A.'s stock,	\$300	
B.'s "	400	
Entire stock	\$700	: 300 :: \$210 : \$90 A.'s gain.
"	"	700 : 400 :: \$210 : 120 B.'s "

SOLUTION BY PERCENTAGE.

Since the entire amount invested is \$700, and the gain \$210, the gain on every \$1 of investment will be represented by the number of times that 700 is contained in \$210, which is .30 or 30 cents on the dollar. Now if each man's stock be multiplied by .30 it will represent his share of the gain thus :

$$\begin{array}{rcl} \$300 \times .30 & = & \$90 \text{ A.'s gain.} \\ 400 \times .30 & = & 120 \text{ B.'s "} \end{array}$$

Entire stock.....	700	210	Entire gain.
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Hence,—To find each partner's share of the profit or loss, when there is no reference to time, we have the following

RULE.

As the whole stock is to each partner's stock, so is the whole gain or loss to each partner's gain or loss; or, divide the whole gain or loss by the number denoting the entire stock, and the quotient will be the gain or loss on each dollar of stock; which multiplied by the number denoting each partner's share of the entire stock, will give his share of the entire gain or loss.

EXERCISES.

1. Three persons, A., B., and C., enter into partnership. A. advances \$500, B. \$550, and C. \$600; they gain by trade \$412.50. What is each partner's share of the profit?

Ans. A.'s \$125; B.'s \$137.50; C.'s \$150.

2. A, B, C and D purchase an oil well. A pays for 6 shares, B for 5, C for 7, and D for 8. Their net profits at the end of three months have amounted to \$7800; what sum ought each to receive?

Ans. A, \$1800; B, \$1500; C, \$2100; D, \$2400.

3. A and B purchased a lot of land for \$4500. A paid $\frac{1}{3}$ of the price, and B the remainder; they gained by the sale of it 20 per cent.; what was each man's share of the profit?

Ans. A, \$300; B, \$600.

4. A captain, mate, and 12 sailors, won a prize of \$2240, of which the captain took 14 shares, the mate 6, and the remainder was equally divided among the sailors; how much did each receive?

Ans. The captain, \$980; the mate, \$420; each sailor, \$70.

5. A and B invest equal sums in trade, and clear \$220, of which A is to have 8 shares on account of transacting the business, and B only 3 shares; what is each man's gain, and what allowance is made A for his time? Ans. Each man's gain \$60; A \$100 for his time.

6. A, B, C and D enter into partnership with a joint capital of \$4000, of which A furnishes \$1000; B \$800; C \$1300, and D the balance; at the end of nine months their net profits amount to \$1700; what is each partner's share of the gain, supposing B to receive \$100 for extra services?

Ans. A, \$400; B, \$320; C, \$520; D, \$360.

7. Six persons, A, B, C, D, E and F, enter into partnership, and gain \$7000, which is to be divided among them in the following manner:—A to have $\frac{1}{5}$; B, $\frac{1}{7}$; C, $\frac{1}{3}$ as much as A and B, and the remainder to be divided between D, E and F, in the proportion of 2, $2\frac{1}{2}$ and $3\frac{1}{2}$; how much does each partner receive?

Ans. A, \$1400; B, \$1000; C, \$800; D, \$950; E, \$1187.50; F, \$1662.50.

8. A, B and C enter into partnership with a joint stock of \$30,000, of which A furnished an unknown sum; B furnished $1\frac{1}{2}$, and C $1\frac{1}{4}$ times as much. At the end of six months their profits were 25 per cent. of the investment; what was each man's share of the gain? Ans. A's, \$2000; B's, \$3000; and C's, \$2500.

9. A, B, C and D trade in company with a joint capital of \$3000; on dividing the profits, it is found that A's share is \$120; B's, \$255; C's, \$225; and D's, \$300; what was each partner's stock?

Ans. A's, \$400; B's, \$850; C's, \$750; and D's, \$1000.

10. Three labouring men, A, B and C, join together to reap a certain field of wheat, for which they agree to take the sum of

\$19.84; A and B calculate that they can do $\frac{4}{5}$ of the work; A and C $\frac{2}{3}$; B and C $\frac{3}{5}$ of it; how much should each receive according to these estimates? Ans. A, \$8.32; B, \$7.04; and C, \$4.48.

To find each partner's share of the gain or loss, when the capital is invested for different periods.

EXAMPLE.

Two merchants, A and B, enter into partnership. A invests \$700 for 15 months, and B \$800 for 12 months; they gain \$603; what is each man's share of the profits?

SOLUTION.

$$\$700 \times 15 = \$10500$$

$$\$800 \times 12 = 9600$$

$$20100 : 10500 :: \$603 : \$315 \text{ A's gain.}$$

$$20100 : 9600 :: \$603 : \$288 \text{ B's gain.}$$

The reason for multiplying each partner's stock by the time it was in trade, is evident from the consideration that \$700 invested for 15 months would be equivalent to $\$700 \times 15$ equal to \$10500 for one month, that is \$10500 would yield, *in one month*, the same interest that \$700 would *in fifteen months*. Likewise \$800 invested for 12 months would be the same as \$9600 for one month; hence the question becomes one of the previous case, that is, their investments are the same as if they had invested respectively \$10500 and \$9600 for equal times; hence the

RULE.

Multiply each man's stock by the time he continues it in trade; then say, as the sum of the products is to each particular product, so is the whole gain or loss to each man's share of the gain or loss.

EXERCISES.

11. A, B and C are associated in trade. A furnished \$300 for 6 months; B, \$350 for 7 months, and C, \$400 for 8 months. Their profits amounted to \$1490 at the time of dissolution; what was the profit belonging to each partner?

Ans. A, \$360; B, \$490; C, \$640.

12. A, B and C contract to perform a certain piece of work ; A employs 40 men for $4\frac{1}{2}$ months ; B 45 men for $3\frac{1}{2}$ months, and C 50 men for $2\frac{1}{4}$ months. Their profits, after paying all expenses, are \$850 ; how much of this belongs to each ?

Ans. A, \$340 ; B, \$297.50 ; C, \$212.50.

13. Four men, A, B, C and D, hired a pasture for \$27.80 ; A puts in 18 sheep for 4 months ; B, 24 for 3 months ; C, 22 for 2 months ; and D, 30 for 3 months ; how much ought each to pay ?

Ans. A and B each, \$7.20 ; C, \$4.40 ; D, \$9.

14. On the first day of January A began business with a capital of \$760, and on the first of February following he took in B, who invested \$540 ; and on the first of June following they took in C, who put into the business \$800. At the end of the year they found they had gained \$872 ; how much of this was each man entitled to ?

Ans. A, \$384.93 ; B, \$250.71 ; C, \$236.36.

15. Three merchants, A, B and C, entered into partnership with a joint capital of \$5875, A investing his stock for 6 months, B his for 8 months, and C his for 10 months ; of the profits each partner took an equal share ; how much of the capital did each invest ?

Ans. A, \$2500 ; B, \$1875 ; C, \$1500.

16. Two merchants, A and B, entered into partnership for two years ; A at first furnished \$800, and at the end of one year, \$500 more ; B furnished at first \$1000, at the end of 6 months, \$500 more, and after they had been in business one year, he was compelled to withdraw \$600. At the expiration of the partnership their net profits were \$2550 ; how much must A pay B who wishes to retire from the business ?

Ans. \$2190.

17. Three persons, A, B and C, form a partnership for one year, commencing January 1st, 1865 ; A puts in \$4000 ; B, \$3000 ; and C, \$2500 ; April 1st, A withdraws \$500, and B withdraws \$600 ; June 1st, C puts in \$800 more ; September 1st, A furnishes \$700 more, and B \$400 more. At the end of the year they find they have gained \$1500 ; what is each partner's share of it ?

Ans. A, \$608.68 ; B, \$423.31 ; C, \$468.01.

18. John Adams commenced business January first, 1865, with a capital of \$10000, and after some time formed a partnership with William Hickman, who contributed to the joint stock the sum of \$2800 cash. In course of time they admitted into the firm Joseph Williams, with a stock worth \$3600. On making a settlement January first, 1866, it was found that Adams had gained \$2250 ;

Hickman, \$420 ; and Williams, \$405 ; how long had Hickman's and Williams' money been employed in the business, and what rate of interest per annum had each of the partners gained on their stock ?

Ans. Hickman's, 8 months ; Williams', 6 months. Gain, $22\frac{1}{2}$ per cent. interest.

BANKRUPTCY.

WHEN a trader in Lower Canada, and any person in Upper Canada, is unable to meet his liabilities, he may make an *assignment* of all his property to an *Official Assignee*, to be by him distributed for the benefit of the creditors generally. Official Assignees are appointed by the different Boards of Trade for that purpose.

Creditors may also, under certain circumstances, compel a dishonest debtor to part with all his property for their benefit, and place his estate in bankruptcy.

The shares of the property, which are divided among the creditors, are called *dividends*.

The property to be divided is called *assets*.

EXAMPLE.

A bankrupt owes A \$400 ; B, \$350, and C, \$600 ; his net assets amount to \$810 cash ; how much is he able to pay on the \$1, and how much will each creditor receive ?

SOLUTION:

$\$400 + \$350 + \$600 = \$1350$, total liabilities. Now, if he has \$1350 to pay, and only \$810 to pay it with, he will only be able to pay $\$810 \div 1350 = .60$ or 60 cents on the \$1. Therefore, A will receive $\$400 \times .60 = \$240$; B, $\$350 \times .60 = \$210$, and C, $\$600 \times .60 = \$360$. Hence the

RULE.

Divide the net assets by the number denoting the total amount of the debts, and the quotient will be the sum to be paid on each dollar, then multiply each man's claim by the sum paid on the dollar, and the product will be the amount he is to receive.

EXERCISES.

1. A becomes bankrupt. He owes B, \$800; C, \$500; D, \$1100, and E, \$600. The assets amount to \$1110; how much can he pay on the dollar, and how much does each creditor receive?

Ans. He can pay 37 cents on the dollar, and B receives \$296; C, \$185; D, \$407, and E, \$222.

2. A house becomes bankrupt; its liabilities are \$17940; its assets are \$8970; what is the dividend, and what is the share of the chief creditor to whom \$1282 are due?

Ans. The dividend is 50 cents on the dollar, and the principal creditor gets \$641.

3. A shipbuilder becomes bankrupt, and his liabilities are \$303000; the premises, buildings and stock are worth \$220000, and he has in cash and notes \$12875; the creditors allow him \$3000 for maintenance of his family; the costs are $3\frac{1}{2}$ per cent. of the amount available for the creditors; what is the dividend, and how much does a creditor get to whom \$1360.60 are due?

Ans. The dividend is 75 cents on the dollar, and the creditor specified gets \$1020.

4. Foster & Co. fail. They owe in Toronto, \$22000; in Montreal, \$18000; in Hamilton, \$17100; in Kingston, \$16000; in London, \$4400, and in Quebec, \$4200. Their assets are: house property, \$14000; farms, \$2200; Cash in bank, \$4400; railway stock, \$4200; sundry sums due to them, \$20135. What is the dividend, and how much goes to each city?

Ans. The dividend is 55 cents in the dollar; \$12100 are paid in Toronto; \$9900 are sent to Montreal; \$9405 to Hamilton; \$8800 to Kingston; \$2420 to London, and \$2310 to Quebec.

5. The firm of Reuben Ring & Nephews becomes bankrupt. It owes to Buchanan & Ramsay, \$1080; to Kinneburgh & McNabb, \$850; to Collier, Bros., \$1720; to David Bryce & Son, \$1580; to Sinclair & Boyd, \$970. The assets are; house and store, valued at \$848; merchandise in stock, \$420; sundry debts, \$220. What can the estate pay, and what is the share of each creditor?

Ans. The estate pays 24 cents on the dollar, and the payments are: to Buchanan & Ramsay, \$259.20; to Kinneburgh & McNabb, \$204; to Collier, Bros., \$412.80; to David Bryce & Son, \$379.20; to Sinclair & Boyd, \$232.80.

EQUATION OF PAYMENTS.

Equation of Payments is the process of finding the average or mean time at which the payment of several sums, due at different times, may all be made at one time, so that neither the debtor nor creditor shall be at any loss.

The date to be found is called the *equated time*.

The mode of finding equated time almost universally adopted is very simple, though, as we shall show in the sequel, not altogether correct. It is known as *the mercantile rule*.

Let us observe, in the first place, that the standard by which men of business reckon the advantage that accrues to them from receiving money before the time fixed for its payment, and the loss they sustain by the payment being deferred beyond the appointed time, is the interest of money for each such period. Thus, if \$50 be a year overdue, the loss is \$3, at 6 per cent.; and, if \$50 be paid a year in advance of the time agreed upon, the gain to the payee is \$3, at the same rate. In the former case, the person receiving the money charges the payer \$3 interest for the inconvenience of lying out of his money, but, in the latter case, he deducts \$3 from the debt, for the advantage of having the money in hand. If, on the 1st May, A gives B two notes, one for \$50, at a term of three months, and the other for \$80, at a term of seven months, the first will be legally due on the 1st August, and the 2nd on the 1st December; but A is not able to meet the first at August, and it is held over till the 1st November, when A finds himself in a position to pay both at once. The first is then three months over-due, and accordingly B claims interest for that time, which, at 6 per cent., is 75 cents, but as A tenders payment of the whole debt at once, and the second note will not be due for another month, A claims a deduction of one month's interest, which, at the same rate, is 40 cents, and accordingly A, in addition to the debt, pays B 35 cents.

Let us now suppose another case. A owes B \$130, as before, and he gives B two notes—one for \$50, on 1st May, at 3 months, and another, on the 6th May, for \$80, at 8 months. The first falls due on 1st August, and the other on the 6th January, but A and B agree to settle at such a time that neither shall have interest to pay, but that A shall simply have to pay the principal. Supposing that a settlement is made on 6th November, we find that the 1st note is

3 months and 6 days over due, and the interest on it for that period is 80 cents, while the second will not be due for 2 months, and the interest on it for that period is also 80 cents; consequently, the interest that A should pay, and that which B should allow being equal, they balance each other, and the principal only has to be paid.

There are, then, three methods for the payment of several debts, or a debt to be paid by instalments. The first is to pay each instalment as it becomes due. This needs no elucidation, nor is it often practised, except in the case of small debts, due by persons of contracted means.

The second is what has been illustrated above by the first example, viz., that interest is added for overdue money, and deducted for sums paid in advance of the stipulated time.

The third has been illustrated by the second example, viz., to fix on such a time that the interests on the overdue and underdue sums shall be equal, so that the debtor has only to give the principal to the creditor. If, in this last case, the time should come out as a mixed number, the fraction must be taken as another day, or thrown off, making the payment fall due a day earlier. The principle on which all such settlements are made is, that the interest of any sum paid *in advance* of a stipulated time is equivalent to the interest of the same sum *overdue* for a like time.

With these explanations we are now ready to investigate a rule for the Equation of Payments. For this purpose let us suppose a case. R. Evans owes J. Jones \$200, which he undertakes to pay by two instalments of \$100 each, with interest at six per cent. ; the first payment to be made at once, and the second at the expiration of two years. But the first payment is not made till the end of the first year, at which time R. E. tenders payment of the whole amount. For the accommodation of having the first payment deferred for one year he is to pay \$6, *i. e.*, \$106 in all, and in return for making the second payment a year before it is due, he claims a discount at the same rate, which gives \$6. He has therefore, by the mercantile rule, to pay $106 + 94 = 200$, so that the \$6 in the latter case balances the \$6 in the former. This takes one year as the equated time, and is the mode usually adopted on account of its simplicity, though not strictly accurate.

To find the equated time when there are several payments to be made at different dates.

If A owes B \$300, payable at the end of 4 months; \$500, payable at the end of 6 months, and \$400, payable at the end of $10\frac{1}{2}$ months, to find at what time the whole may be paid, so that interest shall be chargeable to neither party. The interest of \$300 for 4 months is the same as the interest of \$1 for 1200 months; the interest of \$500 for 6 months is the same as the interest of \$1 for 3000 months, and the interest of \$400 for $10\frac{1}{2}$ months is the same as the interest of \$1 for 4200 months. The sum of all these is 8400 months, and the interest of the whole is the same as the interest of \$1 for 8400 months, and if \$1 requires 8400 months to produce a certain interest, the sum of all the principals will require only the $\frac{1}{1200}$ part of 8400 months to produce the same interest, and $8400 \div 1200 = 7$, and hence the equated time is 7 months.

R U L E .

Multiply each payment by the time that must elapse before it becomes due, and divide the sum of these products by the sum of the payments.

E X A M P L E .

To find the equated time for the payment of three debts, the first for \$45, due at the end of 6 months; the second for \$70, due at the end of 11 months, and the third for \$75, due at the end of 13 months.

\$45	×	6	=	\$270
70	×	11	=	770
75	×	13	=	975
190				2015

and $2015 \div 19 = 10\frac{3}{8}$, so that the equated time will be 10 months and 18 days, the small remaining fraction being rejected.

Let us suppose that nothing is paid until the end of the 13 months, and all paid at once, then the amount to be paid will be, at 6 per cent.,

For first debt overdue 7 months, $\$45 + 1.57\frac{1}{2}$, interest for 7 months	\$46.57 $\frac{1}{2}$
For second debt overdue 2 months, $\$70 + .70$, interest for 2 months	70.70
For third debt just due, \$75, no interest.....	75.00
	\$192.27 $\frac{1}{2}$

The work may often be somewhat shortened by counting the differences of time from the date at which the first payment becomes due, the mean time between the dates when the first and last become due being alone required.

If a person owes \$1200 to be paid in four instalments, \$100 in 3 months; \$200 in 10 months; \$300 in 15 months, and \$600 in 18 months, then the excesses of time of the last three above the first are 7, 12 and 15 months, and the work will stand as below.

$$\begin{array}{r}
 \$100 \quad (\text{no time.}) \\
 200 \times 7 = 1400 \\
 300 \times 12 = 3600 \\
 600 \times 15 = 9000 \\
 \hline
 1200) \quad 14000(11\frac{2}{3}
 \end{array}$$

and $11\frac{2}{3} \times 3 = 14\frac{2}{3}$ months. This gives the

R U L E .

Multiply each debt, except the one first due, by the difference between its term and the term of the first; divide the sum of the products by the sum of the debts, the quotient with the term of the first added to it will be the equated time.

Another method, which is often convenient, may be illustrated by the example already given, as the two operations will give the same result.

Interest on \$300 for 4 months = \$ 6.00

Interest on 500 for 6 " = 15.00

Interest on 400 for $10\frac{1}{2}$ " = 21.00

Interest on 1200 for 1 month = 6) 42.00 (7 months as before.

R U L E .

Find the interest on each instalment for the given time, and divide the sum of these by the interest of the whole debt for one month, and the quotient will be the equated time.

As the sum of the instalments is equal to the debt, the result will be the same for any rate of interest.

For the first instalment, \$300, overdue 3 months, A has to

pay..... \$4 50

For the second instalment, \$500, overdue 1 month, A has

to pay..... 2 50

\$7 00

For the third instalment, \$400, not due for $3\frac{1}{2}$ months, A

has to get..... \$7 00

so that the amounts of interest exactly balance, and the paying of the whole, at the end of 7 months, is precisely equivalent to the paying of each instalment as it falls due. The only difference that could arise is, that it might be inconvenient for the creditor to lie out of the first instalment for the three months. In all other respects the settlement is *strictly equitable*, according to the *understanding* that exists among business men. In the first place, the difference between this and what is called "the accurate rule," is insignificantly small; and, in the second place, the "mercantile rule" saves much time, and time is equivalent to so much capital in mercantile transactions. Independently, however, of any other consideration, we may remark that when the mode of reckoning is *conventionally understood*, it becomes perfectly equitable, because every merchant knows the terms on which he can do business with any other, just as bank discount becomes perfectly equitable, because every man, before going to a bank for the discounting of a note, knows perfectly well on what terms he can have it.

Much warm discussion has been indulged in on this subject; but, as we consider the discussion more subtle than profitable, we shall dismiss the subject in a few words. We shall adopt the usual case, that A owes B \$200, one-half to be paid at the present time, and the remainder at the end of two years. It is perfectly obvious that, at the end of the first year, A should pay \$106, that is, the principal, *plus* the interest agreed upon. Regarding the settlement of the second instalment, if A proffers payment of the whole at once, he is clearly entitled to claim a reduction for the unexpired term. Now, the question is, what ought the reduction to be. By the mercantile rule he should pay \$94, but the true present worth of \$100, due at the end of the year, would be $94.33\frac{5}{3}$, so that he would have to pay \$106 on the instalment over due, and $94.33\frac{5}{3}$ on the one not due, making $200.33\frac{5}{3}$, whereas the object is to find at what time interest should be chargeable to neither party.

As a further illustration of the general rule, let us suppose that J. Smith owes R. Evans \$1300, of which \$700 are to be paid at the end of 3 months, \$100 at the end of 4 months, and the balance at the end of 8 months, to find the equated time.

We shall suppose that J. Smith agrees to pay R. Evans the whole amount at the time the debt was contracted; then J. Smith would

owe R. Evans \$1300, *minus* the discount for the length of time the amount was paid before it became due, viz., three months, equalling the discount on \$210 for 1 month; \$100, less the discount for 4 months, equalling the discount on \$400 for 1 month; \$500, less the discount for 8 months, equalling the discount on \$4000 for 1 month. This gives a total of $\$2100 + \$400 + \$4000 = \$6500$, for 1 month.

Now, it is evident that if J. Smith wished to pay the whole amount at such a time that there should be no loss to either party, he must retain this amount for such a length of time as it will take this amount to equal the discount on \$6500 for 1 month, which will be $\frac{1}{1300}$ of \$6500, that is, for 5 months.

To prove that 5 months must be the equated time, we have recourse to the principles laid down under the head of Interest. If a settlement is not made until the expiration of 5 months from the time the debt was contracted, then J. Smith would owe R. Evans \$700, *plus* the interest of that principal during the time it remained unpaid after becoming due, viz., two months, which would give an amount of \$707. So also, \$100, *plus* the interest for 1 month, would be \$100.50, and \$500, *minus* its discount for 3 months (the length of time paid before due), would give \$7.50, leaving \$492.50, and $\$707 + \$100.50 + \$492.50 = \$1300$.

EXERCISES.

1. T. C. Musgrove owes H. W. Field \$900, of which \$300 are due in 4 months; \$400 in 6 months, \$200 in 9 months; what is the equated time for the payment of the whole amount? Ans. 6 months.

2. E. P. Hall & Co. have in their possession 5 notes drawn by G. W. Armstrong, all dated 1st January, 1865; the first is drawn at 4 months, for \$45; the second at 8 months, for \$120; the third at 10 months, for \$75; the fourth at 11 months, for \$60; and the fifth at 15 months, for \$90; for what length of time must a single note be drawn, dated 1st May, 1865, so that it may fall due at the properly equated time? Ans. 6 months.

3. A merchant sold goods as follows, on a credit of 6 months:—May 10, a bill of \$600; June 12, a bill of \$450; September 20, a bill of \$900; at what time will the whole become due?

Ans. January 16.

4. A merchant proposed to sell goods amounting to \$4000 on 8 months' credit, but the purchaser preferred to pay $\frac{1}{2}$ in cash and $\frac{1}{4}$ in 3 months; what time should be allowed him for the payment of the remainder?

Ans. 2 years, 5 months.

5. A gentleman left his son \$1500, to be paid as follows: $\frac{1}{3}$ in 3 months, $\frac{1}{3}$ in 4 months, $\frac{1}{4}$ in 6 months, and the remainder in 8 months; at what time ought the whole to be paid at once?

Ans. 4 mos., 15 days.

6. A merchant bought goods amounting to \$6000. He agrees to pay \$500 in cash, \$600 in six months, \$1500 in 9 months, and the remainder in 10 months; at what time ought he to pay the whole in one payment?

Ans. $8\frac{3}{6}$ months.

7. There is due to a merchant \$800, one-sixth of which is to be paid in 2 months, one-third in 3 months, and the remainder in 6 months; but the debtor agrees to pay one-half in cash; how long may he retain the other half, so that neither party may sustain loss?

Ans. $8\frac{2}{3}$ months.

8. A merchant sold to W. L. Brown, Esq., goods to the amount of \$3051, on a credit of 6 months, from September 25th, 1864. October 4th Brown paid \$476; November 12th, \$375; December 5th, \$800; January 1st, 1865, \$200. When, in equity, ought the merchant to receive the balance?

Ans. Oct. 8th, 1867.

9. A having sold B goods to the amount of \$1200, left it optional with him either to take them on 8 month's credit, or to pay one-half in cash, one-fifth in two months, one-sixth in four months, and the remainder at an equated time, to correspond with the terms first named; what was the time?

Ans. 4 years, 4 mos.

10. A grocer sold 484 barrels of rosin, as follows:

February 6th, 35 barrels @ \$3.12 $\frac{1}{2}$, on 4 months' time.

March 12th, 38 barrels @ 3.00, on 4 months' time.

March 12th, 411 barrels @ 2.62 $\frac{1}{2}$, on 4 months' time.

What is the equated time for the payment of the whole?

Ans. July 8th.

11. Bought of A. B. Smith & Co. 1650 barrels of flour, at different times, and on various terms of credit, as by the following statement; what is the equated time for the payment of the whole?

May 6th, 150 barrels, at \$4.50, on 3 months' credit.

May 20th, 400 barrels, at 4.75, on 4 months' credit.

July 10th, 500 barrels, at 5.00, on 5 months' credit.

August 4th, 600 barrels, at 4.25, on 4 months' credit.

Ans. November 6th.

12. J. B. Smith & Co. bought of A. Hamilton & Son 576 barrels of rosin, as follows:

May 3rd, 62 barrels @ \$2.50, on 6 months' credit.

May 10th, 100 barrels @ 2.50, on 6 months' credit.

May 18th, 10 barrels @ 2.50, as cash.

May 26th, 50 barrels @ 2.75, on 30 days' credit.

May 26th, 345 barrels @ 2.50, on six months' credit.

May 26th, 9 barrels @ 2.00, on six months' credit.

What is the equated time for the payment of the whole?

Ans. November 2nd.

13. Purchased goods of J. R. Worthington & Co., at different times, and on various terms of credit, as by the following statement:

March 1st, 1863, a bill of \$675.25, on 3 months' credit.

July 4th, 1863, a bill of 376.18, on 4 months' credit.

September 25th, 1863, a bill of 821.75, on 2 months' credit.

October 1st, 1863, a bill of 961.25, on 8 months' credit.

January 1st, 1864, a bill of 144.50, on 3 months' credit.

February 10th, 1864, a bill of 811.30, on 6 months' credit.

March 12th, 1864, a bill of 567.70, on 5 months' credit.

April 15th, 1864, a bill of 369.80, on 4 months' credit.

What is the equated time for the payment of the whole?

Ans. March 16th, 1864.

AVERAGING ACCOUNTS.

WHEN one merchant trades with another; exchanging merchandise, or giving and receiving cash, the memorandum of the transactions is called an *Account Current*. If the goods be purchased at different dates, or for different terms of credit, and some are not due while others are overdue, the fixing on a time when all may be settled, so that no interest shall be chargeable to either party, is called *Averaging the Account*.

Since interest is the standard to which is referred the benefit of receiving money before it is due, so that in the meantime it can be used in trade, and also the damage of not getting it when due, it is fair and proper that interest should be charged on all sums overdue, and deducted from all not due. In illustration, let us suppose that A sells goods to B, March 2, on 4 months' credit, and again an equal amount on March 20, on 6 months' credit; the first will be due on July 2, and the second on September 20. Should B tender payment of the whole on June 2, he would be entitled to claim interest for

one month on the first purchase, and for three months and eighteen days on the second. But if payment be delayed till August 2, A would be entitled to one month's interest on the first purchase, and B to the interest on the second for one month and eighteen days, so that there would be in favour of B, on the whole, a balance of interest for eighteen days. Again, supposing the settlement is not made till September 20, when all is due, no interest can be either charged or claimed on the second purchase, the term of credit having just then expired; but as the first debt is two months and eighteen days overdue, A is entitled to interest on it for that period. If neither is paid till after September 20, A has a right to claim interest on each for the period it has been overdue. But this regulates only one side of the account. In order to settle the other, let us suppose that B has, in the meantime, sold goods to A, it is obvious that B's claims on A must be settled on the very same principle, and that therefore the final result must be simply the finding of the balance. It is more usual, however, in accounts current, to fix on a time such that the interest due by A shall exactly balance that due by B. To illustrate this, let us suppose a case corresponding to a ledger account :

R. EVANS.

1865.	DR.
July 21, To Merchandise on 2 months' credit...	\$200
July 25, To Cash.....	150
Aug. 24, To Merchandise on 4 months' credit...	100
Sept. 21, To Merchandise on 3 months' credit...	250
	\$700

1865.	CR.
August 1, By Cash.....	\$100
August 20, By Merchandise at 22 days.....	110
Sept'r 30, By Cash	180
Balance.....	310
	\$700

To find in this case at what time the account may be settled so that interest shall be chargeable to neither party. Equating the time, as in equation of payments, we have the following operation :

days' credit, we write $110 \times 22 = 2420$. The third is cash paid 71 days after August 1, and we write $180 \times 71 = 12780$.

Had the account been settled on September 9, the debits would have been paid 46 days before coming due, and the credit side would have gained and the debit side lost the interest for that time.

Again, we must consider how long it would take the balance, \$310, to produce the same interest that \$390 would produce in 46 days. It is obvious that whatever interest \$390 gives in 46 days will require 46 times \$390 for \$1 to produce the same interest, that is, $390 \times 46 = 17940$ days, and it will require $17940 \div 310 = 58$ days, for \$310 to produce the same interest. If the settlement is made on October 25, the latest date, then the credit has been due 46 days, and therefore bearing interest; and in order that the debit side may be increased by an equal amount, the time must be extended beyond October 25, that is, it must be counted *forward*. For the same reason, if the greater side had become due first, then the balance must be considered as due at a *previous* date, and therefore we must count *backward*.

An account may be averaged from any date, but either the first or the last will be found the most convenient. The first due is generally used.

On the principles now explained may be founded the following

R U L E .

Find the equated time when each side becomes due.

Multiply the amount of the smaller side by the number of days between the two average dates, and divide the product by the balance of the account.

The quotient thus obtained will be the time that the balance becomes due, counted from the average date of the larger side, FORWARD when the amount of that side becomes due LAST, but BACKWARD when it becomes due FIRST.

The cash value of a balance depends on the time of settlement. If the settlement be made *before* the balance is due, the interest for the unexpired time is to be *deducted*; but if the settlement is not made till *after* the balance is due, interest is to be *added* for the time it is overdue.

E X E R C I S E S .

In J. H. Marsden's Ledger, we find the following accounts, which,

on being equated, stand as follows ; at what time should the respective balances commence to draw interest :

1. *Dr.* J. S. PECKHAM. *Cr.*
May 16th, 1865.....\$724.45. | July 29th, 1865.....\$486.80.
Ans. December 15th, 1864.

2. *Dr.* NELSON BOSTFORD. *Cr.*
November 19th, 1865\$635. | December 12th, 1865.....\$950.
Ans. January 27th, 1866.

3. *Dr.* JAMES CROW & Co. *Cr.*
February 24th, 1866....\$512.25. | June 10th, 1865.....\$309.70.
Ans. March 27th, 1867.

4. *Dr.* J. H. BURRITT & Co. *Cr.*
March 17th, 1866.....\$145. | January 15th, 1866.....\$695.60.
Ans. December 30th, 1865.

5. *Dr.* M. McDONALD. *Cr.*
August 27th, 1865.....\$341. | November 7th, 1865.....\$247.

6. *Dr.* JAMES I. MUSGROVE. *Cr.*
July 20th, 1866.....\$711. | April 14th, 1866.....\$1260.
Ans. December 9th, 1865.

7. *Dr.* THOS. A. BRYCE & Co. *Cr.*
June 24th, 1864.....\$1418. | September 7th, 1865.....\$2346.

8. *Dr.* E. R. CARPENTER. *Cr.*
December 2nd, 1865...\$1040.80. | August 13th, 1865....\$1112.40.

9. Required the time when the balance of the following account becomes subject to interest, allowing the merchandise to have been on 8 months' credit ?

Dr. A. B. SMITH & Co. *Cr.*

1864.			1865.		
May 1,	To Mdse.....	\$300.00	Jan. 1,	By Cash....	\$500.00
July 7,	" "	759.96	Feb. 18,	" Mdse. ..	481.75
Sep. 11,	" "	417.20	Mar. 19,	" Cash....	750.25
Nov. 25,	" "	287.70	April 1,	" Draft...	210.00
Dec. 20,	" "	571.10	May 25,	" Cash....	100.00

Ans. August 5, 1865.

10. When will the balance of the following account fall due, the merchandise items being on 6 months' credit?

Dr. J. K. WHITE. *Cr.*

1865.			1865.		
May 1,	To Mdse.....	\$312.40	June 14,	By Cash....	\$200.00
May 23,	" "	85.70	July 30,	" Mdse....	185.90
June 12,	" Cash paid dft..	105.00	Aug. 10,	" Cash....	100.00
July 29,	" Mdse.....	243.80	Aug. 21,	" Mdse....	58.00
Aug. 4,	" "	92.10	Sept. 28,	" "	45.10
Sept. 18,	" Cash.....	50.00			

Ans. January 12, 1866.

11. When does the balance of the following account become subject to interest?

Dr. W. H. MUSGROVE. *Cr.*

1864.			1864.		
Aug. 10,	To Mdse 4 mos.	\$285.30	Oct. 13,	By Cash.....	\$400.00
Aug. 17,	" " 60 days	192.60	Oct. 26,	" "	150.00
Sept. 21,	" " 30 "	256.80	Dec. 15,	" Mdse 2 mos	345.80
Oct. 13,	" Cash p'd dft.	190.00	Dec. 30,	" " 4 "	230.40
Nov. 25,	" Mdse 6 mos.	432.20	1865.		
Nov. 30,	" " 90 days	215.25	Jan. 4,	" Cash.....	340.30
Dec. 18,	" " 2 mos.	68.90	Jan. 21,	" "	180.00
1865.					
Jan. 31,	" Cash.....	100.00			

12. In the following account, when did the balance become due, the merchandise articles being on 6 months' credit?

Dr. R. J. BRYCE in account with D. HICKS & Co. *Cr.*

1864.			1864.		
Jan. 4,	To Mdse.....	\$ 96.57	Jan. 30,	By Cash...	\$240.00
Jan. 18,	" "	57.67	April 3,	" " ...	48.88
Feb. 4,	" Cash paid draft.	80.00	May 22,	" " ...	50.00
Feb. 4,	" Mdse.....	38.96			
Feb. 9,	" Cash paid draft.	50.26			
Mar. 3,	" Mdse.....	154.46			
Mar. 24,	" "	42.30			
April 9,	" "	23.60			
May 15,	" "	28.46			
May 21,	" "	177.19			

Ans. December 22nd, 1864.

13. When, in equity, should the balance of the following account be payable?

<i>Dr.</i>		J. McDONALD & Co.		<i>Cr.</i>	
1865.			1864.		
Jan. 3,	To Cash....	\$200	Sept. 20,	By Mdse, 6 mos...	\$583.17
Jan. 31,	" "	300	Oct. 27,	" " 4 " ..	321.00
Feb. 8,	" "	75	Dec. 5,	" " 6 " ..	137.00
Feb. 21,	" "	100	1865.		
Mar. 10,	" "	350	Jan. 18,	" " 60 days..	98.75
Mar. 24,	" "	25	Feb. 26,	" " 6 mos...	53.98
Apr. 12,	" "	40	Apr. 15,	" " 4 " ..	634.00
June 1,	" "	80	June 12,	" " 2 " ..	97.23
June 20,	" "	125	Sept. 21,	" " 6 " ..	84.00
July 4,	" "	268	Dec. 29,	" " 6 " ..	132.14
Sept. 27,	" "	250			
Dec. 9,	" "	100			

Ans. October 10, 1866.

To find the true cash balance of an account current when each item draws interest.

EXAMPLE.

What is the balance of the following account on January 19th, 1866, a credit of three months being allowed on the merchandise, money being worth 6 per cent. ?

<i>Dr.</i>		MUSGROVE & WRIGHT.		<i>Cr.</i>	
1865.			1865.		
Mar. 12,	To Merchandise...	\$340.00	Apr. 20,	By Mdse ...	\$200.00
Apr. 21,	" " ...	150.00	May 4,	" Cash ...	110.00
May 6,	" Cash paid draft	165.00	June 15,	" " ...	230.00
May 27,	" Mdse.....	215.00	Aug. 10,	" Mdse ...	180.00
July 16,	" Cash.....	100.00	Sept. 23,	" Cash ...	50.00
Sept. 10,	" Mdse.....	310.00	Nov. 12,	" " ...	50.00
Oct. 19,	" "	120.00	Dec. 15,	" " ...	100.00

SOLUTION.

<i>Debits.</i>			<i>Credits.</i>		
Due.			Due.		
June 12,	$\$340 \times 221 =$	75140	July 20,	$\$200 \times 183 =$	36600
July 21,	$150 \times 182 =$	27300	May 4,	$110 \times 260 =$	28600
May 6,	$165 \times 258 =$	42570	June 15,	$230 \times 218 =$	50140
Aug. 27,	$215 \times 145 =$	31175	Nov. 10,	$180 \times 70 =$	12600
July 16,	$100 \times 187 =$	18700	Sept. 23,	$50 \times 118 =$	5900
Dec. 10,	$310 \times 40 =$	12400	Nov. 12,	$50 \times 68 =$	3400
Jan. 19,	$120 \times 0 =$	0	Dec. 15,	$100 \times 35 =$	3500
	<u>\$1400</u>	6)207285		<u>\$920</u>	6)140740
		<u>\$34.547</u>			<u>\$23.456</u>

The different items on the debit and credit sides of the account being on interest from the date on which it becomes due until the time of settlement, the total interest of all the debit items will be the same as the interest of \$207285 for one day, or the interest of \$1 for 207285 days, which is \$34.547. So also, the total interest of all the credit items will be the same as the interest of \$140740 for one day, or the interest of \$1 for 140740 days, which is \$23.456. Now, since each side of the account is to be increased by its interest, the cash balance will be represented by the number denoting the difference between the two sides of the account, after the interest is added; thus, $\$1400 + \$34.547 = \$1434.547$, amount of debit side, and $\$920 + \$23.456 = \$943.456$, amount of credit side, then $\$1434.547 - \$943.456 = \$49.109$, cash balance.

SECOND METHOD.

<i>Debits.</i>			<i>Credits.</i>		
	Days.	Int.		Days.	Int.
Int. on \$340 for	221	\$12.523	Int. on \$200 for	183	\$6.100
" 150 "	182	4.550	" 110 "	260	4.766
" 165 "	258	7.095	" 230 "	218	8.356
" 215 "	145	5.195	" 180 "	70	2.100
" 100 "	187	3.116	" 50 "	118	.983
" 310 "	40	2.066	" 50 "	68	.566
" 120 "	0		" 100 "	35	.583
		<u>\$34.545</u>			<u>\$23.454</u>
\$1400			\$920		

Now, \$34.545 debit interest—\$23.454 credit interest=\$11.09,

the balance of interest, and \$1400, amount of debit items + \$11.09 = \$1411.09, and \$1411.09 — \$920 amount of credit items = \$491.09 the cash balance, which is the same as obtained by the first solution. Hence from the foregoing we deduce the following

R U L E .

Multiply each item of debit and credit by the number of days intervening between its becoming due and the time of settlement. Then consider the sums of the products of the debit and credit items as so many dollars, and find the interest on each for one day, which will be the interest, respectively, of the debit and credit items.

Place the balance of interest on its own side of the account, and the difference then between the two sides will be the true balance ; or,

Find the interest on each item from the date on which it becomes due to the time of settlement. The difference of the sums of interests, on the debit and credit sides of the account will represent the balance of interest, which is placed on its own side of the account, and the difference then between the two sides will be the true balance.

NOTE.—If any item should not come due until after the time of settlement, the side upon which it is, should be diminished, or the opposite side increased by the interest of such item from the time of settlement until due.

E X E R C I S E S .

1. What will be the cash balance of the following account if settled on January 1, 1865, allowing interest at 8 per cent. on each item after it is due ?

Dr. R. EVANS in account with JOHN JONES. *Cr.*

1864.			1864.		
June 11,	To Mdse, 4 mos.	\$315.00	Apr. 15,	By Mdse, 3 mos.	\$350.00
June 29,	“ “ 6 “	180.00	May 10,	“ “ 4 “	120.00
July 18,	“ Cash p'd dft.	200.00	June 12,	“ “ 6 “	240.00
Aug. 25,	“ Cash.....	75.00	June 30,	“ Cash.....	100.00
Aug. 31,	“ Mdse, 2 mos.	50.00	July 15,	“ “ 	90.00
Sept. 3,	“ “ 1 “	100.00	July 27,	“ “ 	80.00
Sept. 20,	“ Cash.....	80.00	Aug. 6,	“ Mdse, as cash	100.00
Oct. 14,	“ “ 	150.00	Aug. 20,	“ Cash.....	175.00
Oct. 19,	“ Mdse, as cash	300.00	Aug. 30,	“ Mdse, 3 mos.	75.00

Ans. \$110.86.

2. A. B. Smith is in account and interest with J. K. Amos & Co., as follows :—Debtor, January 1, 1865, to merchandise, on 6 months,

Ans. \$61.36.

Ans. \$50.64.

ALLIGATION.

is a Latin word, which means *binding to*, or *binding together*.

It is usual to distinguish alligation as being of two kinds, *medial* and *alternate*.

ALLIGATION MEDIAL.

Alligation medial relates to the average value of articles compounded, when the actual quantities and rates are given.

EXAMPLE.

A miller mixes three kinds of grain: 10 bushels, at 40 cents a bushel; 15 bushels, at 50 cents a bushel; and 25 bushels, at 70 cents a bushel; it is required to find the value of the mixture.

10 bushels, at 40 cents a bushel, will be worth 400 cents.,

15 bushels, at 50 cents a bushel, will be worth 750 cents.,

25 bushels, at 70 cents a bushel, will be worth 1750 cents.,

giving a total of 50 bushels and 2900 cents, and hence the mixture is $2900 \div 50 = 58$ cents, the price of the mixture per bushel. Hence the

R U L E .

Find the value of each of the articles, and divide the sum of their values by the number denoting the sum of the articles, and the quotient will be the price of the mixture.

E X E R C I S E S .

1. A farmer mixes 20 bushels of wheat, worth \$2.00 per bushel, with 40 bushels of oats, worth 50 cents per bushel; what is the price of one bushel of the mixture? Ans. \$1.

2. A grocer mixes 10 pounds of tea, at 40 cents per pound; 20 pounds, at 45 cents per pound, and 30 pounds, at 50 cents per pound; what is a pound of this mixture worth? Ans. $46\frac{2}{3}$ cents.

3. A liquor merchant mixed together 40 gallons of wine, worth 80 cents a gallon; 25 gallons of brandy, worth 70 cents a gallon; and 15 gallons of wine, worth \$1.50 a gallon; what was a gallon of this mixture worth? Ans. 90 cents.

4. A farmer mixed together 30 bushels of wheat, worth \$1 per bushel; 72 bushels of rye, worth 60 cents per bushel; and 60 bushels of barley, worth 40 cents per bushel; what was the value of $2\frac{1}{2}$ bushels of the mixture? Ans. \$1.50.

5. A goldsmith mixes together 4 pounds of gold, of 18 carats fine; 2 pounds, of 20 carats fine; 5 pounds, of 16 carats fine; and 3 pounds, of 22 carats fine; how many carats fine is one pound of the mixture? Ans. $18\frac{3}{7}$.

A L L I G A T I O N A L T E R N A T E .

Alligation alternate is the method of finding how much of several ingredients, the quantity or value of which is known, must be combined to make a compound of a given value.

C A S E I .

Given, the value of several ingredients, to make a compound of a given value.

EXAMPLE

How much sugar that is worth 6 cents, 10 cents, and 13 cents per pound, must be mixed together, so that the mixture may be worth 12 cents per pound?

SOLUTION.

12 cents.	{	1 lb., at 6 cents, is a gain of 6 cents.	} Gain.	
		1 lb., at 10 cents, is a gain of 2 cents.		8
		1 lb., at 13 cents, is a loss of 1 cent.	} Loss.	1
		7 lbs. more, at 13 cents, is a loss of.....		7
				—
			Gain	8 Loss 8

It is evident, in forming a mixture of sugar worth 6, 10 and 13 cents per pound so as to be worth 12 cents, that the gains obtained in putting in sugar of *less* value than the average price must exactly balance the losses sustained in putting in sugar of *greater* value than the average price. Hence in our example, sugar that is worth 6 cents per pound when put in the mixture will sell for 12, thereby giving a gain of 6 cents on every pound of this sugar put in the mixture. So also sugar that is worth 10 cents per pound, when in the mixture will bring 12, so that a gain of 2 cents is obtained on every pound of this sugar used in the compound. Again, sugar that is worth 13 cents per pound, on being put into the mixture will sell for only 12 cents, consequently a loss of 1 cent is sustained on every pound of this sugar used in forming the mixture. In this manner we find that in taking *one* pound of each of the different qualities of sugar there is a gain of 8 cents, and a loss of only 1 cent. Now, our losses must equal our gains, and therefore we have yet to lose 7 cents, and as there is only one quality of sugar in the mixture by which we can lose, it is plain that we must take as much more sugar at 13 cents as will make up the loss, and that will require 7 pounds. Therefore, to form a mixture of sugar worth 6, 10 and 13 cents per pound, so as to be worth 12 cents per pound, we will require 1 pound at 6 cents, 1 pound at 10 cents, and 1 pound at the 13 cents + 7 pounds of the same, which must be taken to make the loss equal to the gain.

By making a mixture of any number of times these answers, it will be observed, that the compound will be correctly formed. Hence we can readily perceive that any number of answers may be obtained

to all exercises of this kind. From what has been said we deduce the following

R U L E .

Find how much is gained or lost by taking one of each kind of the proposed ingredients. Then take one or more of the ingredients, or such parts of them as will make the gains and losses equal.

E X E R C I S E S .

1. A grocer wishes to mix together tea worth 80 cents, \$1.20, \$1.80 and \$2.40 per pound, so as to make a mixture worth \$1.60 per pound; how many pounds of each sort must he take?

Ans. 1 lb. at 80 cents; 1 lb. at \$1.20; 2 lbs. at \$1.80, and 1 lb. at \$2.40.

2. How much corn, at 42 cents, 60 cents, 67 cents, and 78 cents per bushel, must be mixed together that the compound may be worth 64 cents per bushel?

Ans. 1 bush. at 42 cts.; 1 bush. at 60 cts.; 4 bush. at 67 cts.; and 1 bush. at 78 cts.

3. It is required to mix wine, worth 60 cents, 80 cents, and \$1.20 per gallon, with water, that the mixture may be worth 75 cts. per gallon; how much of each sort must be taken?

Ans. 1 gal. of water; 1 gal. of wine at 60 cts.; 9 gal. at 80 cts.; and 1 gal at \$1.20.

4. In what proportion must grain, valued at 50 cents, 56 cents, 62 cents, and 75 cents per bushel, be mixed together, that the compound may be 62 cents per bushel?

Give, at least, three answers, and prove the work to be correct.

5. A produce dealer mixed together corn, worth 75 cents per bushel; oats, worth 40 cents per bushel; rye, worth 65 cents per bushel, and wheat, worth \$1 per bushel, so that the mixture was worth 80 cents per bushel; what quantity of each did he take?

Give four answers, and prove the work to be correctly done in each case.

C A S E I I .

When one or more of the ingredients are limited in quantity, to find the other ingredients.

E X A M P L E .

How much barley, at 40 cents; oats, at 30 cents, and corn, at 60

cents per bushel, must be mixed with 20 bushels of rye. at 85 cents per bushel, so that the mixture may be worth 60 cents per bushel ?

SOLUTION.

Bush.	Cents.	Gain.	Loss.
1	at 40, gives.....	.20	...
1	at 30, gives.....	.30	...
1	at 60, gives.....	.00	.00
20	at 85, gives.....	...	5.00
		.50	5.00
9	at 40, gives.....	1.80	...
9	at 30, gives.....	2.70	...
		\$5.00	\$5.00

By taking 1 bushel of barley, at 40 cents, 1 bushel of oats at 30 cents, and 1 bushel of corn at 60, in connection with 20 bushels of rye at 85 cents per bushel, we observe that our gains amount to 50 cents and our losses to \$5.00. Now, to make the gains equal the losses, we have to take 9 bushels more at 40 cents, and 9 bushels more at 30 cents. This gives us for the answer 1 bushel + 9 = 10 bushels of barley, 1 bushel + 9 = 10 bushels of oats, and 1 bushel of corn. From this we deduce the

RULE.

Find how much is gained or lost, by taking one of each of the proposed ingredients, in connection with the ingredient which is limited, and if the gain and loss be not equal, take such of the proposed ingredients, or such parts of them, as will make the gain and loss equal.

EXERCISES.

6. How much gold, of 16 and 18 carats fine, must be mixed with 90 ounces, of 22 carats fine, that the compound may be 20 carats fine ?

Ans. 41 ounces of 16 carats fine, and 8 of 18 carats fine.

7. A grocer mixes teas worth \$1.20, \$1, and 60 cents per pound, with 20 pounds, at 40 cents per pound ; how much of each sort must he take to make the composition worth 80 cents per pound ?

Ans. 20 at \$1.20 ; 10 lbs. at \$1 ; 10 lbs. at 60 cents.

8. How much barley, at 50 cents per bushel, and at 60 cents per bushel, must be mixed with ten bushels of pease, worth 80 cents

per bushel, and 6 bushels of rye, worth 85 cents per bushel, to make a mixture worth 75 cents per bushel?

Ans. 3 bushels, at 50 cents; $2\frac{1}{3}$ bushels, at 60 cents.

9. How many pounds of sugar, at 8, 14, and 13 cents per pound, must be mixed with 3 pounds, worth $9\frac{1}{4}$ cents per pound; 4 pounds, worth $10\frac{1}{2}$ cents per pound; and 6 pounds, worth $13\frac{1}{2}$ cents per pound, so that the mixture may be worth $12\frac{1}{2}$ cents per pound?

Ans. 1 lb., at 8 cts.; 9 lbs., at 14 cts.; and $5\frac{1}{2}$ lbs., at 13 cts

CASE III.

To find the quantity of each ingredient, when the sum of the ingredients and the average price are given.

EXAMPLE.

A grocer has sugar worth 8, 10, 12 and 14 cents per pound, and he wishes to make a mixture of 240 pounds, worth 11 cents per pound; how much of each sort must he take?

SOLUTION.

	Gain.	Loss.
1 lb., at 8 cents, gives.....	3	.
1 lb., at 10 cents, gives.....	1	.
1 lb., at 12 cents, gives.....	.	1
1 lb., at 14 cents, gives.....	.	3
4 lbs.	4	4

$240 \text{ lbs.} \div 4 = 60 \text{ lbs.}$ of each sort.

By taking 60 lbs. of each sort we have the required quantity, and it will be observed that the gains will exactly balance the losses, consequently the work is correct. Hence the

RULE.

Find the least quantity of each ingredient by CASE I., Then divide the given amount by the sum of the ingredients already found, and multiply the quotient by the quantities found for the proportional quantities.

10. What quantity of three different kinds of raisins, worth 15 cents, 18 cents, and 25 cents per pound, must be mixed together to fill a box containing 680 lbs., and to be worth 20 cents per pound?

Ans. 200 lbs., at 15 cents; 200 lbs., at 18 cents; and 280 lbs., at 25 cents.

11. How much sugar, at 6 cents, 8 cents, 10 cents, and 12 cents per pound, must be mixed together, so as to form a compound of 200 pounds, worth 9 cents per pound? Ans. 50 lbs. of each.

12. How much water must be mixed with wine, worth 80 cents per gallon, so as to fill a vessel of 90 gallons, which may be offered at 50 cents per gallon? Ans. $56\frac{2}{3}$ gals. wine, and $33\frac{1}{3}$ gals. water.

13. A wine merchant has wines worth \$1, \$1.25, \$1.50, \$1.75, and \$2 per gallon, and he wishes to form a compound to fill a 150 gallon cask that will sell at \$1.40 per gallon; how many gallons of each sort must he take? Ans. 54 of \$1, and 24 of each of the others.

14. A grocer has sugars worth 8 cents, 10 cents, 12 cents, and 20 cents per pound; with these he wishes to fill a hogshead that would contain 200 pounds; how much of each kind must he take, so that the mixture may be worth 15 cents per pound?

Ans. $33\frac{1}{3}$ lbs. of 8, 10, and 12 cents, and 100 lbs. of 20 cents.

15. A grocer requires to mix 240 pounds of different kinds of raisins, worth 8 cents, 12 cents, 18 cents, and 24 cents per lb., so that the mixture shall be worth 10 cents per pound; how much must be taken of each kind?

Ans. 192 lbs. of 8 cents, and 16 lbs. of each of the other kinds.

MONEY; ITS NATURE AND VALUE.

MONEY is the medium through which the incomes of the different members of the community are distributed to them, and the measure by which they estimate their possessions.

The precious metals have, among almost all nations, been the standard of value from the earliest time. Except in the very rudest state of society, men have felt the necessity of having some article, of more or less intrinsic value, that can at any time be exchanged for different commodities. No other substances were so suitable for this purpose as gold and silver. They are easily divisible, portable, and among the least imperishable of all substances. The work of dividing the precious metals, and marking or coining them, is generally undertaken by the Government of each State.

Money is a commodity, and its value is determined, like that of other commodities, by demand and supply, and cost of production. When there is a large supply of money it becomes cheap; in other words, more of it is required to purchase other articles. If all the

money in circulation were doubled, prices would be doubled. The usefulness of money depends a great deal upon the rapidity of its circulation. A ten-dollar bill that changes hands ten times in a month, purchases, during that time, a hundred dollars' worth of goods. A small amount of money, kept in rapid circulation, does the same work as a far larger sum used more gradually. Therefore, whatever may be the quantity of money in a country, only that part of it will effect prices which goes into circulation, and is actually exchanged for goods.

Money hoarded, or kept in reserve by individuals, does not act upon prices. An increase in the circulating medium, conformable in duration and extent to a temporary activity in business, does not raise prices, it merely prevents the fall that would otherwise ensue from its temporary scarcity.

PAPER CURRENCY.

PAPER CURRENCY may be of two kinds—convertible and inconvertible. When it is issued to represent gold, and can at any time be exchanged for gold, it is called convertible. When it is issued by the sovereign power in a State, and is made to pass for money, by merely calling it money, and from the fact that it is received in payment of taxes, and made a legal tender, it is known as an inconvertible currency. Nothing more is needful to make a person accept anything as money, than the persuasion that it will be taken from him on the same terms by others. That alone would ensure its currency, but would not regulate its value. This evidently cannot depend, as in the case of gold and silver, upon the cost of production, for that is very trifling. It depends, then, upon the supply or the quantity in circulation. While the issue of inconvertible currency is limited to something under the amount of bullion in circulation, it will on the whole maintain a par value. But as soon as gold and silver are driven out of circulation by the flood of inconvertible currency, prices begin to rise, and get higher with every additional issue. Among other commodities the price of gold and silver articles will rise, and the coinage will rise in value as mere bullion. The paper currency will then become proportionably depreciated, as compared with the metallic currency of other countries. It would be

quite impossible for these results to follow the issue of convertible paper for which gold could at any time be obtained.

All variations in the value of the circulating medium are mischievous; they disturb existing contracts and expectations, and the liability to such disturbing influences renders every pecuniary engagement of long date entirely precarious.

A convertible paper currency is, in many respects, beneficial. It is a more convenient medium of circulation. It is clearly a gain to the issuers, who, until the notes are returned for payment, obtain the use of them as if they were a real capital, and that, without any loss to the community.

THE CURRENCY OF CANADA.

IN Canada there are two kinds of currency; the one is called the old or Halifax currency, reckoned in pounds, shillings, pence and fractions of a penny; the other is reckoned by dollars and cents as already explained under the head of Decimal Coinage. The equivalent in gold of the pound currency is 101.321 grains Troy weight of the standard of fineness prescribed by law for the gold coins of the united kingdom of Great Britain and Ireland. The only gold coins now in circulation in Britain are the sovereign, value one pound, or twenty shillings sterling; and the half sovereign, ten shillings. The dollar is one-fourth of the pound currency, and the pound sterling is equal to $\$4.86\frac{2}{3}$. In the year 1786, the congress of the United States adopted the decimal currency, the dollar being the unit, and the system was introduced into Canada in 1858. By the term *legal tender* is meant the proffer of payment of an account in the currency of any country as established by law. Copper is a legal tender in Canada to the amount of one shilling or twenty cents, and silver to the amount of ten dollars. The British sovereign of lawful weight passes current, and is a legal tender to any amount paid in that coin. There is a silver currency proper to Canada, though United States' coins are most in circulation. The gold eagle of the United States, coined before July 1, 1834, is a legal tender for $\$10.66\frac{2}{3}$ of the coin current in this province. The same coin issued after that is a legal tender for $\$10$.

EXCHANGE.

It often becomes necessary to send money from one town or country to another for various purposes, generally in payment for goods. The usual mode of making and receiving payments between distant places is by bills of exchange. A merchant in Liverpool, whom we shall call A. B., has received a consignment of flour from C. D., of Montreal; and another man, E. F., in Liverpool, has shipped a quantity of cloth, in value equal to the flour, to G. H. in Montreal. There arises, in this transaction, an indebtedness to Montreal for the flour, as well as an indebtedness from Montreal for the cloth. It is evidently unnecessary that A. B., in Liverpool, should send money to C. D. in Montreal, and that G. H., in Montreal, should send an equal sum to E. F. in Liverpool. The one debt may be applied in payment of the other, and by this plan the expense and risk attending the double transmission of the money may be saved. C. D. draws on A. B. for the amount which he owes to him; and G. H. having an equal amount *to pay* in Liverpool, buys this bill from C. D., and sends it to E. F., who, at the maturity of the bill, presents it to A. B. for payment. In this way the debt due from Montreal to Liverpool, and the debt due from Liverpool to Montreal are both paid without any coin passing from one place to the other.

An arrangement of this kind can always be made when the debts due between the different places are equal in amount. But if there is a greater sum due from one place than from the other, the debts cannot be simply written off against one another. Indeed, when a person desires to make a remittance to a foreign country, he does not make a personal search for some one who has money to receive from that country, and ask him for a bill of exchange. There are exchange brokers and bankers whose business this is. They buy bills from those who have money to receive, and sell bills to those who have money to pay. A person going to a broker to buy a bill may very likely receive one that has been bought the same day from a merchant. If the broker has not on hand any exchange that he has bought, he will often give a bill on his own foreign correspondent; and to place his correspondent in funds to meet it, he will remit to him all the exchange which he has bought and not re-sold.

When brokers find that they are asked for more bills than are offered to them, they do not absolutely refuse to give them. To enable their correspondents to meet the bills at maturity, as they have no exchange to send, they have to remit funds in gold and silver. There are the expenses of freight and insurance upon the specie, besides the occupation of a certain amount of capital involved in this; and an increased price, or premium, is charged upon the exchange to cover all.

The reverse of this happens when brokers find that more bills are offered to them than they can sell or find use for. Exchange on the foreign country then falls to a discount, and can be purchased at a lower rate by those who require to make payments.

There are other influences that disturb the exchange between different countries. Expectations of receiving large payments from a foreign country will have one effect, and the fear of having to make large payments will have the opposite effect.

AMERICAN EXCHANGE.

EXCHANGE between United States and Canada is a matter of every day importance on account of the proximity of the two countries, and the incessant intercourse between them both of a social and commercial character. Much inconvenience has been felt latterly in Canada on account of the depreciation of American currency. The immediate cause of this was the late war. The exigencies of the Northern States compelled them to issue, to enormous amounts, an inconvertible paper currency known by the name of "Greenbacks." As the value of these depended mainly on the stability of the government, and the success of the war, public confidence wavered, and in consequence the value of this issue sunk materially. Much damage was accordingly suffered by Canada, as all her commercial intercourse with the States was cramped by this depreciation of the currency, as well as by the fluctuation of the money market generally, and the doubtful issue of the struggle.

From these causes the value of gold rose gradually until it reached the enormous premium of nearly two hundred per cent., or quotation of nearly three hundred. But though gold was quoted at nearly two hundred per cent. premium, it must not be understood that

American money, or greenbacks, was at that rate per cent. discount. For example, when gold is quoted at 150, or 50 per cent. premium, greenbacks are not at a discount of 50 per cent., as many might be led to suppose, but only at $33\frac{1}{3}$. The error will be more apparent from the consideration that when gold is quoted at 100 per cent. premium, the discount would not be 100, for if it were, money would be worth absolutely nothing.

CASE I.

To find the value of \$1, American currency, when gold is at a premium.

EXAMPLE.

When gold is quoted at 140, or 40 per cent. premium, what is the value of \$1, American currency?

SOLUTION.

Since gold is at a premium of 40 per cent., it requires 140 cents of American funds to equal in value \$1, or 100 cents in gold. Hence the value of \$1, American money, will be represented by the number of times 140 is contained in 100, which is $.71\frac{3}{7}$, or $71\frac{3}{7}$ cents. Hence to find the value of \$1 of any depreciated currency reckoned in dollars and cents, we deduce the following

RULE.

Divide 100 cents by 100 plus the rate of premium on gold, and the quotient will be the value of \$1.

Subtract this from \$1, and the remainder will be the rate of discount on the given currency.

CASE II.

To find the value of any given sum of American currency when gold is at a premium.

EXAMPLE.

What is the value of \$280, American money, when gold is quoted at 140, or 40 per cent. premium?

SOLUTION.

We find by Case I. the value of \$1 to be $71\frac{3}{7}$ cents. Now it is evident that if $71\frac{3}{7}$ cents be the value of \$1, the value of \$280 will be 280 times $71\frac{3}{7}$ cents, which is \$200, or $\$280 \div 1.40 = \$200$. Hence we have the following

R U L E .

Multiply the value of \$1 by the number denoting the given amount of American money, and the product will be the gold value ; or,

Divide the given sum of American money by 100 (the number of cents in \$1,) plus the premium, and the quotient will be the value in gold.

C A S E I I I .

To find the premium on gold when American money is quoted at a certain rate per cent. discount.

E X A M P L E .

When the discount on American money is 40 per cent., what is the premium on gold ?

S O L U T I O N .

If American money is at a discount of 40 per cent., the discount on \$1 would be 40 cents, and consequently the value of \$1 would be equal to \$1.00—40 cents, equal to 60 cents. Now, if 60 cents in gold be worth \$1 in American currency, \$1 or 100 cents in gold would be worth 100 times $\frac{1}{60}$ of \$1, which is $\$1.66\frac{2}{3}$, from which if we subtract \$1, the remainder will be the premium. Therefore, if American currency be at a discount of 40 per cent., the premium on gold would be $66\frac{2}{3}$ per cent. Hence we deduce the following

R U L E .

Divide 100 cents by the number denoting the gold value of \$1, American currency, and the quotient will be the value, in American currency, of \$1 in gold, from which subtract \$1, and the remainder will be the premium.

C A S E I V .

To find the value in American currency of any given amount of gold.

E X A M P L E .

What is the value of \$200 of gold, in American currency, gold being quoted at 150 ?

S O L U T I O N .

When gold is quoted at 150, it requires 150 cents, in American currency, to equal in value \$1 in gold. Now, if \$1 in gold be worth \$1.50 in American currency, \$200 will be worth 200 times \$1.50, which is \$300. Hence the

R U L E .

Multiply the value of \$1 by the number denoting the amount of gold to be changed, and the product will be the value in American currency ; or,

To the given sum add the premium on itself at the given rate, and the result will be the value in American currency.

E X E R C I S E S .

1. If American currency is at a discount of 50 per cent., what is the value of \$450 ? Ans. \$225.

2. The quotation of gold is 140, what is the discount on American currency ? Ans. $28\frac{4}{7}$ per cent.

3. A person exchanged \$750, American money, at a discount of 35 per cent. for gold ; how much did he receive ? Ans. \$487.50.

4. Purchased a draft on New York for \$1500, at a discount of $31\frac{1}{2}$ per cent. ; what did it cost me ? Ans. \$1027.50.

5. American currency is quoted at $33\frac{1}{3}$ per cent discount ; what is the premium on gold ? Ans. 50 per cent.

6. Purchased a suit of clothes in Toronto for \$35, but on paying for the same in American funds, the tailor charged me 32 per cent. discount ; how much had I to pay him ? Ans. \$51.47.

7. What would be the difference between the quotations of gold, if greenbacks were selling at 40 and 60 per cent. discount. ?

Ans. $83\frac{1}{3}$ per cent.

8. P. Y. Smith borrowed from C. R. King, \$27 in gold, and wished to repay him in American currency, at a discount of 38 per cent. ; how much did it require ? Ans. \$43.55.

9. J. E. Peckham bought of Sidney Leonard a horse and cutter for \$315.50, American currency, but only having \$200 of this sum, he paid the balance in gold, at a premium of 65 per cent. ; how much did it require ? Ans. \$70.

10. An American drover purchased of a farmer a yoke of oxen valued at \$135 in gold, but paid him \$112 in American currency, at a discount of $27\frac{1}{2}$ per cent. ; how much gold did it require to pay the balance ? Ans. \$53.80.

11. W. H. Hounsfield & Co., purchased in New York City, merchandise amounting in value to \$4798.40, on 3 months' credit, premium on gold being $79\frac{3}{8}$ per cent. At the expiration of the three months they purchased a draft on Adams, Kimball and Moore of

New York, for the amount due, at a discount of $57\frac{3}{4}$ per cent. ; what was the gain by exchange ? Ans. \$647.75.

12. A makes an exchange of a horse for a carriage with B ; the horse being valued at \$127.50 in gold, and the carriage at \$210, American currency. Gold being at a premium of 65 per cent ; what was the difference, and by whom payable ?

Ans. B pays A 23 cents.

13. A merchant takes \$63 in American silver to a broker, and wishes to obtain for the same greenbacks which are selling at a discount of 30 per cent. The broker takes the silver at $3\frac{1}{2}$ per cent. discount ; what amount of American currency does the merchant receive ? Ans. \$86.85.

14. I purchased of B. W. Smith a farm in the township of Essa containing 100 acres, at \$15 per acre. Sold 50 acres to an American speculator, at \$23 per acre, American money ; exchanged $\frac{1}{5}$ of the remainder with Isaiah Wright, Esq., for a town lot in Brantford, allowing him for the difference of barter \$400, for which he agreed to take its equivalent in American money, at a discount of 30 per cent. ; the balance I sold to J. R. Forster, at a profit of 20 per cent., receiving in payment his note at 30 days, which I immediately sold to an American for \$900 greenbacks. The amount of American currency I then had on hand I exchanged with my broker for Canada money, which was quoted at a premium of 50 per cent. ; required the amount of profit, and the rate per cent. of gain.

15. A merchant left Toronto for New York City to purchase his stock of spring goods, taking with him to defray expenses \$95 in gold. After purchasing his ticket to the Suspension Bridge for \$2.40, he expended the balance in greenbacks, which were at a discount of $41\frac{1}{2}$ per cent. When in New York he drew from this amount \$23.85 to "square" an old account then past due. On arriving home he found that he still had in greenbacks \$16.40, which he disposed of at a discount of $43\frac{3}{4}$ per cent., receiving in payment American silver at a discount of $3\frac{1}{4}$ per cent., which he passed off at $2\frac{1}{2}$ per cent. discount for gold. What were his expenses in gold ; the actual amount in greenbacks paid for expenses, and the amount of silver received ?

Ans. Total expenses in gold, \$71.76 ; expenses in greenbacks, \$118.04 ; silver received, \$9.53.

STERLING EXCHANGE.

IN Britain money is reckoned by pounds, shillings and pence, and fractions of a penny, and is called *sterling money*, the gold sovereign, consisting of 22 parts gold and 2 alloy, being the standard, and the shilling, one-twentieth part of this, a silver coin of 37 parts silver and 3 copper, and the penny, one-twelfth part of this, a copper coin, the ingredients and size of which have frequently been altered. This is called *sterling money*, and the estimating of that currency in terms of the currency of another country, which in the case of Canada is done by dollars and cents, is called *sterling exchange*. If a house in Canada has transactions with one in Britain, it is necessary that each should be able to estimate what given sum in the currency of the one country is worth in that of the other.

The comparative value of the pound sterling, or gold sovereign and Canadian money, was formerly \$4.44 $\frac{4}{9}$, but as this was less than the intrinsic value, and besides as the commercial value, though fluctuating, was always considerably higher, it was settled by Act of Provincial Parliament that the standard value of the pound sterling, estimated in dollars and cents, should be raised to \$4.86 $\frac{2}{3}$. The increase in the standard value was therefore equal to 9 $\frac{1}{2}$ per cent. of its nominal value.

Sterling exchange is usually quoted in the form : 108, 109, 109 $\frac{1}{2}$, 110, &c., which indicates that it is at 8, 9, 9 $\frac{1}{2}$, or 10 per cent. premium. The REAL PAR VALUE of the British sovereign in Canadian money is \$4.86 $\frac{2}{3}$, the exact equivalent of the pound sterling. The COMMERCIAL VALUE is the amount in dollars and cents required to purchase one pound sterling at any given rate of the money market. If exchange is quoted at 109 $\frac{1}{2}$, this means that it will require £109 $\frac{1}{2}$ at par value to purchase £100 at commercial value. The rate of exchange between Canada and Britain is usually reckoned from the old par value. Though the commercial value fluctuates, yet 9 $\frac{1}{2}$ per cent. above par may be taken as its average, so that if we add the premium of \$4.44 $\frac{4}{9}$ to itself at 9 $\frac{1}{2}$ per cent., we shall have \$4.86 $\frac{2}{3}$, which is nearly the average of the commercial value, and is generally taken as such.

EXAMPLE.

A merchant in Canada wishes to remit £648.17.6 to Britain. The old par value is $\$4.44\frac{4}{9} = \$4\frac{40}{9} = \frac{1}{9}$ of \$40 by reducing to an improper fraction. Then if the quotation is 108, or 8 per cent. above the nominal value, we find the premium on \$40 at 8 per cent., which is \$3.20, which added to \$40 will give \$43.20, and $\$43.20 \div 9 = \$4.80$ to be remitted for every pound sterling, and therefore £648.17.6 multiplied by 4.80 or 4.8 will be the value in Canadian money. 17s. 6d = .875, and the operation is as follows

$$\begin{array}{r}
 648.875 \\
 4.8 \\
 \hline
 5191000 \\
 2595500 \\
 \hline
 \$3114.60
 \end{array}$$

RULE.

To \$40 add the premium on itself at the quoted rate, multiply the sum by the number representing the amount of sterling money, and divide the result by 9, the quotient will be the equivalent of the sterling money in dollars and cents.

NOTE.—If there be shillings, pence, &c., in the sterling money, they are to be reduced to the decimal of £1.

To find the value of Canadian money in sterling money at any given rate above par.

Let it be required to find the value of \$465 in sterling money, at 8 per cent. above its nominal value. Here we have exactly the converse of the last problem, and therefore, having found the value of £1 sterling, we divide the given sum instead of multiplying; thus, the premium on \$40, at 8 per cent., is \$3.20, which added to \$40 makes \$43.20, and $43.20 \div 9 = 4.80$, and $\$465 \div 4.80 = £96.17.6$.

RULE.

Divide the given sum by the number denoting the value of one pound sterling at the given rate above par, and if there be a decimal remaining reduce it to shillings and pence.

EXERCISES.

1. When sterling exchange is quoted at 108, what is the value of £1 ? Ans. \$4.80.

2. If £1 sterling be worth $\$4.84\frac{4}{9}$, what is the premium of exchange between London and Canada ? Ans. 9 per cent.

3. At 10 per cent. above its nominal value, what is the worth of £50 sterling, in Canadian currency ? Ans. \$244.44.

4. When sterling exchange is quoted at $9\frac{1}{4}$ per cent. premium, what is the value of \$1000 ? Ans. £205 18s. $11\frac{3}{4}$ d.

5. At 12 per cent. above its nominal value, what will a bill for £1800 cost in dollars and cents ? Ans. \$8960.

6. A merchant sold a bill of exchange on London for £7000, at an advance of 11 per cent. ; what did he receive for it more than its real value ? Ans. \$466.66 $\frac{2}{3}$.

7. Bought a bill on London for £1266 15s., at $9\frac{1}{2}$ per cent. premium ; what shall I have to pay for it ? Ans. \$6164.85.

8. A merchant sells a bill on London for £4000, at 8 per cent. above its nominal value, instead of importing specie at an expense of 2 per cent. ; what does he save ? Ans. \$122.66 $\frac{2}{3}$.

9. A merchant in Kingston paid \$7300 for a draft of £1500 on Liverpool ; at what per cent. of premium was it purchased ? Ans. $9\frac{1}{2}$.

10. A broker sold a bill of exchange for £2000, on commission, at 10 per cent. above its nominal value, receiving a commission of $\frac{1}{10}$ per cent. on the real value, and 5 per cent. on what he obtained for the bill above its real value ; what was his commission ? Ans. \$11.95 $\frac{3}{4}$.

11. I owe A. N. McDonald & Co., of Liverpool, \$7218, net proceeds of sales of merchandise effected for them, which I am to remit them in a bill of exchange on London for such amount as will close the transaction, less $\frac{1}{4}$ per cent. on the face of the bill for my commission for investing. Bills on London are at 8 per cent. premium. Required the amount of the bill, in sterling, to be remitted.

Ans. £1500 5s. 6d.

TABLE OF FOREIGN MONEYS.

CITIES AND COUNTRIES.	DENOMINATIONS OF MONEY.	VALUR.
London, Liverpool, &c.	12 pence=1 shilling; 20 shillings =1 pound.....=	\$4.86 ² / ₃
Paris, Havre, &c.....	100 centimes=1 franc.....=	.18 ³ / ₄
Amsterdam, Hague, &c.	100 cents=1 guilder or florin...=	.40
Bremen	5 swares=1 grote; 72 grotes=1 <i>rix dollar</i>=	.78 ² / ₄
Hamburg, Lubec, &c...	12 pfennings=1 schilling; 16s.= 1 <i>mark banco</i>=	.35
Berlin, Dantzic.....	12 pfennings=1 groschen; 30 gro. =1 thaler.....=	.69
Belgium.....	100 centimes=1 franc.....=	.18 ³ / ₅
St. Petersburg.....	100 kopecks=1 ruble.....=	.75
Stockholm	12 rundstycks=16 skillings; 48s. =1 <i>rix dollar specie</i>=	1.06 .
Copenhagen.....	16 skillings=1 mark; 6 m.=1 <i>rix</i> <i>dollar</i>=	1.05
Vienna, Trieste, &c....	60 kreutzers=1 florin.....=	.48 ¹ / ₂
Naples	10 grani=1 carlino; 10 car.=1 <i>ducat</i>=	.80
Venice, Milan, &c.....	100 centesimi=1 <i>lira</i>=	.16
Florence, Leghorn, &c.	100 centesimi=1 <i>lira</i>=	.16
Genoa, Turin, &c.....	100 centesimi=1 <i>lira</i>=	.18 ³ / ₅
Sicily.....	20 grani=1 taro; 30 tari=1 oz.=	2.40
Portugal.....	1000 reas=1 millrea.....=	1.12
Spain.....	{ 34 maravedis=1 <i>real vellon</i> = 68 maravedis=1 <i>real plate</i> ..=	.05 .10
Constantinople	100 aspers=1 <i>piaster</i>=	.05
British India.....	12 pice=1 anna; 16 annas=1 <i>rupee</i>=	.44 ¹ / ₂
Canton	100 candarines=1 mace; 10 m.= 1 <i>tael</i>=	1.48
Mexico	8 rials=1 <i>dollar</i>=	1.00
Monte Video.....	100 centesimas=1 rial; 8 rials=1 <i>dollar</i>=	.83 ³ / ₁₀
Brazil.....	1000 reas=1 <i>milrea</i>=	.82 ⁴ / ₅
Cuba	8 reals plate or 20 reals vellon=1 <i>dollar</i>=	1.00
Turkey.....	100 aspers=1 <i>piaster</i>=	.05
United States.....	10 mills=1 cent; 10 cents=1 dime; 10 dimes=1 <i>dollar</i>=	variable.
New Brunswick.....	} 4 farthings=1 penny; 12 pence =1 shilling; 20 shillings=1 pound.*.....=	4.00
Nova Scotia.....		
Newfoundland.....		

* The Government of New Brunswick now issues postage stamps in the decimal currency, but so far as we have been able to ascertain, the currency of

ARBITRATION OF EXCHANGE.

Arbitration of Exchange is the method of finding the rate of exchange between two countries through the intervention of one or more other countries. The object of this is to ascertain what is the most advantageous channel through which to remit money to a foreign country.

Three things have here to be considered. *First*, what is the most secure channel; *secondly*, what is the least expensive, and *thirdly*, the comparative value of the currencies of the different countries. Regarding the two first considerations no general rule can be given, as there must necessarily be a continual fluctuation arising from political and other causes. We are therefore compelled to confine our calculation to the third, viz., the comparative value of the coin current of different countries.

For this purpose we shall investigate a rule, and append tables.

Let us suppose an English merchant in London wishes to remit money to Paris, and finds that owing to certain international relations, he can best do it through Hamburg and Amsterdam, and that the exchange of London on Hamburg is $13\frac{1}{2}$ marcs per pound sterling; that of Hamburg on Amsterdam, 40 marcs for $36\frac{1}{4}$ florins, and that of Amsterdam on Paris, $56\frac{3}{4}$ florins for 120 francs, and thus the question is to find the rate of exchange between London and Paris.

SOLUTION:

We write down the equivalents in ranks, the equivalent of the first term being placed to the right of it, and the other pairs below them in a similar order. Hence the first term of any pair will be of the same kind as the second term of the preceding pair. As the answer is to be the equivalent of the first term, the first term in the last rank corresponds to the third term of an analogy, and is therefore a multiplier, it must be placed below the second rank. The

these three Provinces is, as usual, in pounds, shillings and pence. It is to be hoped that when the Confederation of the British Provinces takes place, the decimal currency will be speedily adopted in the Lower Provinces, and that the efforts now being made in Britain to adapt the same currency will prove successful.

terms being thus arranged, we divide the product of the second rank by that of the first, and the quotient will be the equivalent, as exhibited below :

$$£1 \text{ sterling} = 13\frac{1}{2} \text{ marcs.}$$

$$40 \text{ marcs} = 36\frac{1}{4} \text{ florins.}$$

$$56\frac{3}{4} \text{ florins} = 120 \text{ francs.}$$

$$£1 \text{ stg.}$$

As it is most convenient to express the fractions decimally, we have

$$\frac{13.5 \times 36.25 \times 120 \times 1}{1 \times 40 \times 56.75} = 25.87 \text{ francs.}$$

The foregoing explanations may be condensed into the form of a

R U L E .

Write down the first term, and its equivalent to the right of it, and the other pairs in the same order, the odd term being placed under the second rank, and then divide the product of the second rank by the product of the first, the quotient will be the required equivalent.

NOTE.—The true principle on which this operation is founded is, that each pair consists of the antecedent and consequent which are to each other in the ratio of equality IN POINT OF INTRINSIC VALUE, though not in regard to THE NUMBERS BY WHICH THEY ARE EXPRESSED, and therefore the required term and its equivalent must have the same relation to each other, that is, they will be an antecedent and a consequent in the ratio of equality as regards their *value*, but not as regards the *numbers* by which they are expressed.

E X E R C I S E S .

1. If the exchange of London on Paris is 28 francs per pound sterling, and that of Canada on Paris 18 cents per franc; what is the rate of exchange of Canada on London, through Paris?

Ans. \$5.04 per £ sterling.

2. If exchange between Montreal and London is at 8 per cent. premium, and between London and Paris 25 $\frac{1}{4}$ francs per pound sterling; what sum in Montreal is equal to 7000 francs in Paris?

Ans.

3. When exchange between Quebec and Hamburg is at 34 cents per mark banco, and between Hamburg and St. Petersburg is 2 marks, 8 schillings per ruble; how much must be paid in St. Petersburg for a draft on Quebec for \$650?

Ans. 764 rubles, 70 $\frac{10}{7}$ kopecks.

4. If a merchant buys a bill in London, drawn on Paris, at the rate of 25.87 francs per pound sterling, and if this bill be sold in Amsterdam at 120 francs for $56\frac{3}{4}$ florins, and the proceeds be invested in a bill on Hamburg, at the rate of $36\frac{1}{4}$ florins for 40 marcs; what is the rate of exchange between London and Hamburg, or what is £1 sterling worth in Hamburg? Ans. 13.449 + marcs.

5. A merchant of St. Louis wishes to pay a debt of \$5000 in New York; the direct exchange is $1\frac{1}{2}$ per cent. in favour of New York, but on New Orleans it is $\frac{1}{2}$ per cent. discount, and between New Orleans and New York at a $\frac{1}{4}$ per cent. premium; how much would be saved by the circular exchange compared with the direct? Ans. \$87.56.

6. A merchant in Toronto wishes to remit to J. B. Gladstone & Co., of London, £3600 sterling. Exchange on London, in Toronto, is at a premium of 10 per cent. Exchange on London can be obtained at Halifax, Nova Scotia, for 9 per cent. premium. If Toronto Bills on Halifax are at a discount of $\frac{1}{4}$ per cent., and the merchant remits a draft to Halifax, and pays his agent $\frac{1}{2}$ per cent. for investing it in bills on London; what will he gain over the direct exchange? Ans. \$123.80.

7. A merchant in London remits to Amsterdam £1000, at the rate of 18 pence per guilder, directing his correspondent at Amsterdam to remit the same to Paris at 2 francs, 10 centimes per guilder, less $\frac{1}{2}$ per cent. for his commission; but the exchange between Amsterdam and Paris happened to be, at the time the order was received, at 2 francs, 20 centimes per guilder. The merchant at London, not apprised of this, drew upon Paris at 25 francs per pound sterling. Did he gain or lose, and how much per cent.? Ans. $16\frac{5}{7}\frac{6}{5}$ per cent. gain.

MIXED EXERCISES IN EXCHANGE.

1. When gold is quoted at 150 per cent. premium; what is the reason American money is not at a discount of 50 per cent.?

2. I wish to invest \$3760.80 in a sterling bill of exchange; for how many pounds must the bill be drawn, exchange being at a premium of 8 per cent.? Ans. £783 10s.

3. What sum in Canada money must I pay for a bill on London of £76 14s. 1d., exchange being $9\frac{1}{2}$ per cent. premium, and the broker's commission for negotiating the bill being $\frac{1}{2}$ per cent.?

Ans. \$375.

4. A merchant shipped 2560 barrels of flour to his agent in Liverpool, who sold it at £1 8s. 6d. per barrel, and charged 2 per cent. commission; what was the net amount of the flour in Canada money, allowing exchange to be at a premium of 8 per cent. ?

Ans. \$17160.19.

5. What is the cost of a 30 days' bill on Montreal, at $\frac{1}{2}$ per cent. premium, the face of the bill being \$1500 ?

Ans. \$1507.50.

6. What must be the face of a 60 days' draft on Halifax, Nova Scotia, to yield \$1641.75, when sold at a discount of $\frac{1}{2}$ per cent. ?

Ans. \$1650.

7. What is the cost of a 30 days' bill on Quebec, at $\frac{3}{8}$ per cent. premium, and interest off at 6 per cent. ; the face of the bill being \$9256.40 ?*

Ans. \$9240.20.

8. A merchant paid \$14400.12 for a bill on Havre for 79000 francs; how much was exchange below par ?

Ans. 2 per cent.

9. I have in possession the net proceeds of a sale of cotton amounting to \$3765, which my correspondent desires me to remit to him in New Orleans; exchange on New Orleans is at a discount of $2\frac{1}{2}$ per cent., and I invest the whole in a draft at that rate, which I remit to him; what is the face of the draft ?

Ans. \$3861.54.

10. The proceeds of a sale of goods, consigned to me from Bremen, is \$2764.67, on which I am to charge a commission of 10 per cent., and remit the balance to my consignor in such a way as shall be most advantageous to him. Exchange on Paris can be had at 92 cents per 5 francs, and in Paris exchange on Bremen is 17 francs to 4 thalers. Exchange on Liverpool can be had at 9 per cent. premium, and in Liverpool exchange on Bremen is 6 thalers to the pound sterling. Direct exchange is $80\frac{1}{4}$ cents per thaler. Which course will be the best, allowing $\frac{1}{2}$ per cent. brokerage to correspondents both in Liverpool and Paris ?

Ans. By way of Paris.

11. A, of Hamilton, sent articles to the World's Fair in London, which were afterwards sold by B, of London, on A's account, net proceeds £1266 15s. sterling. B was instructed to invest this amount in bills on Toronto, and remit to A, which was accordingly done. B charged $\frac{1}{4}$ per cent. brokerage on the face of the bills for investing, and purchased the bills at 7 per cent discount. Required

* When there is interest to be computed, it must be reckoned on the face of the bill or draft. When other than the value or cost of the bill is to be found, proceed as in percentage.

the amount of the bill A must receive in Canada money to close the transaction. Ans. \$6037.53 nearly.

12. A merchant in Toronto having to remit £434 15s. to Liverpool, wishes to know which is the most profitable, to buy a set of exchange on Liverpool at $10\frac{1}{2}$ per cent. premium, or send it by way of France; exchange on the latter place being $19\frac{3}{4}$ cents per franc, and exchange on Liverpool can be bought in France at the rate of $24\frac{1}{2}$ francs per pound sterling, and he has to pay his correspondent in France $\frac{3}{4}$ of 1 per cent. for purchasing the bill on Liverpool.

Ans. By way of France, \$15.69.

13. John McDonald & Co. wish to remit to a creditor in London £1241 15s. 9d. Exchange on London can be bought in Toronto at $109\frac{3}{4}$, but exchange on London can be purchased in New York for gold at $108\frac{1}{2}$. In New York it takes \$1.85 greenbacks to equal \$1 in gold. The broker in New York charges $\frac{3}{4}$ per cent. on the greenback value for investing. If exchange on New York is at 47 per cent. discount, at which place would it be the most advantageous to purchase, and how much gain, and if the remittance be made by the way of New York, what would be the face of the draft?

Ans. New York by \$141.72; face of draft, \$11161.21.

In the above exercise, suppose that instead of purchasing a draft on New York, they remit specie at an expense of $\frac{5}{8}$ per cent., the New York broker's commission being $\frac{3}{4}$ per cent. on the gold value of the bill; what would be his gain or loss? Ans. Loss \$13.64.

14. Hughes, Bros. & Co., purchase of E. Chaffey & Co., a sterling bill at 60 days on Gladstone & Hart, of London, for £3956 10s. They remit this bill to James Aldler, in London, where it is accepted by Gladstone & Hart, and falls due on the 20th November, at which time it is protested, causing an expense of £2 19s. Gladstone & Hart having failed, E. Chaffey & Co.'s agent in London pays James Aldler on the 20th November, £2000 on account. How much must E. Chaffey & Co. pay to Hughes, Brothers & Co., on the 24th December, to cover the amount still due in London, allowing interest at the rate of 10 per cent. from November 20th, to the maturity of a 60 days' bill at date of 24th December, and $\frac{1}{4}$ of 1 per cent. commission for their trouble in negotiating a new bill? Ans. \$9815.91.

INVOLUTION.

Involution is the process of finding a given power of a given number.

We have noted already, under the head of multiplication, that the product of any number of equal factors is called the second, third, fourth, &c., power of the number, according as the factor is taken two, three, four, &c., times. Thus: $9=3\times 3$ is the second power of 3; $27=3\times 3\times 3$ is the third power of three; $81=3\times 3\times 3\times 3$ is the fourth power of 3. These are often written thus: 3^2 , 3^3 , 3^4 , &c. The small figures, 2, 3, 4, indicate the number of factors, and therefore each is called the *index* or *exponent* of the power. Hence to find any required power of a given quantity, we have the

RULE.

Multiply the quantity continually by itself until it has been used as a factor as often as there are units in the index.

Since the first multiplication exhausts two factors, the number of operations will be *one* less than the number of factors.

Involution, then, is nothing more than multiplication, and for any power above the second, it is a case of continual multiplication. For the sake of uniformity the original quantity is called the *first power*, and also the root in relation to higher powers. Again, if we multiply 3×3 by $3\times 3\times 3$, we have five factors, or $3\times 3\times 3\times 3\times 3$, but this being an inconvenient form, it is written briefly 3^5 , the 5 indicating the number of times that 3 is to be repeated as a factor. Hence, since 3×3 is written 3^2 , and $3\times 3\times 3$ is written 3^3 , it follows that $3^2\times 3^3=3^5$, and therefore we may multiply quantities so expressed by *adding their indices*, and so also we may divide such quantities by *subtracting the index of the divisor from that of the dividend*. For example $3^3\div 3^2=3$ or 3^1 . If we divide 3^1 by 3^1 by subtracting the index of the divisor from that of the dividend, we obtain 3^0 , but 3 or 3^1 divided by 3 or 3^1 is equal to 1, and therefore any quantity with an index zero is equal to unity.

When high powers are to be found, the operation may be shortened in the following manner:—Let it be required to find the sixteenth power of 2. We first find the second power of 2, which is 4,

then $4 \times 4 = 16$, which is the fourth power, and $16 \times 16 = 256$, the eighth power, and $256 \times 256 = 65536$, the sixteenth power. If we wished to find the nineteenth power, we should only have to multiply the last result by 8, which is the third power of 2, for $2^{16} \times 2^3 = 2^{19}$.

EXERCISES.

1. Find the second power of 697. Ans. 485809.
2. What is the third power of 854 ? Ans. 622835864.
3. What is the second power of 4.367 ? Ans. 19.070689.
4. Find the fourth power of 75. Ans. 31640625.
5. What is the sixth power of 1.12 ? Ans. 1.9738+.
6. What is the second power .7, correct to six places ?
Ans. .060893+.
7. What is the fifth power of 4 ? Ans. 1024.
8. Find the third power of .3 to three places ? Ans. .036963.
9. What is the third power of $\frac{7}{9}$? Ans. $\frac{343}{729}$.
10. What is the fifteenth power of 1.04 ?* Ans. 1.800943.
11. Raise 1.05 to the thirty-first power. Ans. 4.538039.
12. What is the eighth power of $\frac{3}{5}$? Ans. $\frac{6561}{390625}$.
13. What is the second power of $4\frac{7}{8}$? Ans. $23\frac{49}{64}$.
14. Expand the expression 6^5 . Ans. .7776.
15. What is the second power of $5\frac{1}{2}$? Ans. $\frac{121}{4} = 30\frac{1}{4}$.
16. What part of 8^3 is 2^6 ? Ans. $\frac{1}{8}$.
17. What is the difference between 5^6 and 4^6 ? Ans. 11529.
18. Expand $3^5 \times 2^4$. Ans. 3888.
19. Express, with a single index, $47^3 \times 47^5 \times 47^6$? Ans. 47^{14} .
20. How many acres are in a square lot, each side of which is 135 rods ? Ans. 113 acres, 3 roods, 25 rods.
21. What is the sixth power of .1 ? Ans. .000001.
22. What is the fourth power of .03 ? Ans. .00000081.
23. What is the fifth power of 1.05 ? Ans. .1.2762815625.
24. What is the third power of .001 ? Ans. .000000001.
25. What is the second power of .0044 ? Ans. .00001836.

The second power of any number ending with the digit 5 may be readily found by taking all the figures except the 5, and multi-

* This exercise will be most readily worked by finding the sixteenth power, and dividing by 1.04. So in the next exercise, find the thirty-second power, and divide by 1.05. A still more easy mode of working such questions will be found under the head of logarithms.

plying that by itself, increased by a unit, and annexing 25 to the result.

Thus, to find the second power of 15, cut off the 5, and 1 remains, and this increased by 1 gives 2, and $2 \times 1 = 2$, and 25 annexed will give 225, the second power of 15. So also,

$\begin{array}{r} 25 \\ 3 \end{array}$	$\begin{array}{r} 35 \\ 4 \end{array}$	$\begin{array}{r} 65 \\ 7 \end{array}$	$\begin{array}{r} 105 \\ 11 \end{array}$	$\begin{array}{r} 215 \\ 22 \end{array}$	$\begin{array}{r} 575 \\ 58 \end{array}$
625	1225	4225	11025	46225	330625

EXERCISES ON THIS METHOD.

- | | |
|---------------------------------------|--------------|
| 26. What is the second power of 135 ? | Ans. 18225. |
| 27. What is the second power of 205 ? | Ans. 42025. |
| 28. What is the second power of 335 ? | Ans. 112225. |
| 29. What is the second power of 455 ? | Ans. 207025. |
| 30. What is the second power of 585 ? | Ans. 342225. |
| 31. What is the second power of 795 ? | Ans. 632025. |

NOTE.—The square root of any quantity ending in 9, must end in either 3 or 7.

No second power can end in 8, 7, 3 or 2.

The second root of any quantity ending in 6, must end in 4 or 6.

The second root of any quantity ending in 5, must end also in 5.

The second root of any quantity ending in 4, must end either in 8 or 2.

The second root of any quantity ending in 1, must end either in 1 or 9.

The second root of any quantity ending in 0, must also end in 0.

EVOLUTION.

The root of any quantity is a number such that when repeated, as a factor, the specified number of times, will produce that quantity. Thus, 3 repeated twice as a factor gives 9, and therefore 3 is called the *second root* of 9, while 3 taken *three times* as a factor will give 27, and therefore 3 is called the *third root* of 27, and so also it is called the *fourth root* of 81.

There are two ways of indicating this. First, by the mark $\sqrt{}$ which is merely a modified form of the letter r, the initial letter of the English word root, and the Latin word *radix* (root). When no mark is attached, the simple quantity or *first root* is indicated. When the *second root* is meant, the mark $\sqrt{}$ alone is placed before the quantity, but if the third, fourth, &c., roots are to be indicated,

the figures 3, 4, &c., are written in the angular space. Thus:
 $3 = \sqrt{9} = \sqrt[3]{27} = \sqrt[4]{81} = \sqrt[5]{243}$, &c., &c. The other method is to write the index as a fraction. Thus, $9^{\frac{1}{2}}$ means the second root of the first power of 9, *i. e.* 3. So also, $27^{\frac{1}{3}}$ is the third root of the first power of 27. In the same manner $64^{\frac{2}{3}}$ means the third root of the second power of 64, or the second power of the third root of 64. Now the third root of 64 is 4, and the second power of 4 is 16, or the second power of 64 is 4096, and the third root of 4096 is 16, so that both views give the same result.

Evolution is the process of finding any required root of a given quantity.

SECOND OR SQUARE ROOT.

Extracting the square or second root of any number, is the finding of a number which, when multiplied by itself, will produce that number.

To find the second root, or square root of any quantity.

By inspecting the table of second powers, it will be found that the second power of any whole number less than 10, consists of either *one* or *two* digits; the second power of any number greater than 9, and less than 100, will in like manner be found to consist of *three* or *four* digits; and, universally, the second power of any number will consist of either *twice* the number of digits, or *one less than twice* the number of digits that the root itself consists of. Hence, if we begin at the units' figure, and mark off the given number in periods of two figures each, we shall find that the number of digits contained in the root will be the same as the number of periods. If the number of digits is even, each period will consist of two figures, but if the number of digits be odd, the last period to the left will consist of only one figure.

Let it now be required to find the second root of 144. We know by the rule of involution that 144 is the second power of 12. Now 12 may be resolved into *one ten* and *two units*, or $10 + 2$, and $10 + 2$ multiplied by itself, as in the margin, gives $100 + 40 + 4$, and since 100 is the second power of 10, and 4 the second power of 2, and 40 is twice the product of 10 and 2, we conclude that the second

power of any number thus resolved is equal to the sum of the second powers of the parts, *plus* twice the product of the parts. Hence to find the second root of 144, let us resolve it into the three parts $100+40+4$, and we find that the second root of the first part is 10, and since 40 is twice the product of the parts, 40 divided by twice 10 or 20 will give the other part 2, and $10+2=12$, the second root of 144. We should find the same result by resolving 12 into $11+1$, or $9+3$, or $8+4$, or $7+5$, or $6+6$, but the most convenient mode is to resolve into the tens and the units. In the same manner, if it be required to find the second root of 1369, we have by resolution $900+420+49$, of which 900 is the second power of 30, and $30 \times 2=60$, and $420 \div 60=7$, the second part of the root, and $30+7=37$, the whole root.

Again, let it be required to find the second root of 15129. This may be resolved as below :

10000 is the second power of 100.

400 is the second power of 20.

9 is the second power of 3.

4000 is twice the product of 20 and 100.

600 is twice the product of 100 and 3.

120 is twice the product of 20 and 3.

15129 is the sum of all, and hence 1 is the root of the hundreds, 2 the root of the tens, and 3 the root of the units.

Generalizing these investigations, we find that the second power of a number consisting of *units alone* is the product of that number by itself; that the second power of a number consisting of *tens and units* is the second power of the *tens*, *plus* the second power of the units, *plus* twice the product of the tens and units; that the second power of a number, consisting of hundreds, tens and units, is the sum of the squares of the hundreds, the tens, and the units, *plus* twice the product of each pair. Now since the complement of the full second power, to the sum of the second powers of the parts, is twice the product of the parts, it follows that, when the first figure of the root has been found, it must be doubled before used as a divisor to find the second term, and for the same reason each figure, when found, must be doubled to give correctly the next divisor. Hence the

R U L E .

Beginning at the units' figure, mark off the whole line in periods of two figures each ; find the greatest power contained in the left hand period, and subtract it from that period ; to the remainder annex the next period ; for a new dividend, place the figure thus obtained as a quotient, and its double as a divisor, and find how often that quantity is contained in the second partial dividend, omitting the last figure ; annex the figure thus found to both divisor and quotient, multiply and subtract as in common division, and to the remainder annex the next period ; double the last obtained figure of the divisor, and proceed as before till all the periods are exhausted,—if there be a remainder, annex to it two ciphers, and the figure thence obtained will be a decimal, as will every figure thereafter obtained.

E X A M P L E S .

1. To find the second root of 797449.

First, commencing with the units' figure, we divide the line into periods, viz., 49, 74 and 79,—we then note that the greatest square contained in 79 is 64,—this we subtract from 79, and find 15 remaining, to which we annex the next period 74, and place 8, the second root of 64, in the quotient, and its double 16 as a divisor, and try how often 16 is contained in 157, which we find to be 9 times ; placing the 9 in both divisor and quotient, we multiply and subtract as in common division, and find a remainder of 53, to which we annex the last period 49, and proceeding as before, we find 3, the last figure of the root, without remainder, and now we have the complete root 893.

8	797449	893
	64	

169	1574	
1783	1521	

	5349	
	5349	

2. This operation may be illustrated as follows :

To find the second root of 273529.

500	273529	500 + 20 + 3 = 523
500 × 2 = 1000 + 20, or	250000	

1020	23529	
1000 + 2 × 20 + 3 = 1043	20400	

	3129	
	3129	

3. To find the second root of 153687.

Here we obtain, by the same process as in the last example, the whole number 392, with a remainder of 23, which can produce only a fraction.

3	153687	392.029+
	9	
69	636	
782	621	
	1587	
78402	1564	
784049	230000	
	156804	
	7319600	
	7056441	
	263159	

We now annex two ciphers, placing the decimal point after the root already found, but as the divisor is not contained in this new dividend, we place a cipher in both quotient and divisor, and annex two ciphers more to the dividend, and by continuing this process we find the decimal part of the root, and the whole root is 392.029+.

EXERCISES.

1. What is the second root of 279841? Ans. 529.
2. What is the second root of 74684164? Ans. 8642.
3. What is the second root of 459684? Ans. 678.
4. What is the second root of 785? Ans. 28.01785+.
5. What is the second root of 1728? Ans. 41.569219+.
6. What is the second root of 666? Ans. 25.8069+.
7. What is the second root of 123456789? Ans. 11111.11106+.
8. What is the second root of 5 to three places? Ans. 2.236.
9. What is the side of a square whose area is 19044 square feet? Ans. 138 feet.
10. What is the length of each side of a square field containing 893025 square rods? Ans. 945 linear rods.

The second root of a fraction is found by extracting the roots of its terms, for $\sqrt{\frac{16}{25}} = \frac{4}{5} \times \frac{4}{5}$ and therefore $\sqrt{\frac{16}{25}} = \sqrt{\frac{4}{5} \times \frac{4}{5}} = \frac{4}{5}$. So also, $\sqrt{\frac{49}{81}} = \frac{7}{9}$. Again, since $\sqrt{\frac{81}{100}} = \frac{9}{10} = .09$, and $.3 \times .3 = .09$, the second root of .09 is .3. This follows from the rules laid down for the multiplication of decimals.

To find the second root of a decimal or of a whole number and a decimal:

Point off periods of two figures each, from the decimal point towards the right and left, adding a cipher, or a repetend, if the number of figures be odd.

From what has been said, it is plain that every period, except the first on the left, must consist of two digits, and every decimal presupposes something going before, for .5 indicates the half of some unit under consideration, and .5 is equivalent to .50, *and not to .05*, from which it is obvious that the second root of .5 is not the root of .05, but of .50, and therefore the second root of .5 is not .2+, as the beginner would naturally suppose, but .7+, for .2+ is the approximate root of .05.

ADDITIONAL EXERCISES.

11. What is the second root of .7 to five places of decimals ?
Ans. .83666.
12. Find the second root of .07 to six places. Ans. 264575.
13. What is the second root of .05 ? Ans. .2236+.
14. What is the second root of .7 ? Ans. .8819+.
15. Find the second root of .5. Ans. .74535+.
16. What is the second root of .1 ? Ans. .3162277+.
17. What is the second root of .1 ? Ans. .3.
18. What is the second root of 1.375 ? Ans. 1.1726, &c.*
19. What is the second root of .375 ? Ans. 61237, &c.*
20. What is the second root of 6.4 ? Ans. 2.52982+.
21. Find to four decimal places $\sqrt[3]{3\frac{3}{20}}$. Ans. 1.7748.
22. Find $\sqrt[3]{2}$ to four decimal places. Ans. 1.4142.
23. Find the value of $\sqrt[3]{3271.4207}$. Ans. 57.196+.
24. Find the second root of .005 to five places. Ans. 07071.
25. Find the square root of 4.372594. Ans. 2.09107+.
26. What is the second root of .01 ? Ans. .1.
27. What is the second root of .001 ? Ans. 03162+.
28. What is the square root of .0001 ? Ans. .01.
29. What is the second root of .000001 ? Ans. .001.
30. What is the second root of 19.0968 ?

* The young student would naturally expect that the decimal figures of $\sqrt[3]{1.375}$ and $\sqrt[3]{.375}$ would be the same, but it is not so. If it were so, $\sqrt[3]{1+}$ $\sqrt[3]{.375}$ would be equal to $\sqrt[3]{1.375}$. That such is not the case, may be shown by a very simple example. $\sqrt[3]{16+}\sqrt[3]{9}=4+3=7$, but $\sqrt[3]{16+9}=\sqrt[3]{25}=5$. Let it be carefully observed, therefore, that *the sum of the second roots is not the same as the second root of the sum.*

O P E R A T I O N

	4	19.0968	4.37 trial.
	83	16	4.36 true.
		309	
Trial 867		249	
		6068	
Too great by 1		6069	
		6068	
True 866		5196	
		872	

Here we find the remainder, 872, is greater than the divisor, 866, which seems inconsistent with ordinary rules; but it must be observed that we are not seeking an exact root, but only the closest possible approximation to it. If the given quantity had been 19.0969, we should have found an exact root 4.37. The remainder 872 being greater than the divisor, shows that the last figure of the root is too small by $\frac{99}{100}$, whereas 7 would be too great by $\frac{1}{100}$, and that 866 is not a correct divisor but an approximate one, and that the true root lies between 4.36 and 4.37.

When the root of any quantity can be found exactly, it is called a *perfect power* or *rational quantity*, but if the root cannot be found exactly, the quantity is called *irrational* or *surd*.

A number may be rational in regard to one root, and irrational in regard to another. Thus, 64 is rational as regards $\sqrt[1]{64}=8$, $\sqrt[3]{64}=4$ and $\sqrt[6]{64}=2$, but it is irrational regarding any other root expressed by a whole number. But 64, with the fractional index $\frac{2}{3}$, *i. e.*, $64^{\frac{2}{3}}$, is rational, because it has an even root as already shown.

We may call $64^{\frac{2}{3}}$ either the second power of the third root of 64, or the third root of the second power. In the former view, the third root of 64 is 4, and the second power of 4 is 16, and according to the second view, 64^2 is 4096, and the third root of 4096 is 16, the same as before. $\sqrt[4]{81}=3$ is rational, and $\sqrt[1]{81}=9$ is rational, but 81 is not rational regarding any other root; while $\sqrt[1]{25}$ is rational only regarding the second root, and $\sqrt[3]{8}=2$ only regarding the third root.

The second root of an even square may be readily found by resolving the number into its prime factors, and taking each of these

factors once,—the product will be the root. Thus, 441 is $3 \times 3 \times 7 \times 7$ and each factor taken once is $3 \times 7 = 21$, the second root. Here let it be observed, that if we used each factor *twice* we should obtain the *second power*, but if we use each factor half the number of times that it occurs, we shall have the second root of that power. 64 is $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$, *i. e.*, 2 repeated six times as a factor gives the number 64, and therefore half the number of these factors will give the second root of 64, or $2 \times 2 \times 2 = 8$, and $2 \times 2 \times 2$ multiplied by $2 \times 2 \times 2 = 8 \times 8 = 64$.

As this cannot be considered more than a trial method, though often expeditious, we would observe that the smallest possible divisors should be used in every case, and that if the number cannot be thus resolved into factors, it has no even root, and must be carried out into a line of decimals, or those decimals may be reduced to common fractions.

THIRD ROOT OR CUBE ROOT.

As extracting the second root of any quantity is the finding of what two equal factors will produce that quantity, so extracting the third root is the finding of what *three* equal factors will produce the quantity.

By inspecting the table of third powers, it will be seen that no third power has more than three digits for each digit of the first power, nor fewer than two less than three times the number of digits. Hence, if the given quantity be marked off in periods of three digits each, there will be one digit in the first power for each period in the third power. The left hand period may contain only one digit.

From the mode of finding the third power from the first, we can deduce, by the converse process, a rule for finding the first power

from the third. We know by the rule of involution that the third power of 25 is 15625. If we resolve 25 into

we have $20+5$, and perform the multiplication in that form,

$$\begin{array}{r} 400+100 \\ 100+25 \\ \hline 400+200+25=(20+5)^2 \\ 20+5 \\ \hline 8000+4000+500 \\ 2000+1000+125 \\ \hline 8000+6000+1500+125=(20+5)^3=15625 \end{array}$$

Now, 8000 is the third power of 20, and 125 is the third power of 5; also, 6000 is three times the product of 5, and the second power of 20, and 1500 is three times the product of 20, and the second power of 5. Let a represent 20 and b represent 5, then

$$\begin{array}{rcl} a^3=20^3 & = & 8000 \\ 3 a^2 b=3 \times 20^2 \times 5 & = & 6000 \\ 3 a b^2=3 \times 20 \times 5^2 & = & 1500 \\ b^3=5^3 & = & 125 \\ \hline & & 15625 \end{array}$$

By using these symbols we obtain the simplest possible method of extracting the third root of any quantity, as exhibited by the subjoined scheme:

$$\begin{array}{rcl} \text{Given quantity} & \dots\dots\dots & 15625 \\ a^3=20^3=20 \times 20 \times 20 & \dots\dots\dots = & 8000 \\ \hline \text{Remainder} & \dots\dots\dots & 7625 \\ 3 a^2 b=3 \times 20^2 \times 5 & \dots\dots\dots = & 6000 \\ \hline \text{Remainder} & \dots\dots\dots & 1625 \\ 3 a b^2=3 \times 20 \times 5^2 & \dots\dots\dots = & 1500 \\ \hline \text{Remainder} & \dots\dots\dots & 125 \\ b^3=5^3=5 \times 5 \times 5 & \dots\dots\dots = & 125 \end{array}$$

From this and similar examples we see that a number denoted by more than one digit may be resolved into tens and units. Thus, 25 is 2 tens and 5 units, 123 is 12 tens and 3 units, and so of all numbers.

To find the third root of 1860867 :

As this number consists of three periods, the root will consist of three digits, and the first period from the left will give hundreds, the second tens, and the third units, and so also in case of remainder, each period to the right will give one decimal place, the first being tenths, the second hundredths, &c., &c.

We may denote the digits by a , b and c .

$a=100$	1860867(100+20+3=123
$a^3=100^3=$	1000000
$b=3a^2 \ b \div 3 \ a^2=\frac{860867}{30000}=20+$, and $30000 \times 20=$	860867 remainder. 600000
$3 \ a \ b^2=3 \times 100 \times 400=$	260867 remainder. 120000
$b^3=20^3=$	140867 remainder. 8000
Now $(a+b)=120 \ \therefore 3 \ (a+b)^2=132867$ remainder. 43200, which is contained 3 times + in 132867, $\therefore c=3$, and $3 \ (a+b)^2 c^2$ $=3 \times 120^2 \times 3=$	129600
And $3 \ (a+b) \ c^2=3 \times 120 \times 9=$	3267 remainder. 3240
And lastly, $c^3=3^3=$	27 27
	 no remainder.

R U L E .

Mark off the given number in periods of three figures each.

Find the highest third power contained in the left hand period, and subtract it from that period. Divide the remainder and next period by three times the second power of the root thus found, and the quotient will be the second term of the root.

From the first remainder subtract three times the product of the second term, and the square of the first, PLUS three times the product of the first term, and the square of the second, PLUS the third power of the second.

Divide the remainder by three times the square of the sum of the first and second terms, and the quotient will be the third term.

From the last remainder subtract three times the product of the term last found, and the square of the SUM of the preceding terms, PLUS the product of the square of the last found term by the SUM of the preceding ones, PLUS the third power of the last found term, and so on.

EXERCISES.

1. What is the third root of 46656 ? Ans. 36.
2. What is the third root of 250047 ? Ans. 63.
3. What is the third root of 2000576 ? Ans. 126.
4. What is the third root of 5545233 ? Ans. 177.
5. What is the third root of 10077696 ? Ans. 216.
6. What is the third root of 46268279 ? Ans. 359.
7. What is the third root of 85766121 ? Ans. 441.
8. What is the third root of 125751501 ? Ans. 501.
9. What is the third root of 153990656 ? Ans. 536.
10. What is the third root of 250047000 ? Ans. 630.
11. What is each side of a square box, the solid content of which is 59319 ? Ans. 39 inches.
12. What is the third root of 926859375 ? Ans. 975.
13. Find the third root of 44.6. Ans. 3.456+.
14. What is the third root of 9 ? Ans. 2.08008+.
15. What is the length of each side of a cubic vessel whose solid content is 2936.493568 cubic feet ? Ans. 1432 feet.
16. Find the third root of 5. Ans. 1.7099.
17. A store has its length, breadth and height all equal ; it can hold 185193 cubic feet of goods ; what is each dimension ? Ans. 57 feet.
18. How many linear inches must each dimension of a cubic vessel be which can hold 997002999 cubic inches of water ? Ans. 999 inches.
19. What is the third root of 1 ? Ans. 1.
20. What is the third root of 144 ? Ans. 5.241483.

The third root of a fraction is found by extracting the third root of the terms. The result may be expressed either as a common fraction, or as a decimal, or the given fraction may be reduced to a decimal, and the root extracted under that form.

EXERCISES.

1. What is the third root of $\frac{27}{64}$?Ans. $\frac{3}{4}=.75$.

Otherwise :

$$\begin{array}{rcl}
 \frac{27}{64}=.421875. & \text{To find the third root of} & \\
 \text{this we have} & .421875(.70+.05=.75 & \\
 70^3= & 343000 & \\
 \left. \begin{array}{l} 3 \times 70^2 \times 5 = 73500 \\ 3 \times 70 \times 5^2 = 5250 \\ 5^3 = 125 \end{array} \right\} = & \begin{array}{r} 78875 \text{ remainder.} \\ 78875 \end{array} & \\
 & \text{..... no remainder.} &
 \end{array}$$

The third root of a mixed quantity will be most readily found by reducing the fractional part to the decimal form, and applying the general rule.

It has been already explained that the second root of an even power may be obtained by dividing the given number by the smallest possible divisors in succession, and taking half the number of those divisors as factors. The same principle will apply to any root. If the given quantity is not an even power, it may yet be found approximately. If we take the number 46656, we notice that as the last figure is an even number, it is divisible by 2, and by pursuing the same principle of operation we find six *twos* as factors, and afterwards six *threes*; and, as in the case of the second root, we take *each* factor *half* the number of times it occurs, so in the case of the third root, we take *each factor one-third* the number of times it occurs.

The same principle on which the extraction of the second and third depends may be applied to any root, the line of figures being divided into periods, consisting of as many figures as there are units in the index; for the fourth root, periods of four figures each; for the fifth, five, &c., &c. We may remark, however, that these modes are now superseded by the grand discovery of Logarithmic Computation. (See Logarithms.

PROGRESSION.

A *series* is a succession of quantities increasing or decreasing by a *Common Difference*, or a *Common Ratio*.

Progression by a Common Difference forms a series by the addition or subtraction of the same quantity. Thus 3, 7, 11, 15, 19, 23 forms a series increasing by the constant quantity 4, and 28, 21, 14, 7, forms a series decreasing by the constant quantity 7. Such a progression is also called an *equidifferent series*.*

Progression by a Common Ratio forms a series increasing or decreasing by multiplying or dividing by the same quantity. Thus, 3, 9, 27, 81, 243, is a series increasing by a constant multiplier 3, and 64, 32, 16, 8, 4, 2, is a series decreasing by a constant divisor 2.

The quantities forming such a progression are also called *Continual Proportionals*,* because the ratio of 3 to 9 is the same as the ratio of 9 to 27, &c., &c. From this it is plain that in a progression by ratio, each term is a mean proportional between the two adjacent ones, and also between any two that are equally distant from it.

The first and last terms are called the *Extremes*, and all between them the *Means*.

PROGRESSION BY A COMMON DIFFERENCE.

In a series increasing or decreasing by a common difference, the sum of the extremes is always equal to the sum of any two that are equally distant from them. Thus, in the first example $3+23=7+19=11+15=26$, and in the second $28+7=21+14=35$.

If the number of terms be odd, the sum of the extremes is equal to twice the middle term. Thus in the series 3, 7, 11, 15, 19, $3+19=2\times 11=22$, and hence the middle term is half the sum of the extremes.

* The names *Arithmetical Progression* and *Geometrical Progression* are often applied to quantities so related, but these terms are altogether inappropriate, as they would indicate that the one kind belonged solely to arithmetic, and the other solely to geometry, whereas, in reality, each belongs to both these branches of science.

In treating of progressions by difference or equidifferent series, there are five things to be considered, viz., the first term, the last term, the common difference, the number of terms, and the sum of the series. These are so related to each other that when any three of them are known we can find the other two.

Given the first term of a series, and the common difference, to find any other term.

Suppose it is required to find the seventh term of the series 2, 5, 8, &c. Here, as the first term is given, no addition is required to find it, and therefore six additions of the common difference will complete the series on to seven terms. In other words, the common difference is to be added to the first term as often as there are units in the number of terms diminished by 1. This gives $7-1=6$, and $6 \times 3=18$, which added to the first term 2 gives 20 for the seventh term. If we had taken the series on the descending scale, 20, 17, 14, &c., we should have had to subtract the 18 from the first term 20 to find the seventh term 2. The term thus found is usually designated *the last term*, not because the series terminates there, for it does not, but simply because it is the last term *considered* in each question proposed. From these illustrations we derive the

RULE (1.)

Subtract 1 from the number of terms, and multiply the remainder by the common difference; then if the series be an increasing one, add the result to the first term, and if the series be a decreasing one, subtract it.

EXAMPLES.

To find the fifty-fourth term of the increasing series, the first term of which is $33\frac{3}{4}$, and the common difference $1\frac{1}{4}$. Here $54-1=53$, and $53 \times 1\frac{1}{4}=66\frac{1}{4}$, and $66\frac{1}{4}+33\frac{3}{4}=100$, the fifty-fourth term.

Given 64 the first term of a decreasing series, and 7 the common difference, to find the eighth term. Here $8-1=7$, and $7 \times 7=49$, and $64-49=15$, the eighth term.

EXERCISES.

1. Find the eleventh term of the decreasing series, the first term of which is $248\frac{3}{4}$, and the common difference $3\frac{1}{4}$. Ans. $216\frac{1}{4}$.

2. The hundredth term of a decreasing series is $392\frac{2}{5}$, and the common difference is $3\frac{3}{5}$, what is the last term? Ans. 36.

3. What is the one-thousandth term of the series of the odd figures? Ans. 1999.

4. What is the five-hundredth term of the series of even digits? Ans. 1000.

5. What is the sixteenth term of the decreasing series, 100, 96, 92, &c.? Ans. 40.

To find the sum of any equidifferent series, when the number of terms, and either the middle term or the extremes, or two terms equidistant from them, are given.

We have seen already that in any such series the sum of the extremes is equal to the sum of any two terms that are equidistant from them, and when the number of terms is odd, to twice the middle term. Hence the middle term, or half the sum of any two terms equi-distant from the extremes, will be equal to half the sum of those extremes. Thus, in the series $2+7+12+17+22+27+32$, we have $\frac{2+32}{2} = \frac{7+27}{2} = 17$, the middle term. It is plain, therefore, that if we take the middle term and half the sum of each equi-distant pair, the series will be equivalent to $17+17+17+17+17+17+17$, or 7 times 17, which will give 119, the same as would be found by adding together the original quantities. The same result would be arrived at when the number of terms is even, by taking half the sum of the extremes, or of any two terms that are equi-distant from them. From these explanations we deduce the

R U L E (2.)

Multiply the middle term, or half the sum of the extremes, or of any two terms that are equidistant from them, by the number of terms.

NOTE.—If the sum of the two terms be an odd number, it is generally more convenient to multiply by the number of terms before dividing by 2.

E X A M P L E S .

Given 23, the middle term of a series of 11 numbers, to find the sum. Here we have only to multiply 23 by 11, and we find at once the sum of the series to be 253.

Given 7 and 73, the extremes of an increasing series of 12 numbers, to find the sum. The sum of the extremes is 80, the half of which is 40, and $40 \times 12 = 480$, the sum required.

Two equidistant terms of a series, 35 and 70, are given in a series of 20 numbers, to find the sum of the series. In this case, we have $35+70=105$, and $105 \times 20 = 2100$, and $2100 \div 2 = 1050$, the sum required.

EXERCISES.

1. Find the sum of the series, consisting of 200 terms, the first term being 1 and the last 200. Ans. 20100.
2. What is the sum of the series whose first term is 2, and twenty-first 62? Ans. 672.
3. What is the sum of 14 terms of the series, the first term of which is $\frac{1}{2}$ and the last 7? Ans. $52\frac{1}{2}$.
4. Find the sum to 10 terms of the decreasing series, the first term of which is 60 and the ninth 12. Ans. 360.
5. A canvasser was only able to earn \$6 during the first month he was in the business, but at the end of two years was able to earn \$98 a month; how much did he earn during the two years, supposing the increase to have been at a constant monthly rate? Ans. \$1248.
6. If a man begins on the first of January by saving a cent on the first, two on the second, three on the third, four on the fourth, &c., &c., how much will he have saved at the end of the year, not counting the Sabbaths? Ans. \$490.41.
7. How many strokes does a clock strike in 13 weeks? Ans. 14196.
8. If $8\frac{3}{4}$ is the fourth part of the middle term of a series of 99 numbers, what is the sum? Ans. 3465.
9. In a series of 17 numbers, 53 and 33 are equidistant from the extremes; what is the sum of the series? Ans. 731.
10. In a series of 13 numbers, 33 is the middle term; what is the sum? Ans. 429.

To find the number of terms when the extremes and common difference are given:

As in the rule (1), we found the *difference of the extremes* by multiplying by *one less* than the number of terms, and added the first term to the result, so now we reverse the operation and find the

RULE (3.)

Divide the difference of the extremes by the common difference and add 1 to the result.

EXAMPLE.

Given the extremes 7 and 109, and the common difference, 3, to find the number of terms.

In this case we have $109 - 7 = 102$, and $102 \div 3 = 34$, and $34 + 1 = 35$, the number of terms.

EXERCISES.

1. What is the number of terms when the extremes are 35 and 333, and the common difference 2? Ans. 150.

2. Two equidistant terms are 31 and 329, and the common difference 2; what is the number of terms? Ans. 150.

3. The first term of a series is 7, and the last 142, and the common difference $\frac{1}{4}$; what is the number of terms? Ans. 541.

4. The first and last terms of a series are $2\frac{1}{2}$ and $35\frac{1}{2}$, and the common difference $\frac{1}{3}$; what is the number of terms? Ans. 100.

5. The first term of a series is $\frac{1}{2}$ and last $12\frac{1}{2}$, and the common difference $\frac{1}{2}$; what is the number of terms? Ans. 25.

Given one extreme, the sum of the series and the number of terms, to find the other extreme.

This case may be solved by reversing Rule (2), for in it the data are the same, except that there the second extreme was given to find the sum, and now the sum is given, to find the second extreme. Therefore, as in that rule we *multiplied* the sum of the extremes by the number of terms and *halved* the product, so now we must *double* the sum of the series and *divide* by the number of terms to find the sum of the extremes, and from this subtract the given extreme, and the remainder will be the required extreme. This will illustrate the

RULE (4.)

Divide twice the sum of the series by the number of terms, and from the quotient subtract the given extreme, and the remainder will be the required extreme.

EXAMPLE.

Given 5050, the sum of a series, 1 the first term, and 100 the number of terms, to find the other extreme.

Twice the sum is 10100, which, divided by 100, gives 101, and $101 - 1 = 100$, the number of terms.

EXERCISES.

1. Given 50, the greater extreme of a decreasing series, 442, the sum, and 17 the number of terms, to find the other extreme.

Ans. 2.

2. If 121268 be the sum of a series, 8 the less extreme, and 142 the number of terms; what is the greater extreme? Ans. 1700.

3. The sum of a series of 7 terms is 105, the greater extreme is 21, and the number of terms 7; what is the less extreme? Ans. 9.

4. The sum of a series is 576, the number of terms 24, and the greater extreme is 47; what is the less extreme? Ans. 1.

5. The sum of a series is $30204\frac{1}{2}$, the greater extreme 312, and the number of terms 193; what is the less extreme? Ans. 1.

Given the extremes and number of terms, to find the common difference.

As explained in the introduction to Rule (1), the number of common differences must be *one less* than the number of terms. It is obvious also, that the sum of these differences constitutes the difference between the extremes, and that therefore the sum of the differences is the same as 1 less than the number of terms. Therefore the difference of the extremes, divided by *the sum of the differences*, will give *one* difference, *i. e.*, the common difference. This gives us the

RULE (5.)

Subtract 1 from the number of terms, and divide the difference of the extremes by the remainder.

EXAMPLE.

If the extremes of an increasing series be 1 and 47, and the number of terms 24, we can find the common difference thus:— $47-1=46$, and $46\div 23=2$, the common difference.

EXERCISES.

1. If the extremes are 2 and 36, and the number of terms 18; what the common difference? Ans. 2.

2. What is the common difference if the extremes are 58 and 3, and the number of terms 12? Ans. 5.

3. In a decreasing series given 1000 the less extreme, and 1793 the greater, and 367 the number of terms, to find the common difference. Ans. $2\frac{1}{6}$.

4. If 6 and 60 are the extremes in a series of 10 numbers, what is the common difference? Ans. 6.

5. What is the common difference in a decreasing series of 42 terms, the extremes of which are 9 and 50? Ans. 1.

There are fifteen other cases, but they may all be deduced from the five here given.

We subjoin the Algebraic form as it is more satisfactory and complete, and also more easy to persons acquainted with the symbols of that science.

Let a be the first term, d the common difference, n the number of terms, s the sum of the series; the series will be represented by $a + (a + d) + (a + 2d) + (a + 3d) + \&c.$, to $\{ a + (n - 1)d. \}$ By inspecting this series it will be seen that the co-efficient of d is always 1 less than the number of terms, for in the second term where d first appears, its co-efficient is 1, in the third it is 2, and therefore since n represents the number of terms, the co-efficient of d in the last term is $n - 1$, and that term therefore is $a + (n - 1)d$. If the series were a decreasing one, that is, one formed by a succession of subtractions, the last term would be $a - (n - 1)d$.

To find the sum of an equidifferent series.

We have here $s = a + (a + d) + (a + 2d) + (a + 3d) + \&c. \dots \dots \dots + \{ a + (n - 1)d. \}$ Since $a + (n - 1)d$ is the last term, the last but one will be $a + (n - 2)d$, and the last but two will be $a + (n - 3)d$, $\&c.$, $\&c.$ But the sum of any number of quantities is the same in whatever order they may be written. Let us therefore write this series both as above, and also in reversed order :

$$s = a + (a + d) + (a + 2d) + (a + 3d) + (a + 4d) + \&c. \dots \dots \dots + a + (n - 3)d + a + (n - 2)d + a + (n - 1)d.$$

$$s = a + (n - 1)d + a + (n - 2)d + a + (n - 3)d + \&c. \dots \dots \dots (a + 4d) + (a + 3d) + (a + 2d) + (a + d) + a.$$

Adding the two members of the second to those of the first, we obtain $2s = \{ 2a + (n - 1)d \} + \{ 2a + (n - 1)d \} + \{ 2a + (n - 1)d \} + \{ 2a + (n - 1)d \} + \&c.$, to n terms.

In the last expression all the terms are the same, but there are n terms, and therefore the whole will be

$$2s = n \left\{ 2a + (n-1)d \right\} \text{ and therefore}$$

$$s = \frac{n}{2} \left\{ 2a + (n-1)d \right\} \dots\dots (1.)$$

As we have used no single symbol to represent the last term, we must now show how it may be obtained from the other data. We have seen that the last term is $a + (n-1)d$, which we may denote by l , which will give us the formula

$$l = a + (n-1)d.$$

This formula, in the case of a decreasing series, will become

$$l = a - (n-1)d, \text{ and generally}$$

$$l = a \pm (n-1)d. (2.)$$

This formula is the same as Rule (1.)

We may modify (1) by (2) by substituting l for $a + (n-1)d$.

Thus:

$$s = \frac{n}{2}(a + l). (3.)$$

This is a convenient form when the last term is given. Using l for the last term, we have five quantities to consider, viz., a , l , d , n , s , and, as already stated, any three of these being given, the other two can be found from (1) and (2.)

To find d when a , l , n are given:

By (2.)

$$l = a + (n-1)d$$

$$\dots l - a = (n-1)d$$

$$\dots d = \frac{l - a}{n - 1}. (4.)$$

This finds the common difference, when the extremes and number of terms are given, and corresponds to Rule (5.)

If a , n , s are given, we have

$$\text{By (1.)} \quad s = \frac{n}{2} \left\{ 2a + (n-1)d \right\}$$

$$\dots 2s = 2an + n(n-1)d$$

$$\dots dn(n-1) = 2(s - an)$$

$$\dots d = \frac{2(s - an)}{n(n-1)}.$$

If n is to be found from a , d , s , we have

$$\text{by (1.)} \quad s = \frac{n}{2} \{ 2a + (n-1)d \}$$

$$\therefore 2s = 2an = dn^2 - dn$$

$$\therefore dn^2 + n(2a-d) = 2s$$

And by solving this quadratic equation, we find

$$n = \frac{d-2a \pm \sqrt{8ds + (2a-d)^2}}{2d}$$

EXAMPLES.

Given $a=6$, $d=4$, $n=20$, to find s .

$$\begin{aligned} \text{First by (2) } l &= a + (n-1)d \\ &= 6 + (20-1)4 \\ &= 82 \end{aligned}$$

$$\begin{aligned} \text{and hence by (3) } s &= \frac{20}{2}(6+82) \\ &= 880. \end{aligned}$$

Given $a=3$, $l=300$, $n=33$, to find d .

$$\begin{aligned} \text{By (4)} \quad d &= \frac{l-a}{n-1} \\ &= \frac{297}{32} = 9\frac{9}{32}.* \end{aligned}$$

MIXED EXERCISES.

1. Given 70, the less extreme, 10 the common difference, and 44 the number of terms, to find the sum. Ans. 12540.

2. What is the less extreme when the greater is 579, the common difference 9, and the sum of the series 18915? Ans. 3.

3. What is the series when $s=143$, $d=2$, $n=11$?

Ans. 3, 5, 7, 9, 11, 13, 15, &c.

4. Given 4 and 49, the extremes, and 6 the number of terms, to find the series. Ans. 4, 13, 22, 31, 40, &c.

5. If 120 stones are laid in a straight line, on level ground, at a regular distance of a yard and a quarter, how far must a person travel to pick them all up one by one and carry them singly and place them in a heap at the distance of 6 yards from the first, and in the same line with the stones? Ans. 10 m. 7 fur., 27 rds., $1\frac{1}{2}$ yds.

6. Insert three means between the extremes 117 and 477.

Ans. 207, 297, and 387.

* The other variations are left as exercises for the student.

7. A courier agreed to ride 100 miles on condition of being paid 1 cent for the first mile, 5 for the second, 9 for the third, and so on; how much did he get per mile on an average, how much for the last mile, and how much altogether?

Ans. \$1.99 per mile, \$3.97 for the last, and \$199 for all.

8. A man performed a journey in 11 days on horseback—the first day he rode 45 miles, but, his horse getting lame, he was forced to slacken the pace at a certain rate per day, so that on the last day he made only five miles; what was the length of the journey, and at what rate did he slacken his speed?

Ans. The journey was 275 miles, and the slackening of speed 4 m. per day.

9. Find the series of which 72 is the sum, 17 the first term, and number of terms 6.

Ans. 17, 15, 13, 11, 9, 7.

10. The Venetian clocks strike the hours for the whole day; how many strokes will one of these strike in a year. Ans. 109500.

11. An Eastern monarch being threatened with invasion, offered his commander-in-chief a reward equivalent to a mill for the first soldier he would enlist within a month, two for the second, three for the third, and so on; the officer enlisted 999,999 men; what was his reward equal to in our money.

Ans. \$499,999,500.

12. One hundred sailors were drawn up in line at a distance from each other of 2 yards, including the breadth of the body—the paymaster, seated a distance of two yards from the first, sent a lieutenant to hand to the first a sum of prize money, then back again to the second, and so on to each singly; how far had the lieutenant to walk?

Ans. 11 miles, 3 fur., 32 rods, 4 yds.

PROGRESSIONS BY RATIO.

There are in progression by ratio, as in progression by difference, the same five quantities to be considered, except that in place of a common difference we have a common ratio; that is, instead of increase or decrease by addition and subtraction, we have increase or decrease by multiplication or division. If any three of these are known the other two can be found.

We have noticed already that if any quantity, 2, be multiplied by itself, the product, 4, is called the square, or second power of that

quantity; if this be again multiplied by 2, the product, 8, is called the cube, or third power of that quantity; if this again be multiplied by 2, the product is called the fourth power of that quantity, and so on to the fifth, sixth, &c., powers. To show the short mode of indicating this, let us take $3 \times 3 \times 3 \times 3 \times 3 = 243$. For brevity this is written 3^5 , which means that there are 5 factors, all 3, to be continually multiplied together, and 5 is called the *index*, because it *indicates* the number of equal factors.

Given the first term and the common ratio to find the last proposed term.

Let it be required to find the sixth term of the increasing series, of which the first term is 3 and the ratio 4.

This may obviously be found by successive multiplications of the first term, 3, by the ratio, 4,—thus:—

$$\begin{array}{rcl} & & 3 = \text{1st term.} \\ & 3 \times 4 = & 12 = \text{2nd term.} \\ & 12 \times 4 = & 48 = \text{3rd term.} \\ & 48 \times 4 = & 192 = \text{4th term.} \\ & 192 \times 4 = & 768 = \text{5th term.} \\ & 768 \times 4 = & 3072 = \text{6th term.} \end{array}$$

The series, therefore, is 3, 12, 48, 192, 768, 3072. From this, it is plain, that as to find the last of 6 terms, only 5 multiplications of the first are required, in all cases the number of multiplications will be one less than the number of terms. But to multiply five times by 4 is the same as to multiply by 1024, the fifth power of 4, for $4 \times 4 \times 4 \times 4 \times 4 = 1024$, and $1024 \times 3 = 3072$.*

This gives us the general

RULE (1.)

Multiply the first term by that power of the given ratio which is a unit less than the number of terms.

If the series be a decreasing one, divide instead of multiplying.

EXAMPLES.

Given in a series of 12 numbers, the first term 4 and the ratio 2, to find the last term.

Since 11 is one less than the number of terms, we find the 11th power of 2, which is 2048, and this, multiplied by the first term, 4, gives 8192 for the twelfth term.

* For the abbreviated mode see Involution.

Given the ninth term of a decreasing series, 39366, and the ratio 3, to find the first term.

As there are 9 terms, we take the 8th power of the ratio, 3, which we find to be 6561, and the first term $39366 \div 6561 = 6$, the first term.

EXERCISES.

1. What is the ninth term of the increasing series of which 5 is the first term and 4 the ratio? Ans. 327680.

2. What is the twelfth term of the increasing series, the first term of which is 1 and the ratio 3? Ans. 177147.

3. In a decreasing series the first term is 78732, the ratio 3, and the number of terms, 10; what is the last term? Ans. 4.

4. What is the 20th term of an increasing series, the first of which is 1.06, and also the ratio 1.06? Ans. 3.207135.

5. In a decreasing series the first term is 126.2477, the ratio 1.06; what is the last of 5 terms? Ans. 100.

Given the extremes and ratio, to find the sum of the series.

It is not easy to give a direct *proof* of this rule without the aid of Algebra, but the following illustration may be found satisfactory, and, in some sort, be accounted a proof.

Let it be required to find the sum of a series of continual proportions, of which the first term is 5, the ratio 3, and the number of terms 4.

Since 3 is the common ratio, we can easily find the terms of the series by a succession of multiplications. These are—

$$\begin{array}{r} 5+15+45+135, \text{ and the sum is } 200 \\ 15+45+135+405 \\ \hline 400 \end{array}$$

Let us now multiply each term by the ratio, 3, and, for convenience and clearness, place each term of the second line below that one of the first to which it is equal. Let us now subtract the upper from the lower line, and we find that there is no remainder, except the difference of the two extreme quantities, viz., 400. Now, it will be seen that this remainder is exactly double of the sum of the series, 200, and consequently 400 divided by 2, will give the sum 200. Also, 405 is the product of the last term by the ratio, and 400 is the difference between that product and the first term, and the divisor, 2, is a unit less than the ratio, 3. Hence the

R U L E (2.)

Multiply the last term by the ratio, from this product subtract the first term, and divide the remainder by the ratio, diminished by unity.

E X A M P L E .

Given the first term of an increasing series, equal 4, the ratio 3, and the number of terms 6, to find the sum of the series.

By the former rule we find the last term to be 972. This, multiplied by the ratio, gives 2916, and the first extreme, 4, subtracted from this, leaves 2912, and this divided by 2, which is less than the ratio, gives 1456, the sum of the series.

E X E R C I S E S .

1. What is the sum of the series, of which the less extreme is 4, the ratio 3, and the number of terms 10 ? Ans. 118096.

2. What is the sum of the series, of which 1 is the less extreme, 2 the ratio, and 14 the number of terms ? Ans. 16383.

3. What is the sum of the series, of which the greater extreme is 18.42015, the less 1, and the ratio 1.06 ? Ans. 308.755983.

4. A cattle dealer offered a farmer 10 sheep, at the rate of a mill for the first, a cent for the second, a dime for the third, a dollar for the fourth, &c., &c.; in what amount was he "taken in," supposing that each sheep was worth \$11.111 ? Ans. \$1111100.00.

5. What is the sum of six terms of the series, of which the greater extreme is $\frac{1}{2}$ and the ratio $\frac{3}{5}$? Ans. $\frac{3724}{125}$, or $1\frac{599}{125}$.

To find the ratio when the extremes and number of terms are given :

Let it be required to find the ratio when the extremes are 3 and 192, and the number of terms 7. This is effected by simply reversing the first rule, and therefore we divide 192 by 3 and find 64, and take the 6th root of 64, which is 2, the ratio. Hence the

R U L E (3.)

Divide the greater extreme by the less, and find that root of the quotient, the index of which is one less than the number of terms.

E X A M P L E .

If the greater extreme is 1024, and the less 2, and the number of terms 10, we divide 1024 by 2, and find 512, and then by extracting the ninth root of 512, we find the ratio, 2.

EXERCISES.

1. If the first yearly dividend of a joint stock company be \$1, and the dividends increase yearly, so as to form a series of continual proportionals, what will all amount to in 12 years, the last dividend being \$2048, and what will be the ratio of the increase?

Ans. ratio, 2; sum, \$4095.

2. What is the ratio, in the series of which the less extreme is 3 and the greater 98034, and the number of terms 16. Ans. .196605.

3. What is the ratio of a series, the extremes of which are 4 and 324, and the number of terms 5? Ans. 3.

4. What is the ratio of a series, the number of terms being 7 and the extremes 3 and 12288? Ans. 4.

5. In a series of 23 terms the extremes are 2 and 8388608; what is the ratio? Ans. 2.

To insert any number of means between two given extremes:

Find the ratio by Rule (3), and multiply the first extreme by this ratio, and the second will be obtained, and divide the last by the ratio, and the last but one will be obtained; continue this operation until the required term or terms be procured.

NOTE.—A mean proportional is found by taking the square root of the product of the extremes.

EXAMPLE.

Let it be required to insert between the extremes 5 and 1280 *three* terms, so that the numbers constituting the series shall be continual proportionals.

The number of terms here is 5, and hence, by Rule (3), we find the ratio to be 4, and 5 multiplied by this will give the second term, 20, and that again multiplied by 4 will give 80, the third, and that again multiplied by 4 will give the fourth term, 320, so that the full series is found to be 5, 20, 80, 320, 1280. The same result would be found by dividing the greater extreme by 4, and so on downwards, thus: 1280, 320, 80, 20, 5.

EXERCISES.

1. Between 5 and 405 insert three terms, which shall make the whole a series of continual proportionals. Ans. 5, 15, 45, 135, 405.

2. Insert between $\frac{1}{9}$ and 27 four terms to form a series, and give the ratio. Ratio, 3; series, $\frac{1}{9}$, $\frac{1}{3}$, 1, 3, 9, 27.

3. What three numbers inserted between 7 and 4375 will form a series of continual proportionals? Ans. 35, 175, 875.

4. What is the mean proportional between 23 and 8464?

Ans. 441.2164+.

5. Find a mean proportional between $\frac{1}{2}\frac{6}{7}$ and $\frac{4}{3}$.

Ans. $\frac{8}{9}$.

ALGEBRAIC FORM.

Let a represent the first term, l the last, r the ratio, n the number of terms, and s the sum.

Then $s = a + ar + ar^2 + ar^3 + ar^4 + \&c. \dots ar^{n-2} + ar^{n-1}$.

Multiplying the whole equation by r , we obtain

$$rs = ar + ar^2 + ar^3 + ar^4 + ar^5 + \&c. \dots ar^{n-1} + ar^n.$$

But $s = a + ar + ar^2 + ar^3 + ar^4 + ar^5 + \&c. \dots ar^{n-1}$.

Subtracting, we obtain

$$rs - s = s(r-1) = ar^n - a, \text{ and therefore}$$

$$s = \frac{ar^n - a}{r-1} \dots (1.)$$

But we found the last term of the series to be ar^{n-1} , calling this l , we have from (1.) $s = \frac{rl - a}{r-1} \dots (2.)$

If r is a fraction, r^n and ar^n decrease as n increases, as already shown under the head of fractions, so that if n become indefinitely great, ar^n will become unassignably small, compared with any finite quantity, and may be reckoned as nothing. In this case (1) will become $s = \frac{-a}{r-1} = \frac{a}{1-r} \dots (3.)$

By this formula we can find the sum of any infinite series so closely as to differ from the actual sum by an amount less than any assignable quantity. This is called the *limit*, an expression more strictly correct than *the sum*.

From the formula $s = \frac{rl - a}{r-1}$, any three of the quantities a , r , l , s being given, the fourth can be found.

Let it be required to find the sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \&c.$, to infinity.

Here $a=1$ and $r=\frac{1}{2}$. $\therefore s = 1 - \frac{1}{2} = \frac{1}{\frac{1}{2}} = 1 \times 2 = 2$. Therefore, 2 is the number to which the sum of the series continually approaches, by the increase of the number of its terms, and is the limit from which it may be made to differ by a quantity less than any assignable quantity, and is also the limit beyond which it can never pass.

By adding the first two terms, we find $1 + \frac{1}{2} = \frac{3}{2} = 2 - \frac{1}{2} = 1\frac{1}{2}$.

By adding the first three terms, we find $\frac{3}{2} + \frac{1}{4} = \frac{7}{4} = 2 - \frac{1}{4} = 1\frac{3}{4}$.

By adding the first four terms, we find $\frac{7}{4} + \frac{1}{8} = \frac{15}{8} = 2 - \frac{1}{8} = 1\frac{7}{8}$.

By adding the first five terms, we find $\frac{15}{8} + \frac{1}{16} = \frac{31}{16} = 2 - \frac{1}{16} = 1\frac{15}{16}$.

By adding the first six terms, we find $\frac{31}{16} + \frac{1}{32} = \frac{63}{32} = 2 - \frac{1}{32} = 1\frac{31}{32}$.

It will be observed here that the difference from 2 is continually decreasing. The next term would differ from 2 by $\frac{1}{64}$, and the next by $\frac{1}{128}$, &c., &c. Thus, when the series is carried to infinity, 2 may be taken as the sum, because it differs from the actual sum by a quantity less than any assignable quantity.

EXAMPLES.

To find the sum of the first twelve terms of the series $1 + 3 + 9 + 27 + \&c.$:

Here $a=1$, $r=3$,

$$\text{And } s = \frac{r^n - a}{r - 1} = \frac{3^{12} - 1}{3 - 1} = \frac{3 \times 177147 - 1}{2} = 265720.$$

To find the sum of the series $1, -3, 9, -27, \&c.$, to twelve terms,

$$s = \frac{-3 \times \frac{11}{-3} - 1}{-3 - 1} = \frac{-3 \times -177147 - 1}{-4} = -132860.$$

In the case of infinite series, if a is sought, s and r being given, we have from (3) $a = s(1 - r)$, and if r is sought, a and s being given, we have $r = \frac{s - a}{s}$ or $1 - \frac{a}{s}$.

EXERCISES.

1. Find the sum of the series $2, 6, 18, 54, \&c.$, to 8 terms.

Ans. 6560.

2. Find the sum of the infinite series $\frac{1}{2} - \frac{1}{6} + \frac{1}{12} - \frac{1}{24}$. Observe here $r = -\frac{1}{2}$.

Ans. $\frac{2}{9}$.

3. What is the sum of the series $1, \frac{1}{3}, \frac{1}{9}, \&c.$, to infinity?

Ans. $\frac{3}{2}$.

4. Find the sum of the infinite series $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \&c.$

Ans. 3.

5. What is the sum of nine terms of the series $5, 20, 80, \&c.$?

Ans. 436905.

6. Find the sum of $\sqrt{\frac{1}{2}} + \frac{1}{2} + \sqrt{\frac{1}{8}} + \&c.$, to infinity.

Ans. $\sqrt{\frac{1}{2}} - 1$.

7. What is the limit to which the sum of the infinite series $\frac{2}{4}, \frac{1}{2}, \frac{1}{3}, \frac{2}{9}, \&c.$, continually approaches?

Ans. $\frac{9}{4}$.

8. What is the sum of ten terms of the series 4, 12, 36, &c. ?

Ans. 118096.

9. Insert three terms between 39 and 3159, so that the whole shall be a series of continual proportionals.

Ans. 117, .351 and 1053.

10. Insert four terms between $\frac{1}{9}$ and 27, so that the whole shall form a series of continual proportionals.

Ans. $\frac{1}{3}$, 1, 3, 9.

11. The sum of a series of continual proportionals is $10\frac{1}{3}$, the first term $3\frac{3}{8}$; what is the ratio ?

Ans. $\frac{2}{3}$.

12. The limit of an infinite series is 70, the ratio $\frac{3}{7}$; what is the first term ?

Ans. 40.

ANNUITIES.

The word *Annuity* originally denoted a sum paid *annually*, and though such payments are often made half-yearly, quarterly, &c., still the term is applied, and quite properly, because the calculations are made for the year, at what time soever the disbursements may be made.

By the term *annuities certain* is indicated such as have a fixed time for their commencement and termination.

By the term *annuities contingent* is meant annuities, the commencement or termination of which depends on some contingent event, most commonly the death of some individual or individuals.

By the term annuity *in reversion* or *deferred*, is meant that the person entitled to it cannot enter on the enjoyment of it till after the lapse of some specified time, or the occurrence of some event, generally the death of some person or persons.

An annuity *in perpetuity* is one that "lasts for ever," and therefore is a species of hereditary property.

An annuity *forborne* is one the payments of which have not been made when due, but have been allowed to accumulate.

By the *amount* of an annuity is meant the sum that the principal and compound interest will amount to in a given time.

The *present worth* of an annuity is the sum to which it would amount, at compound interest, at the end of a given time, if forborne for that time.

Tables have been constructed showing the present and final values per unit for different periods, by which the value of any annuity may be found according to the following

R U L E S .

To find either the amount or the present value of an annuity,—

Multiply the value of the unit, as found in the tables, by the number denoting the annuity.

If the annuity be in perpetuity,—

Divide the annuity by the number denoting the interest of the unit for one year.

If the annuity be in reversion,—

Find the value of the unit up to the date of commencement, and also to the date of termination, and multiply their difference by the number denoting the annuity.

To find the annuity, the time, rate and present worth being given.

Divide the present worth by the worth of the unit.

Tables are appended varying from 20 to 50 years.

E X A M P L E S .

To find what an annuity of \$400 will amount to in 30 years, at $3\frac{1}{2}$ per cent.

We find by the tables the amount of \$1, for 30 years, to be \$51.622677, which multiplied by 400 gives \$20649.07 nearly.

To find the present worth of an annuity of \$100 for 45 years, at 3 per cent.

By the table we find \$24.518713, and this multiplied by 100 gives \$2451.88.

To find the present worth of a property on lease for ever, yielding \$600, at $3\frac{1}{2}$ per cent.

The rate per unit for one year is .035, and 600 divided by this gives \$17142.86.

To find the present worth of an annuity on a lease in reversion, to commence at the end of three years and to last for 5, at $3\frac{1}{2}$ per cent.

By the table we find the rate per unit for 3 years to be \$2.801637, and for 8 years, the time the lease expires, \$6.873956; the difference is \$4.072319, which, multiplied by 300, gives \$1221.70.

Given \$207.90, the present worth of an annuity continued for 4 years, at 3 per cent., to find the annuity.

By the tables, the value for \$1 is \$3.717098, and \$207.90, divided by this, gives \$55.93.

TABLE,

SHOWING THE AMOUNT OF AN ANNUITY OF ONE DOLLAR PER ANNUM, IMPROVED
AT COMPOUND INTEREST FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000	1.000 000
2	2.030 000	2.035 000	2.040 000	2.050 000	2.060 000	2.070 000
3	3.090 900	3.106 225	3.121 600	3.152 500	3.183 600	3.214 900
4	4.183 627	4.214 943	4.246 464	4.310 125	4.374 616	4.439 943
5	5.309 136	5.362 466	5.416 323	5.525 631	5.637 093	5.750 739
6	6.468 410	6.550 152	6.632 975	6.801 913	6.975 319	7.153 291
7	7.662 462	7.779 408	7.898 294	8.142 008	8.393 838	8.654 021
8	8.892 336	9.051 687	9.214 226	9.549 109	9.897 468	10.259 803
9	10.159 106	10.368 496	10.582 795	11.026 564	11.491 316	11.977 989
10	11.463 879	11.731 393	12.006 107	12.577 893	13.180 795	13.816 448
11	12.807 796	13.141 992	13.486 351	14.206 787	14.971 643	15.783 599
12	14.192 030	14.601 962	15.025 805	15.917 127	16.869 941	17.888 451
13	15.617 790	16.113 030	16.626 838	17.712 983	18.882 138	20.140 643
14	17.086 324	17.676 986	18.291 911	19.598 632	21.015 066	22.550 488
15	18.598 914	19.295 681	20.023 588	21.578 564	23.275 970	25.129 022
16	20.156 881	20.971 030	21.824 531	23.657 492	25.670 528	27.888 054
17	21.761 588	22.705 016	23.697 512	25.840 366	28.212 880	30.840 217
18	23.414 435	24.499 691	25.645 413	28.132 385	30.905 653	33.999 033
19	25.116 868	26.357 180	27.671 229	30.539 004	33.759 992	37.378 965
20	26.870 374	28.279 682	29.778 079	33.065 954	36.785 591	40.995 492
21	28.676 486	30.269 471	31.969 202	35.719 252	39.992 727	44.865 177
22	30.536 780	32.328 902	34.247 970	38.505 214	43.392 290	49.005 739
23	32.452 884	34.460 414	36.617 889	41.430 475	46.995 828	53.436 141
24	34.426 470	36.666 528	39.082 604	44.501 999	50.815 577	58.176 671
25	36.459 264	38.949 857	41.645 908	47.727 099	54.864 512	63.249 030
26	38.553 042	41.313 102	44.311 745	51.113 454	59.156 383	68.676 470
27	40.709 634	42.759 060	47.084 214	54.669 126	63.705 766	74.483 823
28	42.930 923	46.290 627	49.967 583	58.402 583	68.528 112	80.697 691
29	45.218 850	48.910 799	52.966 286	62.322 712	73.639 798	87.346 529
30	47.575 416	51.622 677	56.084 938	66.438 848	79.058 186	94.460 786
31	50.002 678	54.429 471	59.328 335	70.760 790	84.801 677	102.073 041
32	52.502 759	57.334 502	62.701 469	75.298 829	90.889 778	110.218 154
33	55.077 841	60.341 210	66.209 527	80.063 771	97.343 165	118.933 425
34	57.730 177	63.453 152	69.857 909	85.066 959	104.183 755	128.258 765
35	60.462 082	66.674 013	73.652 225	90.320 307	111.434 780	138.236 878
36	63.271 944	70.007 603	77.598 314	95.836 323	119.120 867	148.913 460
37	66.174 223	73.457 869	81.702 246	101.628 139	127.268 119	160.337 400
38	69.159 449	77.028 895	85.970 336	107.709 546	135.904 206	172.561 020
39	72.234 233	80.724 906	90.409 150	114.095 023	145.058 458	185.640 292
40	75.401 260	84.550 278	95.025 516	120.799 774	154.761 966	199.635 112
41	78.663 298	88.509 537	99.826 536	127.839 763	165.047 684	214.609 570
42	82.023 196	92.607 371	104.819 598	135.231 751	175.950 645	230.632 240
43	85.483 892	96.848 629	110.012 382	142.993 339	187.507 577	247.776 496
44	89.048 409	101.238 331	115.412 877	151.143 006	199.758 032	266.120 851
45	92.719 861	105.781 673	121.029 392	159.700 156	212.743 514	285.749 311
46	96.501 457	110.484 031	126.870 568	168.685 164	226.508 125	306.751 763
47	100.396 501	115.350 973	132.945 390	178.119 422	241.098 612	329.224 386
48	104.408 396	120.388 297	139.263 206	188.025 393	256.564 529	353.270 093
49	108.540 648	125.601 846	145.833 734	198.426 663	272.958 401	378.999 000
50	112.796 867	130.999 910	152.667 084	209.347 976	290.335 905	406.528 929

TABLE,

SHOWING THE PRESENT WORTH OF AN ANNUITY OF ONE DOLLAR PER ANNUM, TO
CONTINUE FOR ANY NUMBER OF YEARS NOT EXCEEDING FIFTY.

Years.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	Years.
1	0.970 874	0.966 184	0.961 538	0.952 381	0.943 396	0.934 579	1
2	1.913 470	1.899 694	1.886 095	1.859 410	1.833 393	1.808 017	2
3	2.828 611	2.801 637	2.775 091	2.723 248	2.673 012	2.624 314	3
4	3.717 098	3.673 079	3.629 895	3.545 951	3.465 106	3.387 209	4
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195	5
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537	6
7	6.230 283	6.114 544	6.002 055	5.786 373	5.582 381	5.389 286	7
8	7.019 692	6.873 956	6.732 745	6.463 213	6.209 744	5.971 295	8
9	7.786 109	7.607 687	7.435 332	7.107 822	6.801 692	6.515 228	9
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577	10
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 669	11
12	9.954 004	9.663 334	9.385 074	8.863 252	8.383 844	7.942 671	12
13	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.357 635	13
14	11.296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 452	14
15	11.937 935	11.517 411	11.118 387	10.379 658	9.712 249	9.107 898	15
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632	16
17	13.166 118	12.651 321	12.165 669	11.274 066	10.477 260	9.763 206	17
18	13.753 513	13.189 682	12.659 297	11.689 587	10.827 603	10.059 070	18
19	14.323 799	13.709 837	13.133 939	12.085 321	11.158 116	10.335 578	19
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 421	10.593 997	20
21	15.415 024	14.697 974	14.029 160	12.821 153	11.764 077	10.835 527	21
22	15.936 917	15.167 125	14.451 115	13.163 003	12.041 582	11.061 241	22
23	16.443 608	15.620 410	14.856 842	13.488 574	12.303 379	11.272 187	23
24	16.935 542	16.058 368	15.246 963	13.798 642	12.550 358	11.469 334	24
25	17.413 148	16.481 515	15.622 080	14.093 945	12.783 356	11.653 583	25
26	17.876 842	16.890 352	15.982 769	14.275 185	13.003 166	11.825 779	26
27	18.327 031	17.285 365	16.329 586	14.643 034	13.210 534	11.986 709	27
28	18.764 108	17.667 019	16.663 063	14.898 127	13.406 164	12.137 111	28
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674	29
30	19.600 441	18.392 045	17.292 038	15.372 451	13.764 831	12.409 041	30
31	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814	31
32	20.338 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555	32
33	20.765 792	19.390 208	18.147 646	16.002 549	14.230 230	12.753 790	33
34	21.131 837	19.700 684	18.411 198	16.192 204	14.368 141	12.854 009	34
35	21.487 220	20.000 661	18.664 613	16.374 194	14.498 246	12.947 672	35
36	21.832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208	36
37	22.167 235	20.570 525	19.142 579	16.711 287	14.736 780	13.117 017	37
38	22.492 462	20.841 087	19.367 864	16.867 893	14.846 019	13.193 473	38
39	22.808 215	21.102 500	19.584 485	17.017 041	14.949 075	13.264 928	39
40	23.114 772	21.355 072	19.792 774	17.159 086	15.046 297	13.331 709	40
41	23.412 400	21.599 194	19.993 052	17.294 368	15.138 016	13.394 120	41
42	23.701 359	21.834 883	20.185 627	17.423 208	15.224 543	13.452 449	42
43	23.981 902	22.062 689	20.370 795	17.545 912	15.306 173	13.506 962	43
44	24.254 274	22.282 791	20.548 841	17.662 773	15.383 182	13.557 908	44
45	24.518 713	22.495 450	20.720 040	17.774 070	15.455 832	13.605 522	45
46	24.775 449	22.700 918	20.884 654	17.880 067	15.524 370	13.650 020	46
47	25.024 708	22.899 438	21.042 936	17.981 016	15.589 028	13.691 608	47
48	25.266 707	23.091 244	21.195 131	18.077 158	15.650 027	13.730 474	48
49	25.501 657	23.276 564	21.341 472	18.168 722	15.707 572	13.766 799	49
50	25.729 764	23.455 618	21.482 185	18.255 925	15.761 861	13.800 746	50

PARTNERSHIP SETTLEMENTS.

The circumstances under which partnerships are formed, the conditions on which they are made, and the causes that lead to their dissolution, are so varied that it is impossible to do more than give general directions deduced from the cases of most common occurrence. In forming a partnership, the great requisite is to have the terms of agreement expressed in the most clear and yet concise language possible, setting forth the sum invested by each, the duration of partnership, the share of gains or losses that fall to each, the sum that each may draw from time to time for private purposes, and any other circumstances arising out of the peculiarities of each case. The ease and satisfaction of making an equitable settlement, in case of dissolution, depends mainly on the clearness of the original agreement, and hence the necessity for its being distinct and explicit. Even when no dissolution is contemplated, settlements should be frequently made, in order that the parties may know how they stand to each other, and how the business is succeeding. This is of great importance in preserving unanimity and securing vigour and regularity in all the transactions of a mercantile house.

A dissolution may take place from various causes. If the partnership is formed for a term of years, the expiration of those years necessarily involves either a dissolution or a new agreement. The death of one of the partners may or may not cause dissolution, for the deceased partner may have, by his will, left his share in the business to his son, or some other relative or friend. In no case, however, can an equitable settlement be made, except by the mutual consent of the parties, or else in exact accordance with the terms of agreement. It is also necessary that when a dissolution takes place public notice should be given thereof, in order that all parties having dealings with the firm may be apprized of the change, and have their accounts arranged. For the same reason, it is necessary that some one of the partners, or some trustworthy accountant appointed by them, should be authorized to collect all debts due to the firm, and pay all accounts owing by it.

Partnerships are sometimes formed for a specific speculation, and therefore, of course, cease with the completion of the transaction, and a settlement must necessarily be then made. No matter for what

time the partnership has been made, any partner is at liberty, at any time, to withdraw, on showing sufficient cause and giving proper notice. This is a just provision, for the circumstances of any partner may so change, from various causes, as to make it undesirable for him to remain in the business. If one partner is deputed to settle the accounts of the house, it would be reckoned fraudulent for any other partner to collect any moneys due, except that on receipt of them he hands them directly over to the person so deputed.

The resources and liabilities, with the net investment on commencing business, being given, to find the net gain or loss.

1. W. Smith and R. Evans are partners in business, and invested when commencing \$1000 each. On dissolving the partnership, the assets and liabilities are as follows:—Merchandise valued at \$1295; cash, \$344; notes against sundry individuals, \$790; W. H. Monroe owes on account \$86.40; E. R. Carpenter owes \$132.85, and C. F. Musgrove owes \$67.50. They owe on sundry notes, as per bill book, \$212.40; E. G. Conklin, on account, \$29.45, and H. C. Wright, on account, \$41.30. What has been the net gain?

SOLUTION.

<i>Assets.</i>		<i>Liabilities.</i>	
Merchandise on hand...	\$1295.00	Bills Payable.....	\$212.40
Cash on hand	344.00	Amt. due E. G. Conklin.	29.45
Bills Receivable.....	790.00	Amt. due H. C. Wright.	41.30
Amt. due from W. H. Monroe	86.40	W. Smith's investment...	1000.00
Amt. due from E. R. Carpenter.....	132.85	R. Evan's investment....	1000.00
Amt. due from C. F. Musgrove	67.50		
			\$2283.15
Total amount Assets....	\$2715.75		
“ “ Liabilities,	2283.15		
Net gain.....	\$432.60		

RULE.

Find the sum of the assets and liabilities; from the assets subtract the liabilities, (including the net amount invested) and the difference will be the net gain; or, if the liabilities be the larger, subtract the assets from the liabilities, and the difference will be the net loss.

2. Harvey Miller and James Carey are partners in a dry goods business; Harvey Miller investing \$1400, and James Carey \$1250. When closing the books, they have on hand—cash, \$1125.30; merchandise, as per inventory book, \$1855.75; amount deposited in Bank of Toronto, \$1200; amount invested in oil lands, \$963; a site of land for building purposes, valued at \$1600; Adam Dudgeon owes them, on account, \$104.92; William Fleming owes \$246.80; a note against Alfred Mills for \$69.43, and a due bill for \$30, drawn by James Laing. They owe W. S. Hope & Co., on account, \$849.21; R. J. King & Co., \$608.12, and on notes, \$1326.14. What has been the net gain or loss? Ans. \$1759.73 gain.

3. James Henning and Adam Manning have formed a co-partnership for the purpose of conducting a general dry goods and grocery business, each to share gains or losses equally.

At the end of one year they close the books, having \$1280 worth of merchandise on hand; cash, \$714.27; Royal Bank stock, \$500; deposited in Royal Bank, \$320.60; store and fixtures valued at \$3100; amount due on notes and book accounts, \$3471.49. The firm owes on notes \$3400, and on open accounts \$747.10.

James Henning invested \$1200, and Adam Manning, \$1000; what is each partner's interest in the business at closing?

Ans. James Henning's interest, \$2719.63; Adam Manning's interest, \$2519.63.

NOTE.—Where the interest of each partner at closing is required, the gain or loss is first found, as in former examples, then the share of gain or loss is added to or subtracted from each partner's investment, and the sum, or difference, is the interest of each partner. If a partner has withdrawn anything from the business, the amount thus withdrawn must be deducted from the sum of his investment, *plus* his share of the gain, or *minus* his share of the loss, and the remainder will be his net capital or interest.

4. F. A. Clarke, W. H. Marsden, and J. M. Musgrove, are conducting business in partnership; F. A. Clarke is to be $\frac{1}{2}$ gain or loss, W. H. Marsden and J. M. Musgrove, each $\frac{1}{4}$.

On dissolving the partnership, they have cash on hand \$712.90; merchandise, as per Inventory Book, \$4360; bills receivable, as per Bill Book, \$1450.75; amount deposited in Bank of Montreal, \$3475; merchandise shipped to Montreal, to be sold on own account and risk, valued at \$995; debts due from individuals on book account, \$2644.67. They owe on notes \$3760, and to Manning & Munson, \$1312.60.

F. A. Clarke invested \$5750, and has drawn out \$875; W. H. Marsden invested \$2500, and has drawn out \$500; J. M. Musgrove invested \$3000, and has drawn out \$750. What has been the net gain or loss, and what is each partner's interest in the business?

Ans. Net loss, \$559.28; F. A. Clarke's interest, \$4595.36; W. H. Marsden's interest, \$1860.18; J. M. Musgrove's interest, \$2110.18.

NOTE.—In this and succeeding examples, net interest is to be allowed on investment, or charged on amounts withdrawn, unless so specified.

5. A, B, and C are partners. The gains and losses are to be shared as follows: A, $\frac{5}{12}$; B, $\frac{3}{12}$; and C, $\frac{4}{12}$. A invested \$3000, and has withdrawn \$2,500, with the consent of B and C, upon which no interest is to be charged; B invested \$2700, and has withdrawn \$1150; C invested \$2500, and has withdrawn \$420. After doing business 14 months, C retires. Their assets consist of bills receivable, \$2937.20; merchandise, \$1970; cash, \$1243.80; 50 shares of the Canada Permanent Building and Savings' Society Stock, the par value of which is \$50 per share; cash deposited in the Ontario Bank, \$1850; store and furniture, \$3130; amount due from W. Smith, \$360.80; G. S. Brown, \$246.40; and E. R. Carpenter, \$97.12. Their liabilities are as follows: amount due Samuel Harris, \$1675; unpaid on store and furniture, \$935; and notes unredeemed, \$3388.76. C, in retiring, agrees to allow the firm 10 per cent. advance on the Savings' Bank stock. What is the amount due C, and what is A's, and what is B's interest in the business?

Ans. Due C, \$815.52; A's interest, \$2356.90; B's interest, \$2664.14.

6. E, F, G and H are partners in business, each to share $\frac{1}{4}$ of profits and losses. The business is carried on for one year, when E and F purchase from G and H their interest in the business, allowing each \$100 for his good will. Upon examination, their resources are found to be as follows: cash deposited in Quebec Bank, \$3645; cash on hand, \$1422; bills receivable, \$1685; bonds and mortgages, \$2746, upon which there is interest due \$106; Royal Canadian Bank stock, \$1000; Quebec Bank stock, \$500; store and fixtures, \$3500; house and lot, \$1800; span of horses, carriages, harness, &c., \$495; outstanding book debts due the firm, \$4780. Their liabilities are: notes payable, \$2345, upon which there is interest due \$57; due on book debts, \$1560. E invested \$5000; F, \$4500;

G, \$4000; and H, \$3000. E has drawn from the business \$1200, upon which he owes interest \$32; F has drawn \$1000—owes interest \$24.50; G has drawn \$950—owes interest \$12; and H has drawn nothing. In the settlement a discount of 10 per cent., for bad debts, is allowed, on the book debts due the firm and on the bills receivable. G takes the Royal Bank stock, allowing on the same a premium of 5 per cent.; and H takes the Quebec Bank stock, at a premium of 8 per cent; E and F take the assets and assumes the liabilities, as above stated. What has been the net gain or loss, the balances due G and H, and what are E and F each worth after the settlement?

Ans. Due G, \$3057.75; due H, \$3529.75; E's net capital, \$4637.75; F's net capital, \$4345.25.

7. H. C. Wright, W. S. Samuels, and E. P. Hall, are doing business together—H. C. W. to have $\frac{1}{2}$ gain or loss; W. S. S. and E. P. H. each $\frac{1}{4}$. After doing business one year, W. S. S. and E. P. H. retire from the firm. On closing the books and taking stock, the following is found to be the result: merchandise on hand, \$3216.50; cash deposited in Quebec Bank, \$1627.35; cash in till, \$134.16; bills receivable, \$940.60; G. Brown owes, on account, \$112.40; Thos. A. Bryce owes \$94.12; W. McKee owes \$143.95; J. Anderson owes \$54.20; R. H. Hill owes \$43.60; and S. Graham owes \$260.13. They owe on notes not redeemed \$1864; H. T. Collins, on account, \$124.45; and W. F. Curtis, \$79.40. H. C. Wright invested \$3200, and has drawn from the business \$350. W. S. Samuels invested \$2455, and has drawn \$140; E. P. Hall invested \$2100, and has drawn \$2000. A discount of 10 per cent. is to be allowed on the bills receivable and book accounts due the firm, for bad debts. H. C. Wright takes the assets and assumes the liabilities, as above stated. What has been the net gain or loss, and what does H. C. Wright pay W. S. Samuels and E. P. Hall on retiring?

Ans. Net loss, \$970.74; H. C. W. pays W. S. S., \$2072.31 $\frac{1}{2}$; and E. P. Hall pays H. C. W. \$142.68 $\frac{1}{2}$.

8. T. P. Wolfe, J. P. Towler and E. R. Carpenter have been doing business in partnership, sharing the gains and losses equally. After dissolution and settlement of all their liabilities they make a division of the remaining effects without regard to the proper proportion each should take. The following is the result according to their ledger:—T. P. Wolfe invested \$3495, and has drawn \$2941;

J. P. Towler invested \$2900, and has drawn \$2200 ; E. R. Carpenter invested \$3150, and has drawn \$3000. How will the partners settle with each other ?

Ans. E. R. Carpenter pays T. P. Wolfe \$86, and J. P. Towler \$232.

9. I, J, K, L and M have entered into co-partnership, agreeing to share the gains and losses in the following proportion :—I, $\frac{4}{15}$; J, $\frac{3}{15}$; K, $\frac{2}{15}$; L, $\frac{5}{15}$; and M, $\frac{1}{15}$. When dissolving the partnership the resources consisted of cash, \$4700 ; merchandise, \$9855 ; notes on hand, \$7680 ; debentures of the city of Toronto, valued at \$6780, on which there is interest due, \$123 ; horses, waggons, &c., \$1280 ; Montreal bank stock, \$5000 ; Ontario bank stock, \$5000 ; mortgages and bonds, \$3600 ; interest due on mortgages, \$345.80 ; store and fixtures, \$8000 ; amount due from W. P. Campbell & Co., \$2418 ; due from R. B. Smith, \$712.60 ; due from J. W. Jones, \$1000. The liabilities are :—Mortgage on store and fixtures, \$5000 ; interest due on the same, \$212.25 ; due the estate of R. M. Evans, \$14675 ; notes and acceptances, \$11940, on which interest is due, \$85 ; sundry other book debts, \$7500 ; I invested \$7800, interest on his investment to date of dissolution, \$702 ; J invested \$6400, interest on investment, \$576 ; K invested \$6100, interest on investment, \$549 ; L invested \$5800, interest on investment, \$522 ; M invested \$5000, interest on investment, \$450. I has withdrawn from the firm at different times, \$2425, upon which the interest calculated to time of dissolution is \$183.40 ; J has drawn \$2960, interest, \$267.85 ; K has drawn \$1850, interest \$87.30 ; L has drawn \$3000, interest, \$460 ; M has drawn \$895, interest, \$63.45. What is the net gain or loss of each partner, and what is the net capital of each partner ?

Ans. I's net loss, \$1233.29 ; I's net capital, \$4660.31. J's net loss, \$924.97 ; J's net capital, \$2823.18. K's net loss, \$616.65 ; K's net capital, \$4095.05. L's net loss, \$1541.62 ; L's net capital, \$1320.38. M's net loss, \$308.32 ; M's net capital, \$4183.23.

10. A, B, C and D are partners. At the time of dissolution, and after the liabilities are all cancelled, they make a division of the effects, and upon examination of their ledger it shows the following result :—A has drawn from the business \$3465, and invested on commencement of business, \$4240 ; B has drawn \$4595, and invested \$3800 ; C has drawn \$5000, and invested \$3200 ; D has drawn \$2200, and invested \$2800. The profit or loss was to be divided in

proportion to the original investment. What has been each partner's gain or loss, and how do the partners settle with each other?

Ans. A's net gain, \$368.43; B's net gain, \$330.20; C's net gain, \$278.06; D's net gain, \$243.31. B has to pay in \$464.80; C has to pay in \$1521.94. A receives \$1143.43; D receives \$843.31.

11. Three mechanics, A. W. Smith, James Walker and P. Ranton, are equal partners in their business, with the understanding that each is to be charged \$1.25 per day for lost time. At the close of their business, in the settlement it was found that A. W. Smith had lost 14 days, James Walker 21 days, and P. Ranton 30 days. How shall the partners properly adjust the matter between them?

Ans. P. Ranton pays A. W. Smith, \$9.58 $\frac{1}{3}$, and James Walker, 83 $\frac{1}{3}$ cents.

12. There are 5 mechanics on a certain piece of work in the following proportions:—A is $\frac{3}{20}$; B, $\frac{2}{20}$; C, $\frac{4}{20}$; D, $\frac{6}{20}$, and E, $\frac{5}{20}$. A is to pay \$1.25 per day for all lost time; B, \$1; C, \$1.50; D, \$1.75, and E, \$1.62 $\frac{1}{2}$. At settlement it is found that A has lost 24; B, 19; C, 34; D, 12; and E, 45 days. They receive in payment for their joint work, \$2500. What is each partner's share of this amount according to the above regulations?

Ans. A's share, \$374.12; B's, \$250.41; C's, \$487.83; D's, \$787.24; E's, \$600.40.

13. A. B. Smith and T. C. Musgrove commenced business in partnership January 1st, 1864. A. B. Smith invested, on commencement, \$9000; May 1st, \$2400; June 1st, he drew out \$1800; September 1st, \$2000, and October 1st, he invested \$800 more. T. C. Musgrove invested on commencing, \$3000; March 1st, he drew out \$1600; May 1st, \$1200; June 1st, he invested \$1500 more, and October 1st, \$8000 more. At the time of settlement, on the 31st December, 1864, their merchandise account was—*Dr.* \$32000; *Cr.* \$27000; balance of merchandise on hand, as per inventory, \$10500; cash on hand, \$4900; bills receivable, \$12400; R. Draper owes on account, \$2450. They owe on their notes, \$1890, and G. Roe on account, \$840. Their profit and loss account is, *Dr.* \$866; *Cr.* \$1520. Expense account is, *Dr.* \$2420. Commission account is, *Cr.* \$2760. Interest account is *Dr.* \$480; *Cr.* \$950. The gain or loss is to be divided in proportion to each partner's capital, and in proportion to the time it was invested. Required each partner's share of the gain or loss, the net balance

due each, and a ledger specification exhibiting the closing of all the accounts, and the balance sheet.

Ans. A. B. S.'s net gain, \$6671.73; his net balance, \$15071.73.

T. C. M.'s net gain, \$2748.27; his net balance, \$12448.27.

PROPERTIES OF NUMBERS.

The term *Integer*, or *Whole Number*, is used in contradistinction to the term *Fraction*. All numbers expressed by the natural series 1, 2, 3...10...20...100, &c., are called integers, so that 3 and 4 are integers, but $\frac{3}{4}$ is a fraction.

All numbers in the natural series 1, 2, 3, &c., that can be resolved into factors, are called *Composite*, while those that cannot be so resolved are called *Prime*. Since $4=2 \times 2$, it is called composite, and so 6, 8, 9, 10, &c., but 1, 2, 3, 5, 7, 11, &c., are called prime because they cannot be resolved into factors. Thus, 11 can only be resolved into 11×1 , or 1×11 , and these are not factors in the strict meaning of the word.

A *Prime Factor* is a prime number, which is a factor of a composite number. The factors of 10 are 2 and 5, both prime numbers.

A composite number may have composite factors, as 36, which has 4 and 9 as factors, and both of these are composite.

When any number will divide two or more others, it is called a *Common Factor*. Thus, 3 is called a common factor of 6, 9, 12, 15, &c.

Numbers that have no common factor, as 4, 5, 9, are said to be *prime to each other*.

To resolve a composite number into its prime factors, *divide it by the least possible factor that it contains, and repeat the process till a prime number is obtained*.

EXAMPLES.

$$\begin{array}{r} 2)96 \\ \hline \end{array}$$

$$\begin{array}{r} 2)48 \\ \hline \end{array}$$

$$\begin{array}{r} 2)24 \\ \hline \end{array}$$

$$\begin{array}{r} 2)12 \\ \hline \end{array}$$

$$\begin{array}{r} 2)6 \\ \hline \end{array}$$

$$3$$

so that the prime factors of 96 are $2 \times 2 \times 2 \times 2 \times 2 \times 3$.

Also, because $5 \times 7 \times 11 = 385$, we see that 5, 7 and 11 are the prime factors of 385.

EXERCISES.

1. What are the prime factors of 2310? Ans. 2, 3, 5, 7, 11.
2. What are the prime factors of 1764? Ans. 2, 2, 3, 3, 7, 7.
3. What are the prime factors of 180642? Ans. 2, 3, 7, 11, 17, 23.
4. What are the prime factors of 95? Ans. 5, 19.
5. What are the prime factors of 51? Ans. 3, 17.
6. What are the prime factors of 99? Ans. 3, 3, 11.
7. What are the prime factors of 651? Ans. 3, 7, 31.
8. What are the prime factors of 362880? Ans. 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 5, 7.
9. What factors are common to 84, 105, and 147? Ans. 3, 7.
10. What are the prime factors of 308? Ans. 4, 7, 11.

Whether a number is prime or composite can only be found by trial.

The only even prime number is 2; for 4, 6, 8, 10, &c., are all multiples of 2.

The only prime number ending in the digit 5 is 5 units, and all other numbers ending in either 5 or 0 are multiples of 5.

ADDITIONAL EXERCISES.

11. Is 101 prime or composite? Ans. Prime.
12. Is 198 prime or composite? Ans. It has the factors 2, 3, 3, 11.
13. Is 171 prime or composite? Ans. It has the factors 3, 3, 19.
14. Is 473 prime or composite? Ans. Prime.
15. Is 477 prime or composite? Ans. Composite.
16. Is 549353259 prime or composite? Ans. Composite.
17. Is 674041 prime or composite? Ans. Composite.
18. Is 199 prime or composite? Ans. Prime.
19. What are the prime factors of 210? Ans. 5, 6, 7.
20. What are the prime factors of 51051? Ans. 3, 7, 11, 13, 17.

NOTE.—We have thought it sufficient under this head to give only the leading and most useful principles.

LOGARITHMS.

A logarithm is the index of the power of any number which that index denotes.

Thus $9=3^2$, where the index 2 is the logarithm of 9. Taking 3 as a common root, we have

$$\begin{array}{ll} 3=3^1 & 1=3^0 \\ 9=3^2 & \frac{1}{3}=3^{-1} \\ 27=3^3 & \frac{1}{9}=3^{-2} \\ 81=3^4 & \frac{1}{27}=3^{-3} \end{array}$$

These indices may, however, be the logarithms of any root. Thus :

$$\begin{array}{ll} 4=4^1 & 1=4^0 \\ 16=4^2 & \frac{1}{4}=4^{-1} \\ 64=4^3 & \frac{1}{16}=4^{-2} \end{array}$$

Also, $8=4^{\frac{3}{2}}$, which means that 8 is the second root of the third power of 4. The third power of 4, or $4 \times 4 \times 4=64$, and 8 is the second root of 64, for $8 \times 8=64$. Again, since $8=2^3$, and $16=2^4$, it follows that the index of any number between 8 and 16, when 2 is the root, will be 3, *plus* some fraction, that is, the index will be some number greater than 3, and less than 4.

If 4 be the root, then, as above, $8=4^{\frac{3}{2}}$, where the index is $\frac{3}{2}$, or 1.5, greater than 1, but less than 2. In this case, the index expressed decimally is a terminator, but even when the decimal does not terminate, it may be found approximately with sufficient accuracy. From this it is obvious that logarithms may be calculated for all numbers. The root from which logarithms are calculated is called the *Base*. The bases we have taken for illustration are 2, 3, and 4, but any number may be assumed as base. The base of the original system, invented by Baron Napier, and hence called *Napierian logarithms*, was 2.718+, but the base that has been found most convenient is 10, as it falls in so exactly with the decimal notation. These, by way of distinction, are called *Common Logarithms*. The common logarithm, then, of a number, is the index of the power of 10, which is equal to that number.

Hence starting from the base 10, we have the following ascending and descending scales :

Power.	No.	Log.	Power.	No.	Log.
$10^0=1$	0		$10^{-1}=\frac{1}{10}$		.1
$10^1=10$	1		$10^{-2}=\frac{1}{100}$		.01
$10^2=100$	2		$10^{-3}=\frac{1}{1000}$		.001
$10^3=1000$	3		$10^{-4}=\frac{1}{10000}$		.0001
$10^4=10000$	...4				

Now we must observe that, since it is proved by Algebra that any quantity with an index zero is always equal to unity, the Logarithm of 1 is 0, for $x^0=1$, $a^0=1$, $m^0=1$, and $10^0=1$, as exhibited in the above scheme. Again, since $\log. 1=0$, and $\log. 10=1$, it follows that the logarithms of all the numbers between 1 and 10 will be less than 1, but above zero, and also that the $\log.$ of any base will be 1. Also, since $\log. 10=1$, and $\log. 100=2$, it follows that the logarithms of all numbers between 10 and 100 will be greater than 1, but less than 2, and that the logarithms of all numbers between 100 and 1000 will be between 2 and 3, and so on.

Logarithms are always expressed in the decimal form.

The integral part is called the *Characteristic*.

The decimal part is called the *Mantissa*.

It is not usual in constructing tables to notice the characteristic, because, as the above explanations will show, it will always be denoted by *as many units as there are digits in the given number, less one*. Thus, the characteristic of 24796 will be 4, and of 879 it will be 2, and therefore it can always be found by counting the digits. It is also obvious that the characteristics of numbers less than unity must be negative, for $\log. \frac{1}{10}=\log. 10^{-1}=-1$, and $\log. \frac{1}{100}=\log. 10^{-2}=\log. .01=-2$. Since the characteristic is always 1 less than the number of digits, we have the same characteristic for 100 as for 199, but for 100 we have simply 2, while for 199 we have 2 *plus* some fraction, and since for 4.297, where we have only one digit, the characteristic is zero, ONE less than the number of digits, it follows that for .297, where there is no whole number, the characteristic must be -1 *plus* some fraction. Hence the mantissa is always positive, and is the same for decimals as for whole numbers. This fraction is .472756, and therefore we have $-1+.472756$, which, for convenience is written $\overline{1}.472756$, the negative sign being written

above the characteristic, an artifice which is extremely convenient in the arranging of tables, and also in operations.

Since $100 \times 100 = 10000$, and $100 = 10^2$, it follows that $10^2 \times 10^2 = 10000 = 10^4$. Now the last index 4 is the sum of the two preceding ones, $2 + 2 = 4$. Again, $10^2 \times 10^3 = 100 \times 1000 = 100000 = 10^5$. Hence we see that quantities can be multiplied by adding their indices, and therefore by adding their logarithms. In the same manner it may be shown that one quantity may be divided by another, by subtracting the logarithm of the divisor from the logarithm of the dividend. Also, that a quantity can be raised to any power by multiplying its logarithm by the index of the power, and that any root may be extracted by dividing the logarithm of the number by the index.

EXPLANATION OF THE TABLES.

In the first column are marked the natural numbers for three digits, and along the top of the page the ten digits. Opposite the three digits, on the margin and under the zero column, will be found the mantissa for the number represented by those digits, or those digits with a cipher annexed, for the annexing of a cipher makes no difference in the mantissa, but adds a unit to the index. If the given number consists of four digits, the required logarithm will be found under the fourth figure on the top of the page, and opposite the other three on the margin. The numbers in the column marked D denote the closely approximate difference between any two contiguous numbers, or the increment of the logarithms as the natural numbers increase by the successive additions of a unit.

To find by the tables the logarithm of any number.

If the number does not exceed 100 look for it at the beginning of the table in the vertical column marked N. or No., and both characteristic and mantissa will be found opposite to it in the column usually marked log. or L.

Thus the logarithm of 98 will be found by simple inspection to be 1.991126.

If the number consist of four digits, find the first three in the vertical column marked N. or No., and from opposite that, in the horizontal column, and under the fourth along the top, take the mantissa, and before it write the characteristic, found by taking a unit for every digit in the given number, less one.

EXAMPLES.

To find the logarithm of 4389. In the vertical column marked No. will be found 438, and opposite this, in the column headed 9, will be found the mantissa .642366, before which must be placed the characteristic 3, a unit less than the number of digits, giving 3.642366.

It is usual to record only four digits in tables of logarithms, but if it be required to find the logarithm of a number expressed by more than four digits, it can readily be found in the following manner. If it is required to find the logarithm of 43896.24. We first find, as in the last example, the mantissa of 4389, which is .642366, then we find the number opposite to it, in the column marked diff. or D., which in this case is 99, the difference between this and the succeeding logarithm. We then multiply the remaining figures of the given number by 99, which gives 617.76, and as .76 approaches so nearly to another unit, we write 618, and add this to the logarithm for 4389, and place the characteristic before the result thus :

$$\begin{array}{r} .642366 \\ \quad 618 \\ \hline 4.642984 \end{array}$$

EXERCISES.

- | | |
|------------------------------------|------------------------------|
| 1. Find the logarithm of 41. | Ans. 1.612784. |
| 2. Find the logarithm of 75. | Ans. 1.875061. |
| 3. Find the logarithm of 100. | Ans. 1.000000. |
| 4. Find the logarithm of 1. | Ans. 0.000000. |
| 5. Find the logarithm of 117. | Ans. 2.068186. |
| 6. Find the logarithm of 786111. | Ans. 5.895484. |
| 7. Find the logarithm of 7861.11. | Ans. 3.895484. |
| 8. Find the logarithm of 46782.79. | Ans. 4.670086. |
| 9. Find the logarithm of .9876. | Ans. $\overline{1}$.994581. |
| 10. Find the logarithm of .08734. | Ans. $\overline{2}$.941213. |

To find the natural number corresponding to any logarithm :

Let it be required to find the natural number corresponding to 5.890197. By inspecting the table, we find in the first column of logarithms the figures 890, but the succeeding figures, 421, are too

great; but by running the eye along the horizontal column above this, we find the exact logarithm, and opposite to it, under No., we find 776, and above it, at the top of the column, we find 6, so that 7766 are the first four figures of the number; but the index, 5, shows that the number must consist of six digits, and therefore we take the next lower logarithm and subtract it from the given one, getting a remainder of 56. We now annex two ciphers, and divide by the tabular difference, 99, which gives the other two figures, 5 and 6, and the whole, 776656, and if we annex more ciphers to the remainder we obtain decimals. This operation will be, when expressed as a

R U L E .

Find the next less logarithm, and the three figures in the column marked No. will be the three first figures, and the one at the top of the page, above the logarithm, will be the fourth figure.

Subtract this logarithm from the given one, and divide the remainder with ciphers by the annexed tabular difference, until one digit more than the number of units in the index be obtained. This will give the whole number, and all figures following that will be decimals.

NOTE.—The logarithms of all numbers consisting of not more than four figures will be found in the tables.

E X E R C I S E S .

11. What is the natural number corresponding to the logarithm 2.371068 ? Ans. 235.

12. What is the natural number corresponding to the logarithm 1.776992 ? Ans. 59.84.

13. What is the natural number corresponding to the logarithm 5.895484 ? Ans. 786111.

14. Multiply by logarithms 104777.7 by .045.

The product is 8765.

15. Divide .00987 by .082. Ans. .12036.

16. Find the second power of 818. Ans. 663124.

17. Find the fifth power of 19. Ans. 2276099.

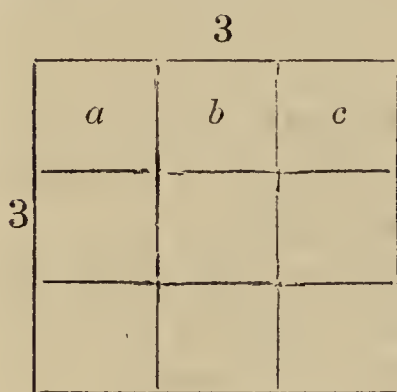
18. Find the second root of 7569. Ans. 87.5.

19. Find the third root of 92.61. Ans. 21.

20. Find the fifth root of 759375. Ans. 15.

MENSURATION.*

We have already observed that no solid body can have more than *three* dimensions, viz. : length, breadth, and thickness, or depth, and that a line is length, or breadth, or depth, or it is a line or unit repeated a certain number of times. A foot in length is a line measured by repeating the linear unit called an inch 12 times, and a yard is the linear unit called a foot, repeated 3 times, and so on. Thus, $\underbrace{1 \text{ ft. } 1 \text{ ft. } 1 \text{ ft.}}_3 = 3 \text{ feet.}$ But there may be two such lines drawn at right angles to each other, and each three feet long, and if



the figure be completed it is a square. Also, if lines be drawn, each an inch apart from the other, and parallel to the two first-mentioned lines, we shall find that there are three small figures, each an inch square, between the two upper horizontal lines, and 3 of the same extent between the two intermediate lines, and 3 between the two lower lines, making 9 in all, or 3 times 3. This is the origin of the expression that 9 is the square of 3. Let the learner mark the difference between 3 square feet and 3 feet square. *a, b* and *c* are 3 square feet, but the whole figure is 3 feet square, and therefore three feet square must be equal to 9 square feet. Three feet square, then, is a square, each of whose sides measures 3 linear feet; but 3 square feet would denote 3 squares, each side of each measuring one linear foot. The space thus inclosed is called the *area*.

This is the principle on which surfaces are measured.

PROBLEM I.

To find the area of a parallelogram :

RULE.

Multiply the length by the perpendicular breadth. If the figure be rectangular, one of the sides will be the perpendicular breadth.

* We have taken for granted that those studying mensuration have learned, at least, the elementary principles of geometry. We have, therefore, only given the rules, as our space would not admit of our giving demonstrations as this would require a separate treatise

If the figure be not rectangular, either the perpendicular breadth must be given or data from which to find it.

EXERCISES.

1. How many acres are there in a square, each side of which is 24 rods ?
Ans. 3 acres, 2 roods, 16 rods.

2. What is the area of a square picture frame, each side of which is 5 ft. 9 in. ?
Ans. 33 ft. 9 in.

3. How many acres are there in a rectangular field, the length of which is $13\frac{3}{4}$ chains, and the breadth $9\frac{1}{2}$?

Ans. 130.625 square chains, or 13 acres, 0 roods, 10 rods.

4. What is the area of a rectangle, whose sides are 14 ft. 6 in. and 4 ft. 9 in. ?
Ans. 68 ft., 126 sq. in.

5. What does the surface of a plank measure, which is 12 ft. 6 in. long and 9 in. broad ?
Ans. 9 sq. ft. 54 sq. in.

6. What is the area of a rhomboidal field, the length of which is 10.52 chains and the perpendicular breadth 7.63 chains ?

Ans. 8 acres, 0 roods, 4.2816 rods.

7. What is the area of a rhomboidal field, the length of which is 24 rods and the perpendicular breadth 24 rods ?

Ans. 3 acres, 2 roods, 16 rods.

8. What is the length of each side of a square field, the area of which is 788544 square yards ?
Ans. 888 yards.

9. The area of a rectangular garden is 1848 square yards, and one side is 56 yards ; what is the other ?
Ans. 33 yards.

10. The area of a rhomboidal pavement is 205, and the length is 20 feet ; what is the perpendicular breadth ?
Ans. $10\frac{1}{4}$ feet.

PROBLEM II.

To find the area of a triangle.

1. If the base and perpendicular, or data to find them, be given, we have the

RULE.

Multiply the base by the perpendicular, and take half the product ; or, multiply half the one by the other.

2. If the three sides are given

RULE.

From half the sum of the sides subtract each side successively, and the square root of the continual product of the half sum, and these three remainders will be the area.

Expressed algebraically this area $= \sqrt{s(s-a)(s-b)(s-c)}$.

EXERCISES.

11. What is the area of a triangle, the base of which is 17 inches, and the altitude 12 inches? Ans. 102 square inches.

12. What is the area of a triangular garden, the length of which is 46 rods, and the breadth 19 rods? Ans. 437 square rods.

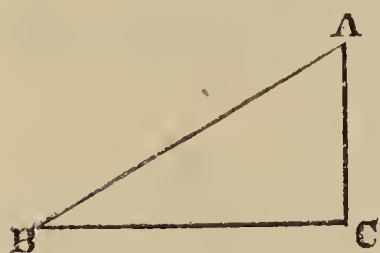
13. Find how many acres, &c., are in a triangular field, the length of which is 49.75 rods, and the breadth $34\frac{1}{2}$ rods.

Ans. 5 acres, 1 rood, $18\frac{3}{16}$ rods.

14. The area of a triangular inclosure is 150 square rods, and the base is 30 linear rods; what is the altitude? Ans. 10 rods.

15. The area of a triangle is 400 rods, and the altitude 40 rods, what is the base? Ans. 20 rods.

16. Three trees are so planted that the lines joining them form a right angled triangle; the two sides containing the right angle are 33 and 56 yards; what is the area in square yards? Ans. 924.



17. Let the position of the trees, as in the last example, be represented by the triangle A B C, and let the distance from A to B be 50 rods, and from B to C 30 rods. Required the area.—(See Euclid I. 47.)

Ans. 600 square rods.

18. In the figure annexed to 17, suppose A B to represent the pitch of a gallery in a church, inclined to the ground at an angle of 45° ; how many more persons will the gallery contain than if the seats were made on the flat B C, supposing B C to be 20 feet and the frontage 60 feet in length? Ans. None.



We have introduced this question and the next to correct a common misapprehension on this point. Because the distance from B to A is greater than the distance from B to C, it is commonly supposed that more persons can be accommodated on the slant A B, than on the flat B C. By inspecting the annexed diagram it will be seen that the seats are not perpendicular to A B, but to B C, and that precisely the same number of seats can be made, and the same number of persons accommodated on B C as on A B.

19. If BC be half the base of a hill, and AB one of its sloping sides, and $BC=30$ yards, and $AB=50$ yards; how many more rows of trees can be planted on AB , than on BC , at 1 yard apart?

Ans. None, because the trees being all perpendicular to the horizon, are parallel to each other as represented by the vertical lines in the last figure.

20. How many acres, &c., are there in a triangular field of which the perpendicular length and breadth are 12 chains, 76 links and 9 chains, 43 links?

Ans. 6 acres, 0 roods, $2\frac{1}{2}$ rods.

21. A ship was stranded at a distance of 40 yards from the base of a cliff 30 yards high; what was the length of a cable which reached from the top of the cliff to the ship?

Ans. 50 yds.

22. A cable 100 yards long was passed from the bow to the stern of a ship through the cradle of a mast placed in midships at the height of 30 yards; what was the length of the ship?

Ans. 80 yards.

23. A man attempts to row a boat directly across a river 200 yards broad, but is carried 80 yards down the stream by the current; through how many yards was he carried?

Ans. $215.4+$ yards.

24. Let the three sides of a triangle be 30, 40, 20; to find the area in square feet.

Ans. 290.4737 square feet.

25. What is the area of an isosceles triangle, each of the equal sides being 15 feet, and the base 20 feet?*

Ans. 111.803 sq. feet.

26. What is the area of a triangular space, of which the base is 56, and the hypotenuse 65 yards?

Ans. 924 square yards.

27. What is the area of a triangular clearing, each side of which is 25 chains?

Ans. 27.0632 acres.

28. What is the area of a triangular clearing, of which the three sides are 380, 420 and 765?

Ans. 9 acres, $37\frac{1}{2}$ perches.

29. A lot of ground is represented by the three sides of a right angled triangle, of which the hypotenuse is 100 rods, and the base 60 rods; what is the area?

Ans. 15 acres.

30. What is the area of a triangular field, of which the sides are 49, 34 and 27 rods respectively?

Ans. 2 acres, 3 roods+.

31. What is the area of a triangular orchard, the sides of which are 13, 14 and 15 yards?

Ans. 84 square yards.

32. Three divisions of an army are placed so as to be represented

* This question, and some others may be solved by either rule, and it will be found a good exercise to solve by both.

by three sides of a triangle, 12, 18 and 24; how many square miles do they guard within their lines?

Ans. Between 104 and 105 square miles.

33. A ladder, 50 feet long, was placed in a street, and reached to a parapet 28 feet high, and on being turned over reached a parapet on the other side 30 feet high; what was the breadth of the street?

Ans. 76.123+feet.

PROBLEM III.

To find the area of a regular Polygon.

1. When one of the equal sides, and the perpendicular on it from the centre, are given.

Multiply the perimeter by the perpendicular on it from its centre, and take half the product; or, multiply either by half the other.

2. When a side only is given.

Multiply the square of the side by the number found opposite the number of sides in the subjoined table.

NOTE.—This table shows the area when the side is unity; or, which is the same thing, the square is the unit.

SIDES.	REGULAR FIGURES.	
3	Triangle	0.4330127.
4	Square.....	1.0000000.
5	Pentagon	1.7204774.
6	Pexagon.....	2.5980762.
7	Heptagon.....	3.6339125.
8	Octagon	4.8284272.
9	Nonagon.....	6.1818241.
10	Decagon.....	7.6942088.
11	Heredecagon.....	9.3656395.
12	Dodecagon.....	11.1961524.

34. If the side of a pentagon is 6 feet and the perpendicular 3 feet, what is the area?

Ans. 45 feet.

35. What is the area of a regular polygon, each side of which is 15 yards?

Ans. 387.107325 sq. yds.

36. If each side of a hexagon be 6 feet, and a line drawn from the centre to any angle be 5 feet, what is the area?

Ans. 72 sq. feet.

37. The side of a decagon is 20.5 rods; what is the area?

Ans. 20 acres, 0 roods, 33.5 rods, nearly.

38. A hexagonal table has each side 60 inches, and a line from the centre to any corner is 50 inches; how many square feet in the surface of the table?

Ans. 38 feet, 128 inches.

39. What is the area of a regular heptagon, the side being $19\frac{1}{5}$ and the perpendicular 10?

Ans. 678.3.

40. An octagonal enclosure has each side 6 yards, what is its area?

Ans. 3 acres, 2 roods, 14 rods. 19 yards.

41. Five divisions of an army guard a certain tract of country—each line is 20 miles; how many square miles are guarded?

Ans. 688.2, nearly.

42. Find the same if there are 6 divisions, and each line extends 5 miles?

Ans. 64.95+ miles.

43. The area of a hexagonal table is $73\frac{3}{4}$ feet; what is each side?

Ans. $5\frac{1}{3}$ feet.

PROBLEM IV.

To find the area of an irregular polygon.

Divide it into triangles by a perpendicular on each diagonal from the opposite angle.

Find the area of each triangle separately, and the sum of these areas will be the area of the trapezium.

NOTE.—Either the diagonals and perpendiculars must be given, or data from which to find them.

44. The diagonal extent of a four-sided field is 65 rods, and the perpendiculars on it from the opposite corners are 28 and 33.5 rods; what is the area?

Ans. 1 acre, 1 rood, 22.083 rods.

45. A quadrangle having two sides parallel, and the one is 20.5 feet long and the other 12.25 feet, and the perpendicular distance between them is 10.75 feet; what is the area?

Ans. 176.03125 sq. feet.

46. Required the area of a six-sided figure, the diagonals of which are as follows: the two extreme ones, 20.75 yards and 18.5, and the intermediate 27.48; the perpendicular on the first is 8.6, on the second 12.8, and those on the intermediate one 14.25 and 9.35?

Ans. 531.889 yards.

47. If the two sides of a hexagon be parallel, and the diagonal parallel to them be 30.15 feet, and the perpendiculars on it from

the opposite angles are, on the left, 10.56, and on the right 12.24, and the part of the diagonal cut off to the left by the first perpendicular, 8.26, and to the right by the second, 10.14; on the other side, the perpendicular and segment of the diagonal to the left are 8.56 and 4.54, and on the right 9.26 and 3.93; what is the area?

Ans. 470.4155 sq. feet.

PROBLEM V.

To find the area of a figure, the boundaries of which are partly right lines and partly curves or salients.

Find the average breadth by taking several perpendiculars from the nearest and most remote points, from a fixed base, and dividing the sum of these by their number, the quotient, multiplied by the length, will be a close approximation to the area.

Let the perpendiculars 9.2, 10.5, 8.3, 9.4, 10.7, their sum is 48.1, then $48.1 \div 5 = 9.62$, and if the base is 20, we have $9.62 \times 20 = 192.4$, the area.

When practicable, as large a portion of the space as possible should be laid off, so as to form a regular figure, and the rest found as above.

A field is to be measured, and the greater part of it can be laid off in the form of a rectangle, the sides of which are 20.5 and 10.5, and therefore its area is 215.25, and the offsets of the irregular part are 10.2, 8.7, 10.9, and 8.5, the sum of which, divided by their number, is 7.66, and $7.66 \times 20.5 = 157.03$, the area of the irregular part, and this, added so the area of the rectangles, gives 372.28, the whole area.

48. The length of an irregular clearing is 47 rods, and the breadths at 6 equal distances are 5.7, 4.8, 7.5, 5.1, 8.4 and 6.5; what is the area? Ans. 1 acre, 1 rood, 29.86 rods.

PROBLEM VI.

To find the circumference of a circle when the diameter is known, or the diameter when the circumference is known.*

The most accurate rule is the well-known theorem that the diameter is to the circumference in the ratio of 113 to 355, and

* In strictness the circumference and diameter are not like quantities, but we may suppose that a cord is stretched round the circumference, and then drawn out into a straight line, and its linear units compared with those of the diameter.

consequently the circumference to the diameter as 355 to 113. Now, $355 \div 113 = 3.1416$ nearly, and for general purposes, sufficient accuracy will be attained by this.

R U L E .

To find the circumference from a given diameter, *multiply the diameter by 3.1416*; and to find the diameter from a given circumference, *divide by 3.1416*.

49. What is the length round the equator of a 15-inch globe?

Ans. 47.124 inches.

50. If a round log has a circumference of 6 feet, 10 inches; what is its diameter?

Ans. 2 feet, $2\frac{1}{10}$ inches nearly.

51. If we take the distance from the centre of the earth to the equator to be 3979; what is the number of miles round the equator?

Ans. 25001 nearly.

P R O B L E M V I I .

To find the area of a circle.

1. If the circumference and diameter are known,—

Multiply the circumference by the diameter, and take one-fourth of the product.

2. If the diameter alone is given,—

Multiply the square of the diameter by .7854.

3. If the circumference alone be given,—

Multiply the square of the number denoting the circumference by .07958.

52. If the diameter of a circle is 7, and the circumference 22; what is the area?

Ans. $38\frac{1}{2}$.

53. What is the area of a circle, the radius of which is $3\frac{5}{8}$ yds?

Ans. $3\frac{5}{8}$ square yards.

54. If a semicircular arc be denoted by 10.05; what is the area of the circle?

Ans. 289.36.

55. If the diameter of a grinding stone be 20 inches; what superficial area is left when it is ground down to 15 inches diameter, and what superficial area has been worn away?

Ans. 176.715 sqr. inches left, and 137.445 worn away.

56. If the chord of an arc be 24 inches, and the perpendicular on it from the centre 11.9; what is the area of the circle?

Ans. 2.689804.

MENSURATION OF SOLIDS.

To find the solid contents of a parallelopiped, or any regularly box-shaped body :

Let it be required to find the number of cubic feet in a box 8 feet long, $4\frac{1}{2}$ feet broad, and $6\frac{3}{4}$ feet deep.

In the first place, the length being 8 feet and the breadth $4\frac{1}{2}$, the area of the base is $8 \times 4\frac{1}{2} = 36$ square feet, and therefore every foot of altitude, or depth, or thickness, will give 36 cubic feet, and as there are $6\frac{3}{4}$ feet of depth, the whole solid content will be 36 times $6\frac{3}{4}$, or 243 cubic feet. Hence the

R U L E .

Take the continual product of the length, breadth, and depth.

NOTE.—Let it be carefully observed that the unit of measure in the case of solids is to be taken as a cube, the base of which is a superficial unit used in the measurement of surfaces. The solid content is indicated by the repetition of this unit a certain number of times. If the body is of uniform breadth the rule needs no modification, but if it is rounded or tapering, as a globe, cone, or pyramid, the calculation becomes virtually to find how much the rounded or tapering body differs from the one of uniform breadth. Suppose, for example, we take a piece of wood 6 feet high, in the form of a pyramid, and having the length and breadth of the base each 6 feet, then the area of the base is 36 ; but if, at the height of 1 foot, the dimensions have each diminished by 1 foot, the area is 25 ; at another foot higher it is 16 ; at the next 9 ; at the next 4 ; at the next 1 ; and at the 6th 0, i. e., it has come to a point, and the calculation is, how much remains from the solid cube after so much has been cut off each side as to give it this form.

This gives rise to the following varieties :

I. To find the solid contents of a cone or pyramid :

Multiply the area of the base by the perpendicular height, and take one-third of the product.

II. To find the solid contents of a cylinder or prism :

Multiply the area of the base by the perpendicular height.

III. To find the surface of a sphere :

Multiply the square of the diameter by 3.1416.

IV. To find the solid contents of a globe or sphere :

Multiply the third power of the diameter by .5236.

V. To find the volume of a spheroid, the axes being given :

Multiply the square of the axis of revolution by the fixed axis, and the product by .5236.

EXERCISES.

57. If the diameter of the base of a cylinder be 2 feet, and its height 5 feet, what is the solid content ? Ans. 25.708 feet.

58. If the diameter of the base of a cone be 1 foot.6 inches, and the altitude 15 feet, what are the solid contents ?

Ans. 8 feet, 120 inches.

59. If the diameter of the base of a cylinder be 7 feet, and the height 5 feet, what is the solid content ? Ans. 245 cubic feet.

60. What are the solid contents of a hexagonal prism, each side of the base being 16 inches and the height 15 feet ?

Ans. 69.282 cubic feet.

61. A triangular pyramid is 30 feet high, and each side of the base is 3 feet ; required the solid contents. Ans. 39.98 cubic feet.

62. What are the solid contents of the earth, the diameter being taken as 7918.7 miles ? Ans. 259992732079.87.

63. In a spheroid the less axis is 70 and the greater 90 ; what are the solid contents ? Ans. 230907.6.

PILING OF BALLS AND SHELLS.

Balls are usually piled on a base which is either a triangle, or square, or rectangle, each side of each course containing one ball less than the one below it.

If the base is an equilateral figure, the vertex of a complete pile will be a single ball ; but if one side of the base be greater than the contiguous one, the vertex will be a *row* of balls. Hence, if the base be an equilateral figure, the pile will be a pyramid, and as the side of each layer contains one layer less than the one below it, the number of balls in height will be the same as the number of balls in one side of the lowest layer. If the pile be rectangular, each layer must also be rectangular, and the number of balls in height will be the same as the number in the less side of the base. If the base be triangular, we have the

R U L E .

Multiply the number on one side of the bottom row by itself PLUS one, and the product by the same base row PLUS two, and divide the result by six.

For a complete square pile we have the

R U L E .

Multiply the number of balls in one side of the lowest course by itself PLUS one, and this product by double the first multiplier PLUS one, and take one-sixth of the result.

If the pile be rectangular, we have the

R U L E .

From three times the number of balls in the length of the lowest course subtract one less than the number in the breadth of the same course ; multiply the remainder by the breadth, and this product by one-sixth the breadth PLUS one.

If the pile be incomplete, find what it would be if complete ; find also what the incomplete one would be as a separate pile, and subtract the latter from the former.

E X E R C I S E S .

64. In a complete triangular pile each side of the base is 40 ; how many balls are there ? Ans. 11480.

65. In each side of the base of a square pile there are 20 shells ; how many in the whole pile ? Ans. 2870.

66. In a rectangular pile there are 59 balls in the length, and 20 in the breadth of the base ; how many are in all ? Ans. 11060.

67. In an incomplete triangular pile, each side of the lowest layer consists of 40 balls, and the side of the upper course of 20 ; what is the number of balls ? Ans. 10150.

NOTE.—Since the upper course is 20, the first row in the wanting part would be 19.

MEASUREMENT OF TIMBER.

Timber is measured sometimes by the square foot, and sometimes by the cubic foot.

Cleared timber, such as planks, beams, &c., are usually measured by the square foot.

What is called board measure is a certain length and breadth, and a uniform thickness of *one inch*.

Large quantities of round timber are often estimated by the ton.

To find either the superficial extent or board measure of a plank, &c.

R U L E .

Multiply the length in feet by the breadth in inches, and divide by 12.

NOTE.—The thickness being taken uniformly as *one inch*, the rule for finding the contents in square feet becomes the same as that for finding surface. If the thickness be not an inch,—

Multiply the board measure by the thickness.

If the board be a tapering one, take half the sum of the two extreme widths for the average width.

If a one-inch plank be 24 feet long, and 8 inches thick, then we have 8 inches equal $\frac{2}{3}$ of a foot, and $\frac{2}{3}$ of 24 feet=16 feet.

A board 30 feet long is 26 inches wide at the one end, and 14 inches at the other, hence 20 is the mean width, *i. e.*, $1\frac{2}{3}$ feet, and $30 \times 1\frac{2}{3} = 50$; or, $30 \times 20 = 600$, and $600 \div 12 = 50$.

To find the solid contents of a round log when the girt is known.

R U L E .

Multiply the square of the quarter girt in inches by the length in feet, and divide the product by 144.

If a log is 40 inches in girt, and 30 feet long, the solid contents will be found by taking the square of 10, the quarter girt in inches, which is 100, and $100 \times 30 = 3000$, and $3000 \div 144 = 20\frac{5}{6}$.

To find the number of square feet in round timber, when the mean diameter is given.

R U L E .

Multiply the diameter in inches by half the diameter in inches, and the product by the length in feet, and divide the result by 12.

If a log is 30 feet long, and 56 inches mean diameter, the number of square feet is $56 \times 28 \times 30 \div 12 = 1960$ feet.

To find the solid contents of a log when the length and mean diameter are given.

R U L E .

Multiply the square of half the diameter in inches by 3.1416, and this product by the length in feet, and divide by 144.

68. How many cubic feet are there in a piece of timber 14×18 , and 28 feet long? Ans. $49\frac{1}{2}$ cubic feet.

69. How many cubic feet are there in a round log 21 inches in diameter, and 40 feet in length? Ans. $96\frac{1}{9}$.

70. What are the solid contents of a log 24 inches in diameter, and 34 feet in length? Ans. $106.81\frac{1}{2}$ cubic feet.

71. How many feet, board measure, are there in a log 23 inches in diameter, and 12 feet long? Ans. $264\frac{1}{2}$.

72. How many feet, board measure, are there in a log, the diameter of which is 27 inches, and the length 16 feet. Ans. 486.

73. What are the solid contents of a round log 36 feet long, 18 inches diameter at one end, and 9 at the other? Ans. $30.63\frac{1}{2}$ cubic feet.

74. How many feet of square timber will a round log 36 inches in diameter and 10 feet long yield? Ans. 540 solid feet.

75. How many solid feet are there in a board 15 feet long, 5 inches wide, and 3 inches thick? Ans. $1\frac{9}{16}$ cubic feet.

76. What are the solid contents of a board 20 feet long, 20 inches broad, and 10 inches thick? Ans. $27\frac{7}{9}$ feet.

77. What is the solid content of a piece of timber 12 feet long, 16 inches broad, and 12 inches thick? Ans. 16 feet.

78. How many cubic feet are there in a log that is 25 inches in diameter, and 32 feet long?

79. How many feet, board measure, does a log 28 inches in diameter, and 14 feet in length contain? Ans. $457\frac{1}{3}$.

80. How many cubic feet are contained in a piece of squared timber that is 12 by 16 inches, and 47 feet in length? Ans. $62\frac{2}{3}$.

81. How many feet, board measure, are there in 22 one-inch boards, each being 13 inches in width, and 16 feet in length?

Ans. $381\frac{1}{3}$.

BALES, BINS, & C.

As bales are usually of the same form as boxes, the same rule applies.

82. Hence, a bale measuring $4\frac{1}{2}$ inches in length, 33 in width, and $3\frac{1}{3}$ in depth, is, in solid content, $37\frac{1}{2}$ feet.

83. A crate is 5 feet long, $4\frac{3}{4}$ broad, and $3\frac{7}{12}$ deep, what is the solid content?

Ans. $85\frac{5}{8}$.

To find how many bushels are in a *bin* of grain :

R U L E .

Find the product of the length, breadth and depth, and divide by 5150.4.

84. A bin consists of 12 compartments ; each measures 6 feet 3 inches in length, 4 feet 8 inches in width, and 3 feet 9 inches in depth ; how many bushels of grain will it hold ?

Ans. 1055, nearly.

To find how many bushels of grain are in a conical heap in the middle of a floor :

R U L E .

Multiply the area of the base by one-third the height.

The base of such a pile is 8 feet diameter and 4 feet high ; what is the content ?

The area of the base is $64 \times .7854 = 83.777$, and $83.777 \times \frac{4}{3} = 67.02$, the number of bushels.

If it be heaped against a wall take half the above result.

If it be heaped in a corner, take one-fourth the above result.

MISCELLANEOUS EXERCISES.

1. What number is that $\frac{2}{3}$ and $\frac{3}{5}$ of which make 255 ?
Ans. $201\frac{6}{19}$.
2. What must be added to $217\frac{1}{2}$, that the sum may be $17\frac{1}{4}$ times $19\frac{1}{2}$?
Ans. $118\frac{7}{8}$.
3. What sum of money must be lent, at 7 per cent., to accumulate to \$455 interest in 3 months ?
Ans. \$26000.
4. Divide \$1000 among A, B and C, so that A may have \$156 more than B, and B \$62 less than C.
Ans. A. $\$416\frac{2}{3}$; B, $\$260\frac{2}{3}$; C, $\$322\frac{2}{3}$.
5. Where shall a pole 60 feet high be broken, that the top may rest on the ground 20 feet from the stump ?
Ans. $26\frac{2}{3}$ feet.
6. A man bought a horse for \$68, which was $\frac{3}{7}$ as much again as he sold it for, lacking \$1 ; how much did he gain by the bargain ?
Ans. \$12.50.
7. A fox is 120 leaps before a hound, and takes 5 leaps to the hound's 2 ; but 4 of the hound's leaps equal 12 of the fox's ; how many leaps must the hound take to catch the fox ?
Ans. 240.
8. A, B and C can do a certain piece of work in 10 days ; how long will it take each to do it separately, if A does $1\frac{1}{2}$ times as much as B, and B does $\frac{1}{2}$ as much as C ?
Ans. A, 30 days ; B, 45 ; C, $22\frac{1}{2}$.
9. At what time between five and six o'clock, are the hour and minute hands of a clock exactly together ?
Ans. 27 min., $16\frac{4}{11}$ sec. past 5.
10. A courier has advanced 35 miles with despatches, when a second starts with additional instructions, and hurries to overtake the first, travelling 25 miles for 18 that the first travels ; how far will both have travelled when the second overtakes the first ?
Ans. 125 miles.
11. What is the sum of the series $\frac{2}{5} - \frac{4}{15} + \frac{8}{45} - \frac{16}{135} + \frac{32}{405} - \&c.$?
Ans. $\frac{6}{25}$.
12. If a man earn \$2 more each month than he did the month before, and finds at the end of 18 months that the rate of increase will enable him to earn the same sum in 14 months ; how much did he earn in the whole time ?
Ans. \$4032.
13. How long would it take a body, moving at the rate of 50

miles an hour, to pass over a space equal to the distance of the earth from the sun, *i. e.*, 95 millions of miles, a year being 365 days?

Ans. 216 years, 326 days, 16 hours.

14. Two soldiers start together for a certain fort, and one travels 18 miles a day, and after travelling 9 days, turns back as far as the second had travelled during those 9 days, he then turns, and in $22\frac{1}{2}$ days from the time they started, arrives at the fort at the same time as his comrade; at what rate did the second travel?

Ans. 18 miles a day.

15. What quantity must be subtracted from the square of 48, so that the remainder may be the product of 54 by 16? Ans. 1440.

16. A father gave $\frac{3}{8}$ of his farm to his son, the son sold $\frac{2}{3}$ of his share for \$1260; what was the value of the whole farm?

Ans. \$5040.

17. There were $\frac{5}{8}$ of a flock of sheep stolen, and 672 were left; how many were there in all? Ans. 1792.

18. A boy gave 2 cents each for a number of pears, and had 42 cents left, but if he had given 5 cents for each, he would have had nothing left. Required the number of pears. Ans. 14.

19. Simplify $\frac{1}{1 + \frac{1}{2 + \frac{1}{2}}}$. Ans. $\frac{5}{7}$.

20. A man contracted to perform a piece of work in 60 days, he employed 30 men, and at the end of 48 days it was only half finished; how many additional hands had to be employed to finish it in the stipulated time?

21. A gentleman gave his eldest daughter twice as much as his second, and the second three times as much as the third, and the third got \$1573; how much did he give to all? Ans. \$15730.

22. The sum of two numbers is 5643, and their difference 125; what are the numbers? Ans. 2884 and 2759.

23. How often will all the four wheels of a carriage turn round in going 7 miles, 1 furlong, and 8 rods, the hind wheels being each 7 feet 6 inches in circumference, and the fore wheels 5 feet $7\frac{1}{2}$ inches? Ans. 23716.

24. What is the area of a right angled triangular field, of which the hypotenuse is 100 rods and the base 60? Ans. 2400 sq. rds.

25. Simplify $\frac{5\frac{4}{5} - 2\frac{1}{6}}{3\frac{3}{4} + \frac{9}{20}}$ of $\frac{4\frac{1}{2} + 5\frac{1}{2}\frac{9}{5}}{4\frac{1}{20}}$ of $\frac{2\frac{3}{5} + 1\frac{2}{3}}{7\frac{19}{24} - 2\frac{1}{4}}$. Ans. $1\frac{5}{7}\frac{3}{5}$.

26. Find the value of $1 + \frac{1}{1 + \frac{1}{2}}$. Ans. $\frac{3}{5}$.

27. If $\frac{2}{3}$ of A's age is $\frac{5}{8}$ of B's, and A is $37\frac{1}{2}$, what age is B? Ans. 40.

28. What is the excess of $\frac{1}{99} \div \frac{1}{101}$ above $\frac{1}{999} \div \frac{1}{1001}$? Ans. $\frac{200}{10989}$.

29. The sum of two numbers is 5330 and their difference 1999; what are the numbers? Ans. $3664\frac{1}{2}$ and $1665\frac{1}{2}$.

30. A person being asked the hour of the day, replied that the time past noon was equal to one-fifth of the time past midnight; what was the time? Ans. 3 P.M.

31. A snail, in getting up a pole 20 feet high, climbed up 8 feet every day, but slipped back 4 feet every night; in what time did he reach the top? Ans. 4 days.

32. What number is that whose $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ parts make 48? Ans. $44\frac{4}{3}$.

33. A merchant sold goods to a certain amount, on a commission of 4 per cent., and, having remitted the net proceeds to the owner, received $\frac{1}{4}$ per cent. for immediate payment, which amounted to \$15.60; what was the amount of his commission? Ans. \$260.

34. A criminal has 40 miles the start of the detective, but the detective makes 7 miles for 5 that the fugitive makes; how far will the detective have travelled before he overtakes the criminal? Ans. 140 miles.

35. A man sold 17 stoves for \$153; for the largest size he received \$19, for the middle size \$7, and for the small size \$6; how many did he sell of each size? Ans. 3 of the large size, 12 of the middle, 2 of the small.

36. A merchant bought goods to the amount of \$12400; \$4060 of which was on a credit of 3 months, \$4160 on a credit of 8 months and the remainder on a credit of 9 months; how much ready money would discharge the debt, money being worth 6 per cent.? Ans. \$12000.

37. If a regiment of soldiers, consisting of 1000 men, are to be clothed, each suit to contain $3\frac{3}{4}$ yards of cloth that is $1\frac{7}{8}$ yards wide, and to be lined with flannel $1\frac{1}{4}$ yards wide; how many yards will it take to line the whole? Ans. 5625.

38. Taking the moon's diameter at 2180 miles, what are the solid contents? Ans. $5424617475 +$ sq. miles.

39. A certain island is 73 miles in circumference, and if two men start out from the same point, in the same direction, the one walking at the rate of 5 and the other at the rate of 3 miles an hour; in what time will they come together? Ans. 36 hours, 30 minutes.

40. A circular pond measures half an acre; what length of cord will be required to reach from the edge of the pond to the centre?
Ans. 83263+ feet.

41. A gentleman has deposited \$450 for the benefit of his son, in a Savings' Bank, at compound interest at a half-yearly rate of $3\frac{1}{2}$ per cent. He is to receive the amount as soon as it becomes \$1781.66 $\frac{1}{2}$. Allowing that the deposit was made when the son was 1 year old, what will be his age when he can come in possession of the money? Ans. 21 years.

42. The select men of a certain town appointed a liquor agent, and furnished him with liquor to the amount of \$825.60, and cash, \$215. The agent received cash for liquor sold, \$1323.40. He paid for liquor bought, \$937; to the town treasurer, \$300; sundry expenses, \$29; his own salary, \$265; he delivered to indigent persons, by order of the town, liquor to the amount of \$13.50. Upon taking stock at the end of the year, the liquor on hand amounted to \$616.50. Did the town gain or lose by the agency, and how much; has the agent any money in his hands belonging to the town; or does the town owe the agent, and how much in either case?

Ans. The town lost \$103.20; the agent owes the town \$7.40.

43. A holds a note for \$575 against B, dated July 13th, payable in 4 months from date. On the 9th August, A received in advance \$62; and on the 5th September, \$45 more. According to the terms of agreement it will be due, adding 3 days of grace, on the 16th November, but on the 3rd of October B proposes to pay a sum which, in addition to the sums previously paid, shall extend the pay day to forty days beyond the 16th of November; how much must B pay on the 3rd of October? Ans. \$111.43.

44. A accepted an agency from B to buy and sell grain for him. A received from B grain in store, valued at \$135.60, and cash, \$222.10; he bought grain to the value of \$1346.40, and sold grain to the amount of \$1171.97. At the end of four months B wished to close the agency, and A returned him grain unsold, valued at \$437.95; A was to receive for services, \$48.12. Did A owe B, or B owe A, and how much? Ans. B owed A 45 cents.

45. A general ranging his men in the form of a square, had 59 men over, but having increased the side of the square by one man, he lacked 84 of completing the square ; how many men had he ?

Ans. 5100.

46. What portion, expressed as a common fraction, is a ponnd and a half troy weight of three pounds avoirdupois ?

Ans. $\frac{72}{175}$.

47. What would the last fraction be if we reckoned by the ounces instead of grains according to the standards ?

Ans. $\frac{3}{8}$.

48. If 4 men can reap $6\frac{2}{3}$ acres of wheat in $2\frac{1}{2}$ days, by working $8\frac{1}{4}$ hours per day, how many acres will 15 men, working equally, reap in $3\frac{3}{4}$ days, working 9 hours per day ?

Ans. $40\frac{10}{11}$ days.

49. Out of a certain quantity of wheat, $\frac{1}{3}$ was sold at a certain gain per cent., $\frac{1}{4}$ at twice that gain, and the remainder at three times the gain on the first lot ; what was the gain on each, the gain on the whole being 20 per cent. ?

Ans. $9\frac{3}{5}$, $19\frac{1}{5}$ and $28\frac{4}{5}$ per cent.

50. If a man by travelling 6 hours a day, and at the rate of $4\frac{1}{2}$ miles an hour, can accomplish a journey of 540 miles in 20 days ; how many days, at the rate of $4\frac{2}{3}$ miles an hour, will he require to accomplish a journey of 600 miles ?

Ans. $21\frac{3}{7}$.

51. Smith in Montreal, and Jones in Toronto, agree to exchange operations, Jones chiefly making the purchases, and Smith the sales, the profits to be equally divided ; Smith remitted to Jones a draft for \$8000 after Jones had made purchases to the amount of \$13682.24 ;—Jones had sent merchandise to Smith, of which the latter had made sales to the value of \$9241.18 ; Jones had also made sales to the worth of \$2836.24 ; Smith has paid \$364.16 and Jones \$239.14 for expenses. At the end of the year Jones has on hands goods worth \$2327.34, and Smith goods worth \$3123.42. The term of the agreement having now expired, a settlement is made, what has been the gain or loss ? What is each partner's share of gain or loss ? What is the cash balance, and in favor of which partner ?

52. In a certain factory a number of men, boys and girls are employed, the men work 12 hours a day, the boys 9 hours and the girls 8 hours ; for the same number of hours each man receives a half more than each boy, and each boy a third more than each girl ; the sum paid each day to all the boys is double the sum paid to all the girls, and for every five shillings earned by all the boys each day, twelve shillings are earned by all the men ; it

is required to find the number of men, the number of boys and the number of girls, the whole number being 59.

Ans. 24 men, 20 boys and 15 girls.

53. A holds B's note for \$575, payable at the end of 4 months from the 13th July; on the 9th August, A received \$62 in advance, as part payment, and on the 5th September \$45 more; according to agreement the note will not be due till 16th November, three days of grace being added to the term; but on the 3rd October B tenders such a sum as will, together with the payments already made, extend time of payment forty days forward; how much must B pay on the 3rd of October?

Ans. \$111.43.

54. If a man commence business with a capital of \$5000 and realises, above expenses, so much as to increase his capital each year by *one tenth* of itself less \$100, what will his capital amount to in twenty years?

Ans. \$27910.

55. A note for \$100 was to come due on the 1st October, but on the 11th of August, the acceptor proposes to pay as much in advance as will allow him 60 days after the 1st of October to pay the balance; how much must he pay on the 11th of August?

Ans. \$54.

56. A person contributed a certain sum in dollars to four charities;—to one he gave one half of the whole and half a dollar; to a second half the remainder and half a dollar; to a third half the remainder and half a dollar; and also to the fourth half the remainder and half a dollar, together with one dollar that was left; how much did he give to each?

Ans. To the first, \$16; to the second, \$8; to the third, \$4; to the fourth, \$3.

57. A farmer being asked how many sheep he had, replied that he had them in four different fields, and that two-thirds of the number in the first field was equal to three-fourths of the number in the second field; and that two-thirds of the number in the second field was equal to three-fourths of the number in the third field; and that two-thirds of the number in the third field was equal to four-fifths of the number in the fourth field; also that there were thirty-two sheep more in the third field than in the fourth; how many sheep were in each field and how many altogether?

Ans. First field, 243; second field, 216; third field, 192; fourth field, 160. Total. 811.

58. How many hours per day must 217 men work for $5\frac{1}{2}$ days to dig a trench $23\frac{1}{4}$ yards long, $3\frac{2}{3}$ yards wide, and $2\frac{1}{3}$ deep, if 24 men working equally can dig one $33\frac{3}{4}$ yards long, $5\frac{3}{5}$ wide, and $3\frac{1}{2}$ deep, in 189 days of 14 hours each. Ans. 16 hours.

59. A man bequeathed one-fourth of his property to his eldest son;—to the second son one-fourth of the remainder, and \$350 besides; to the third one-fourth of the remainder, together with \$975; to the youngest one-fourth of the remainder, and \$1400; he gives his wife a life interest in the remainder, and her share is found to be one-fifth of the whole; what was the amount of the property? Ans. \$20000.

60. Five men formed a partnership which was dissolved after four years' continuance; the first contributed \$60 at first and \$800 more at the end of five months, and again \$1500 at the end of a year and eight months; the second contributed \$600 and \$1800 more at the end of six months; the third gave at first \$400 and \$500 every six months; the fourth did not contribute till the end of eight months, he then gave \$900, and the same sum every six months; the fifth having no capital, contributed by his labor in keeping the books at a salary of \$1.25 per day; at the expiration of the partnership what was the share of each, the whole gain having been \$20000?

Ans. 1st, \$2019.65 nearly; 2nd, \$4871.81 nearly; 3rd, \$4815.81 nearly; 4th, \$6467.74 nearly; 5th, \$1825.00.

LOGARITHMS OF NUMBERS

Numbers from 1 to 100.									
No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0.000000	21	1.322219	41	1.612784	61	1.785330	81	1.908485
2	0.301030	22	1.342423	42	1.623249	62	1.792392	82	1.913814
3	0.477121	23	1.361728	43	1.633468	63	1.799341	83	1.919078
4	0.602060	24	1.380211	44	1.643453	64	1.806180	84	1.924279
5	0.698970	25	1.397940	45	1.653213	65	1.812913	85	1.929419
6	0.778151	26	1.414973	46	1.662758	66	1.819544	86	1.934498
7	0.845098	27	1.431364	47	1.672098	67	1.826075	87	1.939519
8	0.903090	28	1.447158	48	1.681241	68	1.832509	88	1.944483
9	0.954243	29	1.462398	49	1.690196	69	1.838849	89	1.949390
10	1.000000	30	1.477121	50	1.698970	70	1.845098	90	1.954243
11	1.041393	31	1.491362	51	1.707570	71	1.851258	91	1.959041
12	1.079181	32	1.505150	52	1.716003	72	1.857332	92	1.963788
13	1.113943	33	1.518514	53	1.724276	73	1.863323	93	1.968483
14	1.146128	34	1.531479	54	1.732394	74	1.869232	94	1.973128
15	1.176091	35	1.544068	55	1.740363	75	1.875061	95	1.977724
16	1.204120	36	1.556303	56	1.748188	76	1.880814	96	1.982271
17	1.230449	37	1.568202	57	1.755875	77	1.886491	97	1.986772
18	1.255273	38	1.579784	58	1.763428	78	1.892095	98	1.991226
19	1.278754	39	1.591065	59	1.770852	79	1.897627	99	1.995635
20	1.301030	40	1.602060	60	1.778151	80	1.903090	100	2.000000

PP	N.	0	1	2	3	4	5	6	7	8	9	D.
	100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
41	1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
83	2	8300	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
124	3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
166	4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
207	5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
248	6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
290	7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
331	8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
373	9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
	110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
38	1	5323	5714	6105	6495	6885	7275	7664	8053	8441	8830	390
76	2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
113	3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
151	4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
189	5	060398	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
227	6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
265	7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	370
302	8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
340	9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
	120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
35	1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
70	2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
104	3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
139	4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
174	5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
209	6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
244	7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
278	8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
313	9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
	130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
32	1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
64	2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
97	3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
129	4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
161	5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
193	6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
225	7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
258	8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
290	9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
	140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
30	1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
60	2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
90	3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
120	4	8362	8664	8965	9266	9567	9868	160168	160469	160769	161068	301
150	5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
180	6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
210	7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
240	8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
270	9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
	150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
28	1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
56	2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
84	3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
112	4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
140	5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
168	6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
196	7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
224	8	8657	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
252	9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272

PP	N.	0	1	2	3	4	5	6	7	8	9	D.
	160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556	271
26	1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
53	2	9515	9783	210051	210319	210588	210853	211121	211388	211654	211921	267
79	3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
105	4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
132	5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
158	6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
184	7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
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237	9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
	170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	254
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174	7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
198	8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
223	9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
	180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
24	1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
47	2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
71	3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
94	4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
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	190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
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67	3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
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	200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
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162	8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
182	9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198

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39	2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
58	3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
77	4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
97	5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
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74	4	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
93	5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
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148	8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
167	9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
	240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
18	1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
35	2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
53	3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
71	4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
89	5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
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142	8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
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	250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399501	173
17	1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
34	2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
51	3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
68	4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
85	5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
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	260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
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49	3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
66	4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
82	5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
98	6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
115	7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
131	8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
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	270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
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32	2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
47	3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
63	4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
79	5	9333	9491	9648	9806	9964	440122	440279	440437	440594	440752	158
95	6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
111	7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
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15	1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
31	2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
46	3	1786	1940	2093	2247	3400	2553	2706	2859	3012	3165	153
61	4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
77	5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
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122	8	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
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	290	462398	462548	462697	462847	462997	463146	463296	463445	463594	463744	150
15	1	3893	4042	4191	4340	4490	4639	4788	4936	5085	5234	149
29	2	5383	5532	5680	5829	5977	6126	6274	6423	6571	6719	149
44	3	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
59	4	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
74	5	9822	9969	470116	470263	470410	470557	470704	470851	470998	471145	147
88	6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
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43	3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
57	4	2874	3016	3159	3302	3445	3587	3730	3872	4015	4157	143
72	5	4300	4442	4585	4727	4869	5011	5153	5295	5437	5579	142
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129	9	9958	490099	490239	490380	490520	490661	490801	490941	491081	491222	140
	310	491362	491502	491642	491782	491922	492062	492201	492341	492481	492621	140
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28	2	4155	4294	4433	4572	4711	4850	4989	5128	5267	5406	139
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	320	505150	505286	505421	505557	505693	505828	505964	506099	506234	506370	136
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54	4	510545	510679	510813	510947	511081	511215	1349	1482	1616	1750	134
67	5	1883	2017	2151	2284	2418	2551	2684	2818	2951	3084	133
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94	7	4548	4681	4813	4946	5079	5211	5344	5476	5609	5741	133
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	330	518514	518646	518777	518909	519040	519171	519303	519434	519566	519697	131
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26	2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
39	3	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
52	4	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
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78	6	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
91	7	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
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38	3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
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76	6	9076	9202	9327	9452	9578	9703	9829	9954	540079	540204	125
88	7	540329	540455	540580	540705	540830	540955	541080	541205	1320	1454	125
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113	9	2825	2950	3074	3199	3323	3447	3571	3696	3820	3044	124
	350	544068	544192	544316	544440	544564	544688	544812	544936	545060	545183	124
12	1	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
24	2	6543	6666	6789	6913	7036	7159	7282	7405	7529	7652	123
37	3	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
49	4	9003	9126	9249	9371	9494	9616	9739	9861	9984	550106	123
61	5	550228	550351	550473	550595	550717	550840	550962	551084	551206	1328	122
73	6	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
85	7	2668	2790	2911	3033	3155	3276	3398	3519	3640	3762	121
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	360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
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36	3	9907	560026	560146	560265	560385	560504	560624	560743	560863	560982	119
48	4	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
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71	6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
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95	8	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
107	9	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
	370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
12	1	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
23	2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
35	3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
46	4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
58	5	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
70	6	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
81	7	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	115
93	8	7492	7607	7722	7836	7951	8066	8181	8295	8410	8525	115
104	9	8639	8754	8868	8983	9097	9212	9326	9441	9555	9669	114
	380	579784	579898	580012	580126	580241	580355	580469	580583	580697	580811	114
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23	2	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	114
34	3	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
45	4	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	113
57	5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
68	6	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
79	7	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	112
90	8	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	112
102	9	9950	590061	590173	590284	590396	590507	590619	590730	590842	590953	112
	390	591065	591176	591287	591399	591510	591621	591732	591843	591955	592066	111
11	1	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	111
22	2	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	111
33	3	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
44	4	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	110
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77	7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
88	8	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
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21	2	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	108
32	3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
43	4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
54	5	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
64	6	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	107
75	7	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
86	8	610660	610767	610873	610979	1086	1192	1298	1405	1511	1617	106
96	9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
	410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
11	1	3842	3947	4053	4159	4264	4370	4475	4581	4686	4792	106
21	2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
32	3	5950	6055	6160	6265	6370	6476	6581	6686	6790	6895	105
42	4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
53	5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
63	6	9093	9198	9302	9406	9511	9615	9719	9824	9928	620032	104
74	7	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
84	8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
95	9	2214	2318	2421	2525	2628	2732	2835	2939	3043	3146	104
	420	623249	623353	623456	623559	623663	623766	623869	623973	624076	624179	103
10	1	4282	4385	4488	4591	4695	4798	4901	5004	5107	5210	103
20	2	5312	5415	5518	5621	5724	5827	5929	6032	6135	6238	103
31	3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
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61	6	9410	9512	9613	9715	9817	9919	630021	630123	630224	630326	102
71	7	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
82	8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
92	9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
	430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
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30	3	6488	6588	6688	6789	6889	6989	7089	7189	7290	7390	100
40	4	7490	7590	7690	7790	7890	7990	8090	8190	8290	8389	100
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80	8	1474	1573	1672	1771	1871	1970	2069	2168	2267	2366	99
90	9	2465	2563	2662	2761	2860	2959	3058	3156	3255	3354	99
	440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
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20	2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
30	3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
39	4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
49	5	8360	8458	8555	8653	8750	8848	8945	9043	9140	9237	97
59	6	9335	9432	9530	9627	9724	9821	659919	650016	650113	650210	97
69	7	650308	650405	650502	650599	650696	650793	0890	0987	1084	1181	97
78	8	1278	1375	1472	1569	1666	1762	1859	1956	2053	2150	97
88	9	2246	2343	2440	2536	2633	2730	2826	2923	3019	3116	97
	450	653213	653309	653405	653502	653598	653695	653791	653888	653984	654080	96
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19	2	5138	5235	5331	5427	5523	5619	5715	5810	5906	6002	96
29	3	6098	6194	6290	6386	6482	6577	6673	6769	6864	6960	96
38	4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
48	5	8011	8107	8202	8298	8393	8488	8584	8679	8774	8870	95
58	6	8965	9060	9155	9250	9346	9441	9536	9631	9726	9821	95
67	7	9916	660011	660106	660201	660296	660391	660486	660581	660676	660771	95
77	8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
86	9	1813	1907	2002	2096	2191	2286	2380	2475	2569	2663	95

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9	1	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
19	2	4642	4736	4830	4924	5018	5112	5206	5299	5393	5487	94
28	3	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
38	4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
47	5	7453	7546	7640	7733	7826	7920	8013	8106	8199	8293	93
56	6	8386	8479	8572	8665	8759	8852	8945	9038	9131	9224	93
66	7	9317	9410	9503	9596	9689	9782	9875	9967	670060	670153	93
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85	9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
	470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
9	1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
18	2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
28	3	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
37	4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
46	5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
55	6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
64	7	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
74	8	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
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	480	681241	681332	681422	681513	681603	681693	681784	681874	681964	682055	90
9	1	2145	2235	2326	2416	2506	2596	2686	2777	2867	2957	90
18	2	3047	3137	3227	3317	3407	3497	3587	3677	3767	3857	90
27	3	3947	4037	4127	4217	4307	4396	4486	4576	4666	4756	90
36	4	4845	4935	5025	5114	5204	5294	5383	5473	5563	5652	90
45	5	5742	5831	5921	6010	6100	6189	6279	6368	6458	6547	89
54	6	6636	6726	6815	6904	6994	7083	7172	7261	7351	7440	89
63	7	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
72	8	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
81	9	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
	490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
9	1	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
18	2	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
26	3	2847	2935	3023	3111	3199	3287	3375	3463	3551	3639	88
35	4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
44	5	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
53	6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
62	7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
70	8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
79	9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
	500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
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17	2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
26	3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
34	4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
43	5	3291	3377	3463	3549	3635	3721	3807	3893	3979	4065	86
52	6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
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69	8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
77	9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
	510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
8	1	8421	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
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34	4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
42	5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
50	6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
59	7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
67	8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
76	9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84

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	520	716003	716087	716170	716254	716337	716421	716504	716588	716671	716754	83
8	1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
17	2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
25	3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
33	4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
41	5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
50	6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
58	7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
66	8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
75	9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
	530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
8	1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
16	2	5912	5998	6075	6156	6238	6320	6401	6483	6564	6646	82
24	3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
32	4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
41	5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
49	6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
57	7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
65	8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
73	9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
	540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
8	1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
16	2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
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32	4	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
40	5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
48	6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
56	7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
64	8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
72	9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
	550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
8	1	1152	1230	1309	1388	1467	1545	1624	1703	1782	1860	79
16	2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
23	3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
31	4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
39	5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
47	6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
55	7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
62	8	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
70	9	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
	560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
8	1	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
15	2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
23	3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
31	4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
39	5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
46	6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
54	7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
62	8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
69	9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
	570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
8	1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
15	2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
23	3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
30	4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
38	5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
46	6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
53	7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
61	8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
68	9	2679	2754	2829	2904	2978	3053	3128	3203	3278	3353	75

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	580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
7	1	4176	4251	8326	4400	4475	4550	4624	4699	4774	4848	75
15	2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
22	3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
30	4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
37	5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
44	6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
52	7	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
59	8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
67	9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
	590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
7	1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
15	2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
22	3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
29	4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
37	5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
44	6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
51	7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
58	8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
66	9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
	600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
7	1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
14	2	9596	9669	9741	9813	9885	9957	780029	780101	780173	780245	72
22	3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
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36	5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
43	6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
50	7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
58	8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
65	9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
	610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
7	1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
14	2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
21	3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
28	4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
36	5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
43	6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
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	620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
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42	6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
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56	8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
63	9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
	630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
7	1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
14	2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
21	3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
28	4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
35	5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
41	6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
48	7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
55	8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
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7	1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
13	2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
20	3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
27	4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
34	5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
40	6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
47	7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
54	8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
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	650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
7	1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
13	2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
20	3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
26	4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
33	5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
40	6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
46	7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
53	8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
59	9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
	660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
7	1	320201	320267	320333	320399	320464	320530	320595	0661	0727	0792	66
13	2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
20	3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
26	4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
33	5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
39	6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
46	7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
52	8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
59	9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
	670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
6	1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
13	2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
19	3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
26	4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
32	5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
38	6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
45	7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
51	8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
58	9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
	680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
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13	2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
19	3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
25	4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
32	5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
38	6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
44	7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
50	8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
57	9	8219	8282	8345	8408	8471	8534	8597	8660	8723	8786	63
	690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
6	1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
13	2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
19	3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
25	4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
32	5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
38	6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
44	7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
50	8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
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6	1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
12	2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
19	3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
25	4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
31	5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
37	6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
43	7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
50	8	350033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
56	9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
	710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
6	1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
12	2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
18	3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
24	4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
31	5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
37	6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
43	7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
49	8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
55	9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
	720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
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12	2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
18	3	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
24	4	9739	9799	9859	9918	9978	860038	860098	860158	860218	860278	60
30	5	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
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42	7	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
48	8	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
54	9	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
	730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863868	59
6	1	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
12	2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
18	3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
24	4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
30	5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
35	6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
41	7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
47	8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
53	9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
	740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
6	1	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
12	2	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
17	3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
23	4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
29	5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
35	6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
41	7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
46	8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
52	9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
	750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
6	1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
12	2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
17	3	6795	6853	6910	6968	7026	7083	7141	7199	7256	7314	58
23	4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
29	5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
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41	7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
46	8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
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6	1	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
11	2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
17	3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
23	4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
29	5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
34	6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
40	7	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
46	8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
51	9	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
	770	886491	886547	886604	886660	886716	886773	886829	886885	886942	886998	56
6	1	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
11	2	7617	7674	7730	7786	7842	7898	7955	8011	8067	8123	56
17	3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
22	4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
28	5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
34	6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
39	7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
45	8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
50	9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
	780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
6	1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
11	2	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
17	3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
22	4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
27	5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
33	6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
38	7	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
44	8	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
49	9	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
	790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
5	1	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
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17	3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
22	4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
27	5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
33	6	0913	0968	1022	1077	1131	1186	1240	1295	1349	1404	55
38	7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
44	8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
49	9	2547	2601	2655	2710	2764	2818	2873	2927	2981	3036	54
	800	903090	903144	903199	903253	903307	903361	903416	903470	903524	903578	54
5	1	3633	3687	3741	3795	3849	3904	3958	4012	4066	4120	54
11	2	4174	4229	4283	4337	4391	4445	4499	4553	4607	4661	54
16	3	4716	4770	4824	4878	4932	4986	5040	5094	5148	5202	54
22	4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
27	5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
32	6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
38	7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
43	8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
49	9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
	810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
5	1	9021	9074	9128	9181	9235	9289	9342	9396	9449	9503	54
11	2	9556	9610	9663	9716	9770	9823	9877	9930	9984	910037	53
16	3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
21	4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
27	5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
32	6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
37	7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
42	8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
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5	1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
11	2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
16	3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
21	4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
27	5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
32	6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
37	7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
42	8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
48	9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
	830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
5	1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
10	2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
16	3	0645	0697	0749	0801	0853	0905	0958	1010	1062	1114	52
21	4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
26	5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
31	6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
36	7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
42	8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
47	9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
	840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
5	1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
10	2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
15	3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
20	4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
26	5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
31	6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
36	7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
41	8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
46	9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
	850	929413	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
5	1	9930	9981	930032	930083	930134	930185	930236	930287	930338	930389	51
10	2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
15	3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
20	4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
26	5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
31	6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
36	7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
41	8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
46	9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
	860	934498	934549	934599	934650	934700	934751	934801	934852	934902	934953	50
5	1	5003	5054	5104	5154	5205	5255	5306	5356	5406	5457	50
10	2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
15	3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
20	4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
25	5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
30	6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
35	7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
40	8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
45	9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
	870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
5	1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
10	2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
15	3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
20	4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
25	5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
30	6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
35	7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
40	8	3495	3544	3593	3643	3692	3742	3792	3841	3890	3939	49
45	9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49

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5	1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
10	2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
15	3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
20	4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
25	5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
29	6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
34	7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
39	8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
44	9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
	890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
5	1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
10	2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
15	3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
20	4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
24	5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
29	6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
34	7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
39	8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
44	9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
	900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
5	1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
10	2	5207	5255	5303	5351	5399	5447	5495	5543	5592	5640	48
14	3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
19	4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
24	5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
29	6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
34	7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
38	8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
43	9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
	910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
5	1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
9	2	9995	960042	960090	960138	960185	960233	960281	960328	960376	960423	48
14	3	960471	0518	0566	0613	0661	0709	0756	0804	0851	0899	48
19	4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
24	5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
28	6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
33	7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
38	8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
42	9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
	920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
5	1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
9	2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
14	3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
19	4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
23	5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
28	6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
33	7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
38	8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
42	9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
	930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
5	1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
9	2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
14	3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
18	4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
23	5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
28	6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
32	7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
37	8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
41	9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46

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	940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
5	1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
9	2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
14	3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
18	4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
23	5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
28	6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
32	7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
37	8	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	46
41	9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
	950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
5	1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
9	2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
14	3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
18	4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
23	5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
27	6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
32	7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
36	8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
41	9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
	960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
5	1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
9	2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
14	3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
18	4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
23	5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
27	6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
32	7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
36	8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
41	9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
	970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
5	1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
9	2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
14	3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
18	4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
23	5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
27	6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
32	7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
36	8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
41	9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
	980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
4	1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
9	2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
13	3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
18	4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
22	5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
26	6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
31	7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
35	8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
40	9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
	990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
4	1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
9	2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
13	3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
18	4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
22	5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
26	6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
31	7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
35	8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
40	9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

